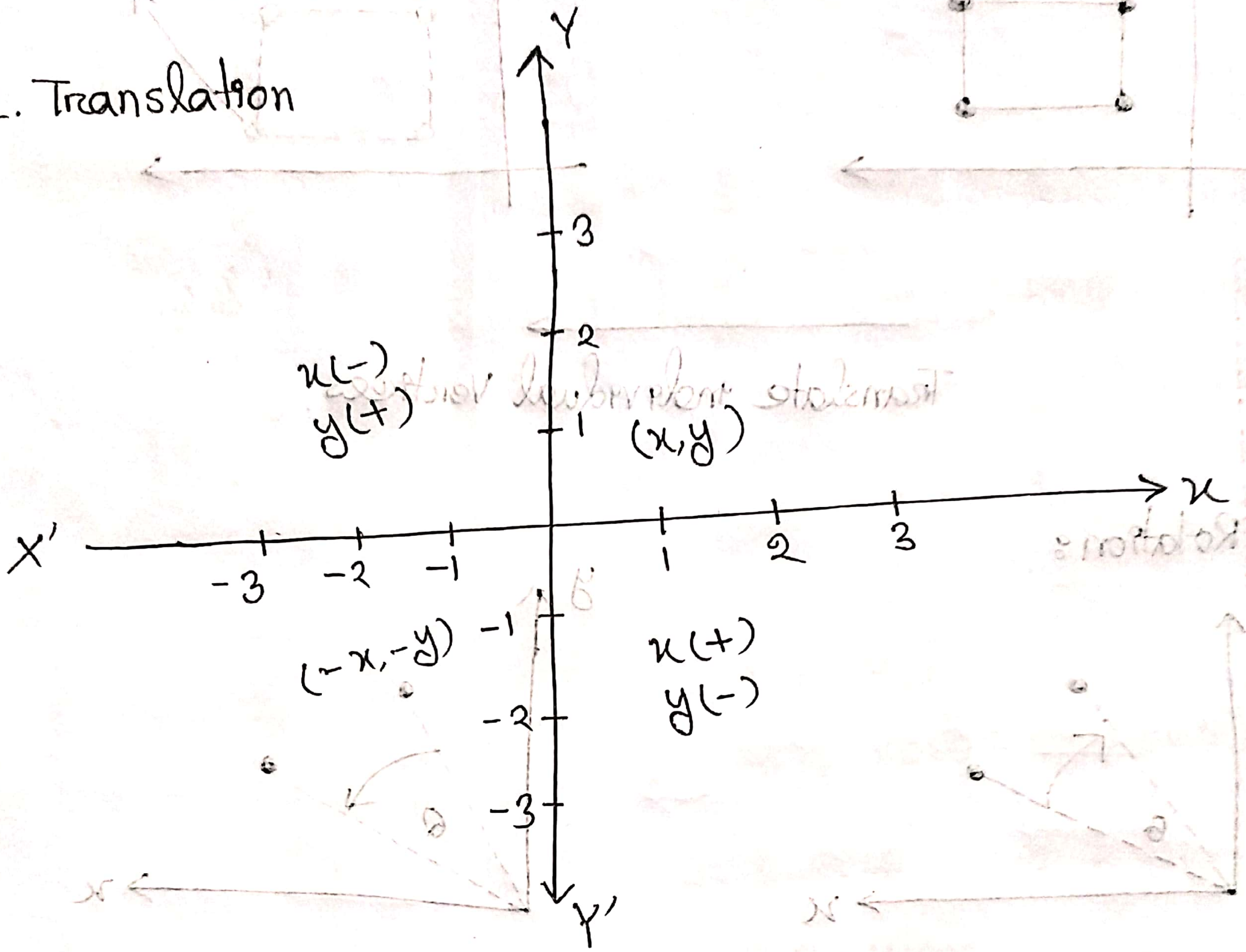
2D Transformation is 5 Types:

1. Translation

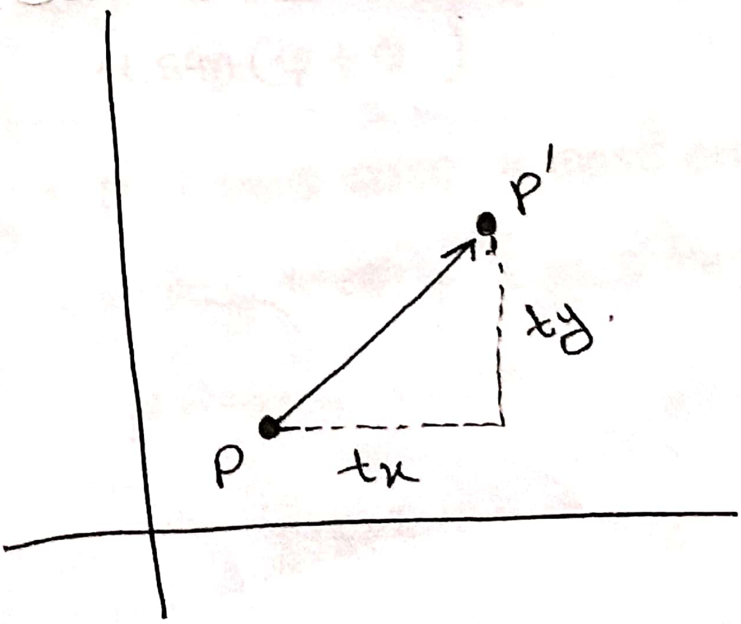


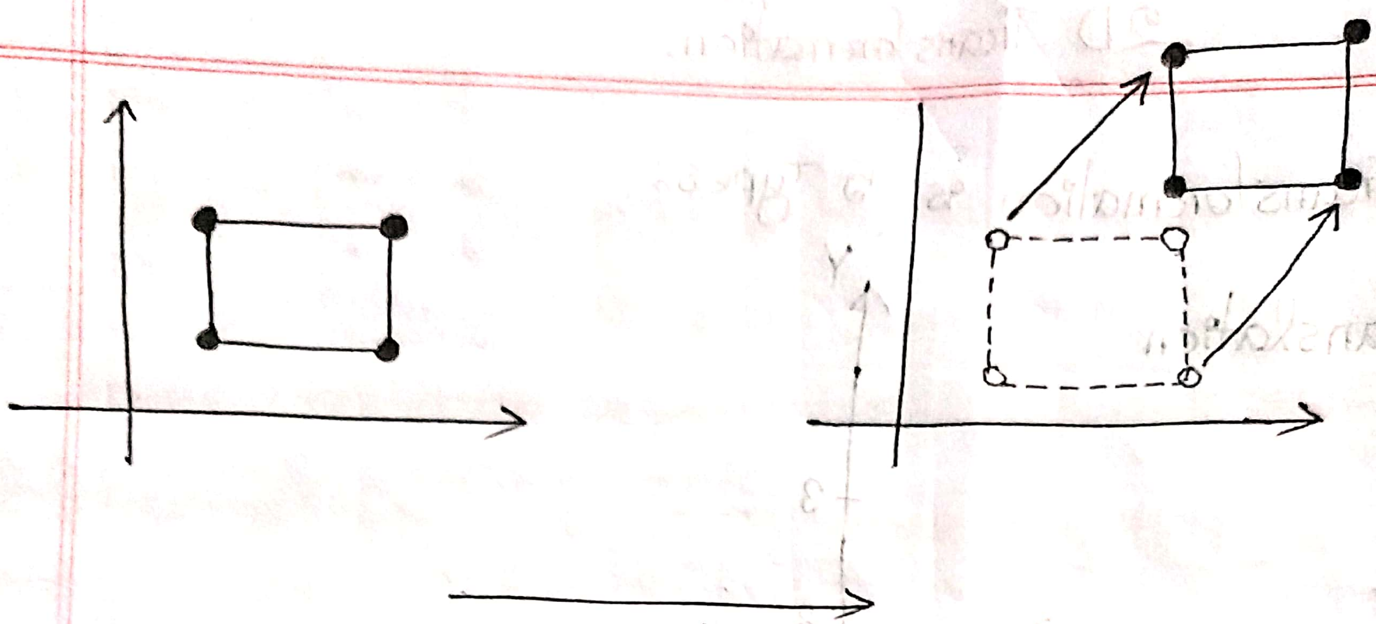
New point: (P'_x, P'_y)

$$P'_x = P_x + t_x$$

$$P'_y = P_y + t_y.$$

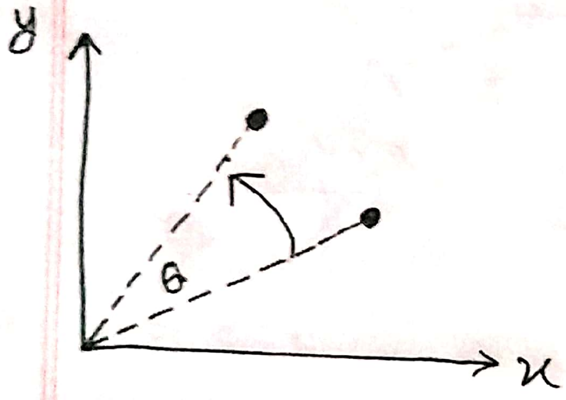
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$



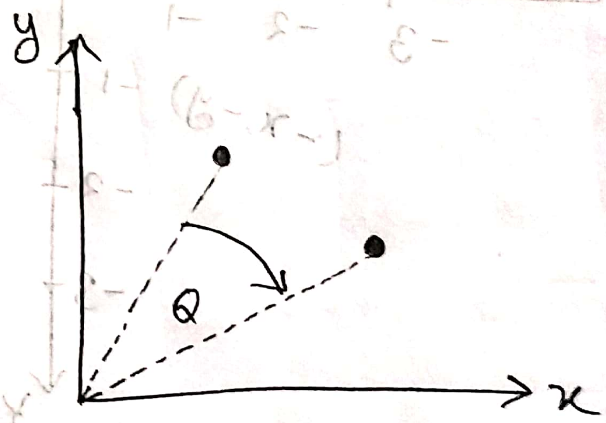


Translate individual vertices.

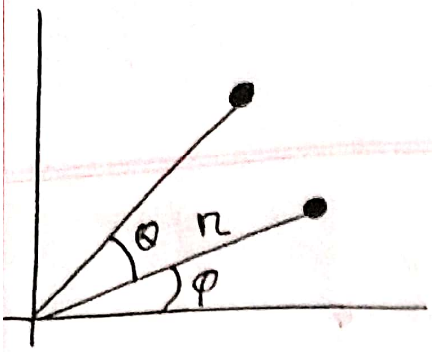
2. Rotation:

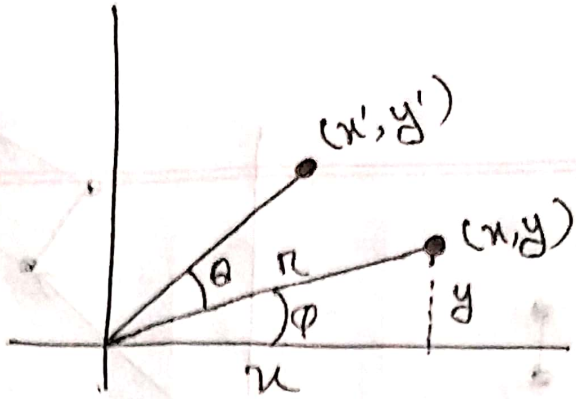


Rotate counter clockwise.



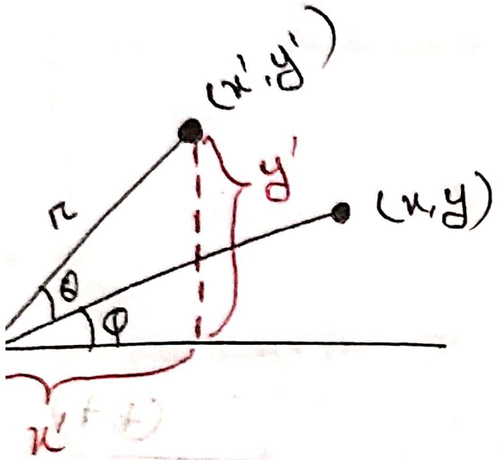
Rotate clockwise





$$\cos\varphi = \frac{x}{r} \quad \therefore \ x = r\cos\varphi$$

$$\sin\varphi = \frac{y}{r} \quad \therefore \ y = r\sin\varphi$$



$$\cos(\varphi+\theta) = \frac{x'}{r} \quad \therefore \ x' = r\cos(\varphi+\theta)$$

$$= r\{\cos\varphi \cdot \cos\theta - \sin\varphi \cdot \sin\theta\}$$

$$= r\cos\varphi \cdot \cos\theta - r\sin\varphi \cdot \sin\theta$$

$$= x\cos\theta - y\sin\theta.$$

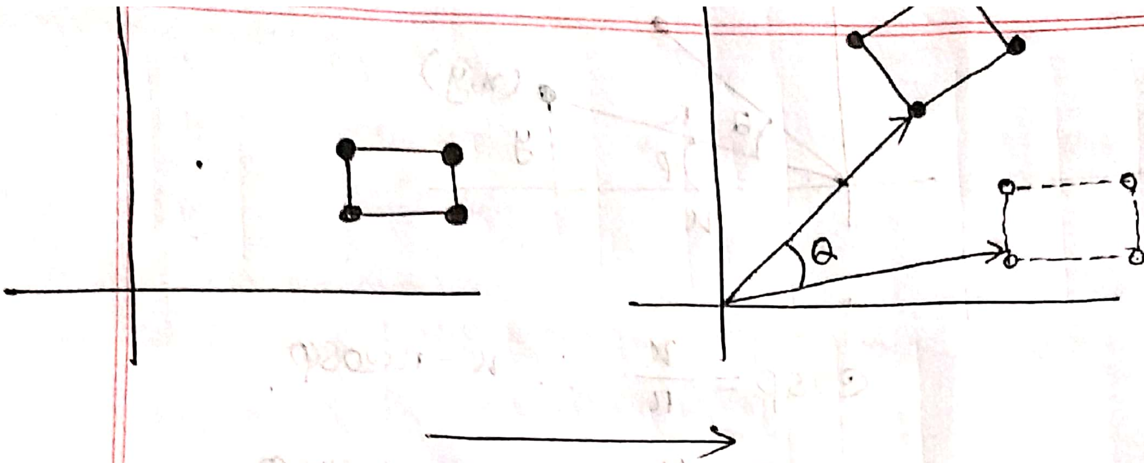
$$\sin(\varphi+\theta) = \frac{y'}{r} \quad \therefore \ y' = r\sin(\varphi+\theta)$$

$$= r\{\sin\varphi\cos\theta + \cos\varphi\sin\theta\}$$

$$= r\sin\varphi\cos\theta + r\cos\varphi\sin\theta$$

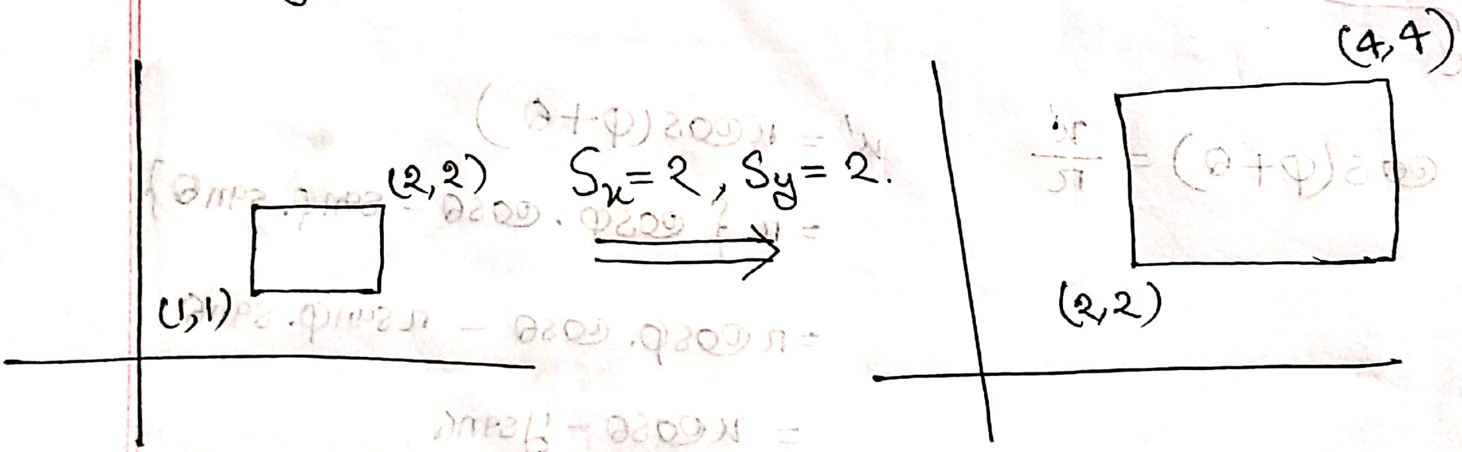
$$= y\cos\theta + x\sin\theta$$

$$\Big] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$



Rotate Individual Vertices.

3. Scaling.

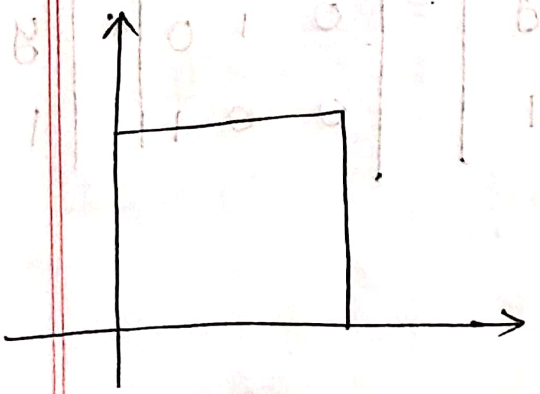


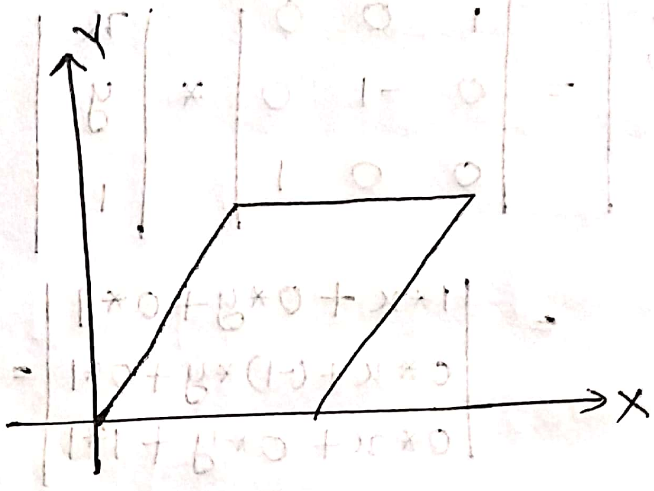
$x' = x \cdot S_x$ * Uniform

$y' = y \cdot S_y$ * Un-uniform.

$$\Rightarrow \begin{vmatrix} x' \\ y' \end{vmatrix} = \begin{vmatrix} S_x & 0 \\ 0 & S_y \end{vmatrix} * \begin{vmatrix} x \\ y \end{vmatrix}$$

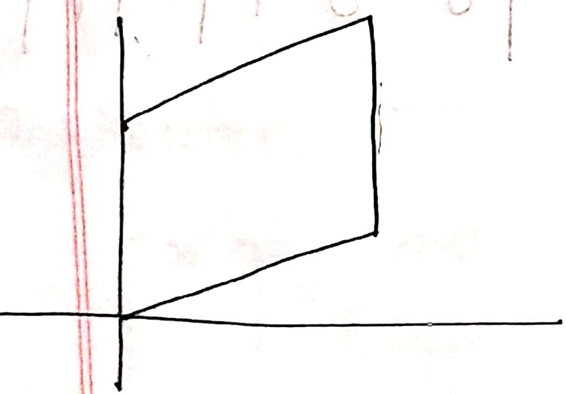
* Shearing





- $Y' = Y$
- $x' = x + y * h$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

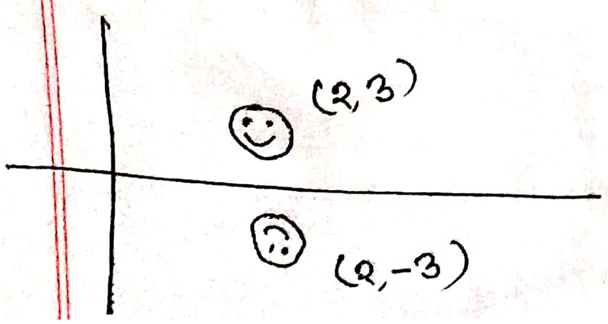


$x' = x$

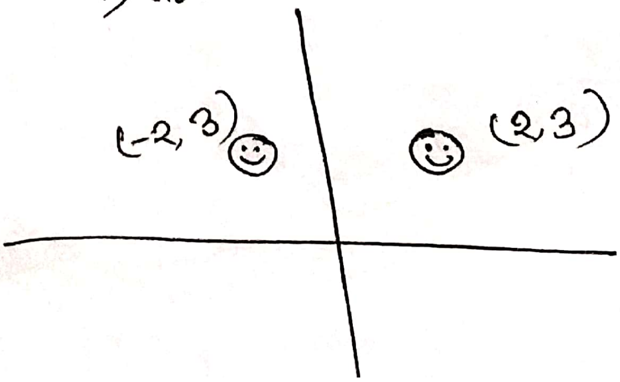
$y' = y + x * g$

Reflection:

1) about x axis.



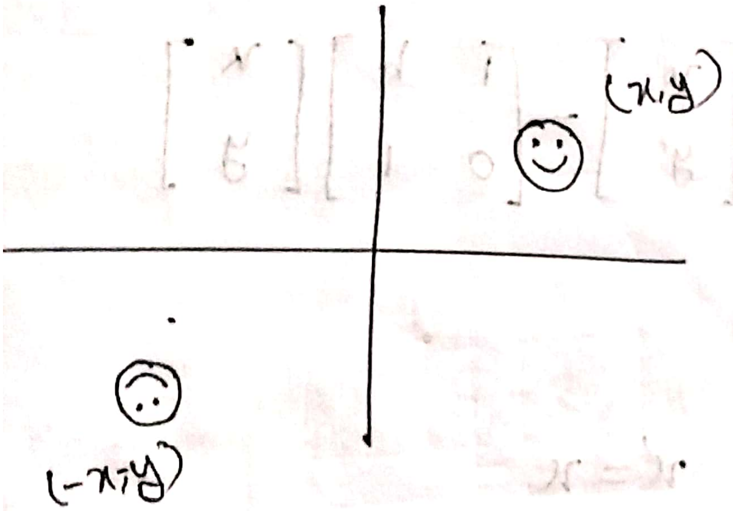
2) about Y axis.



$$\begin{vmatrix} x' \\ y' \\ 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{vmatrix} * \begin{vmatrix} x \\ y \\ 1 \end{vmatrix}$$

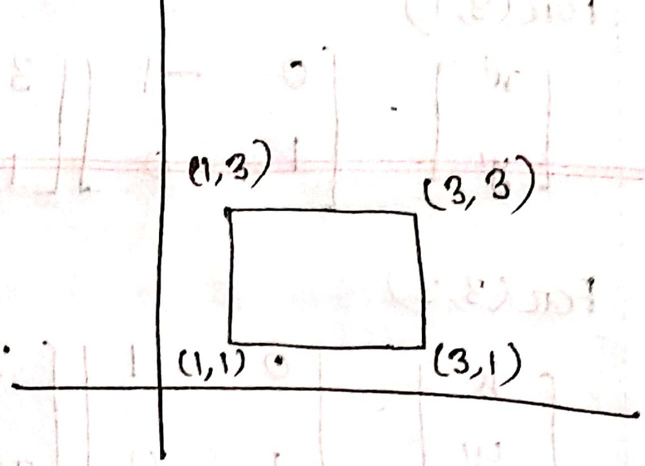
$$= \begin{vmatrix} 1*x + 0*y + 0*1 \\ 0*x + (-1)*y + 0*1 \\ 0*x + 0*y + 1*1 \end{vmatrix} = \begin{vmatrix} x \\ -y \\ 1 \end{vmatrix}$$

$$\begin{vmatrix} x' \\ y' \\ 1 \end{vmatrix} = \begin{vmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} * \begin{vmatrix} x \\ y \\ 1 \end{vmatrix}$$



$$\begin{vmatrix} x' \\ y' \\ 1 \end{vmatrix} = \begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{vmatrix} * \begin{vmatrix} x \\ y \\ 1 \end{vmatrix}$$

Translation



$t_x = 2, \; t_y = 2$

$x' = x + t_x$

$y' = y + t_y$

for (1,1)

$x' = 1 + 2 = 3$

$y' = 1 + 2 = 3$

for (3,3)

$x' = 3 + 2 = 5$

$y' = 3 + 2 = 5$

for (3,1)

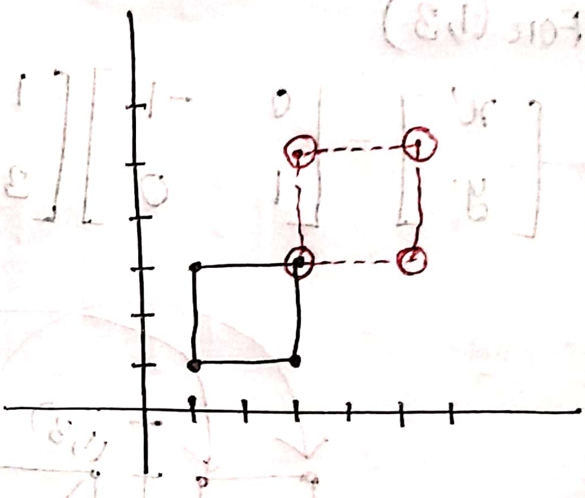
$x' = 3 + 2 = 5$

$y' = 1 + 2 = 3$

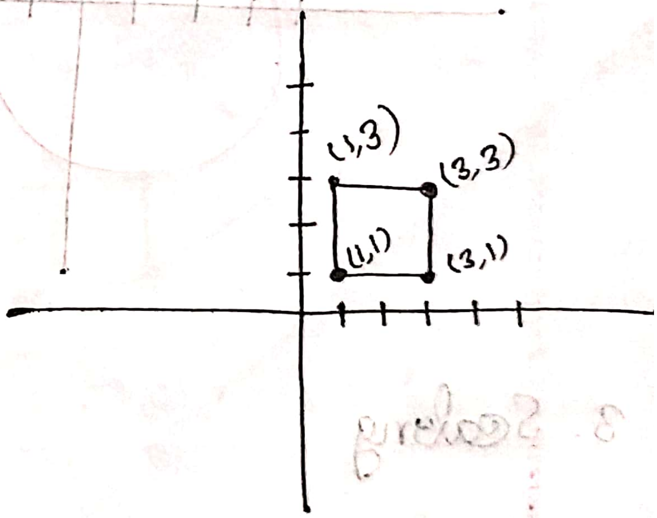
for (1,3)

$x' = 1 + 2 = 3$

$y' = 3 + 2 = 5$



Rotation



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$\boxed{\theta = 90^\circ}$

(1,1)

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos 90^\circ & -\sin 90^\circ \\ \sin 90^\circ & \cos 90^\circ \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0*1 + (-1)*1 \\ 1*1 + 0*1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

For (3, 1).

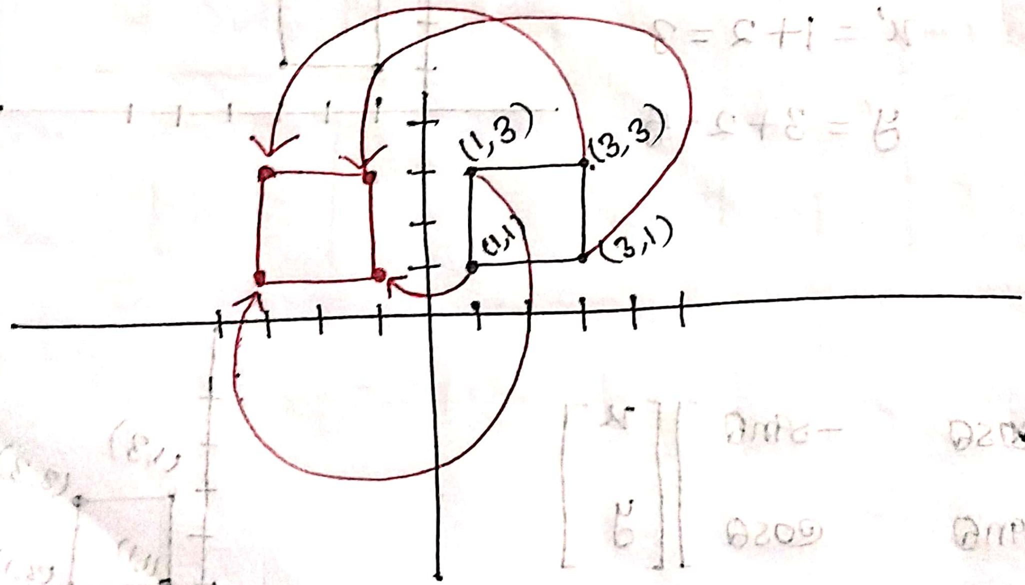
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

For (3, 3)

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \end{bmatrix}$$

For (1, 3)

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

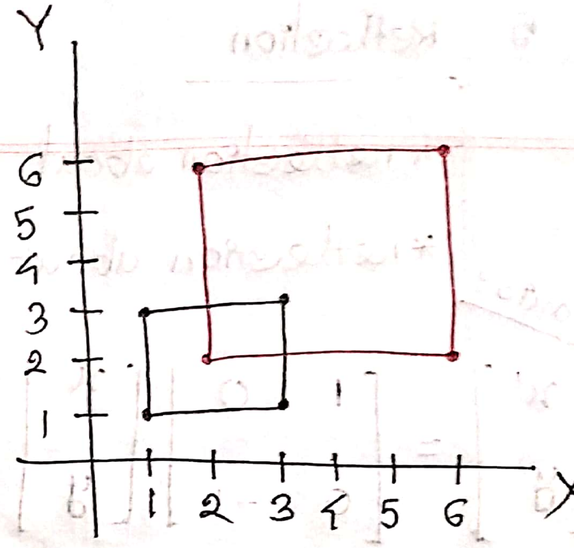


3. Scaling.

Uniform.

$S_x = 2$, $S_y = 2$.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$



for (1,1). $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

for (3,1) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$

for (3,3) $= \begin{bmatrix} 6 \\ 6 \end{bmatrix}$ for (1,3) $= \begin{bmatrix} 2 \\ 6 \end{bmatrix}$

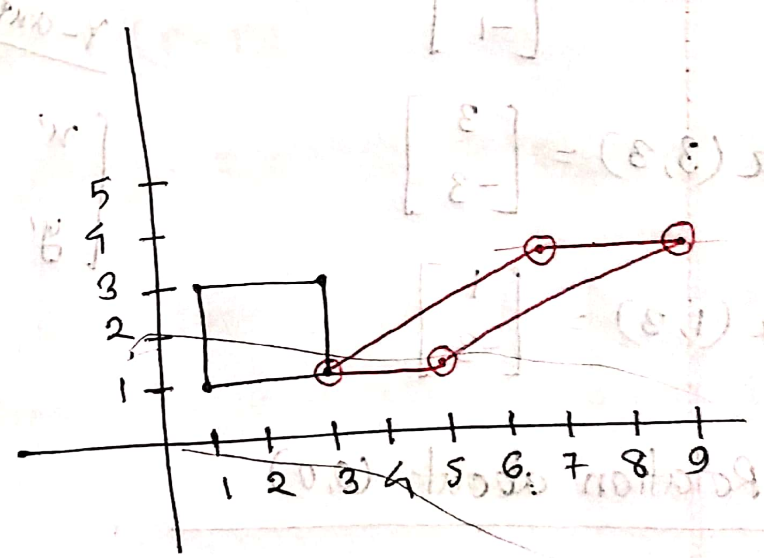
4. Shearing:

x shear

$x' = x + sh_x * y$

$y' = y$

$Sh_x = 2.$



for (1,1) $x' = 1 + (2*1) = 3$

$y' = 1.$

for (3,1) $x' = 3 + (2*1) = 5$

$y' = 1$

for (3,3) $x' = 3 + (2*3) = 9$

$y' = 3$

for (1,3) $x' = 1 + (2*3) = 7$

$y' = 3$

y shear

$x' = x \cancel{+ sh_y * x}$

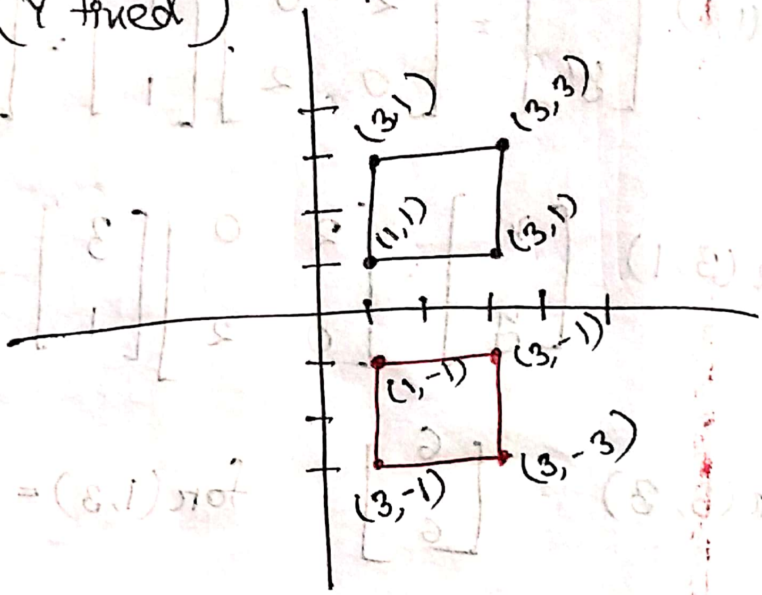
$y' = y + sh_y * x$

5. Reflection

* ~~reflection about X axis (X fixed)~~
* reflection about Y axis (Y fixed)

X axis:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



for (1,1)

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

for (3,1) $= \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

for (3,3) $= \begin{bmatrix} 3 \\ -3 \end{bmatrix}$

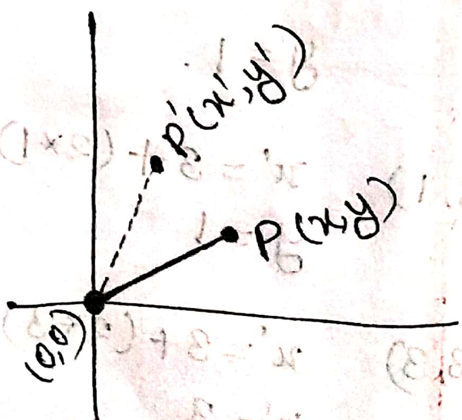
for (1,3) $= \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

Y-axis

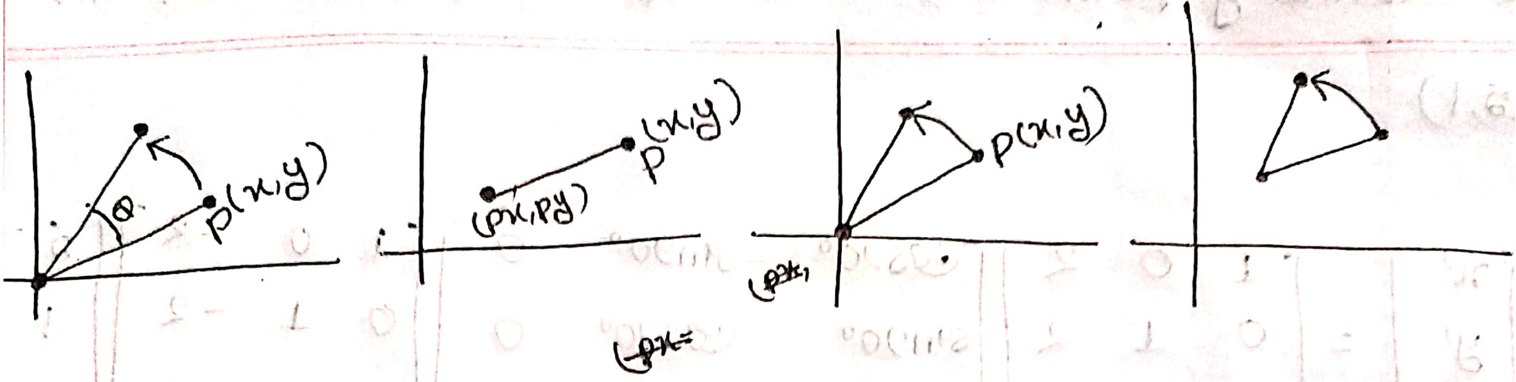
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Rotation about (0,0)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



~~Arb~~ Arbitrary Rotation Centre.



To Rotate about an arbitrary point $P(px, py)$ by θ:

Translate the object so that the pivot point P will coincide the origin: $T(-px, -py)$

Rotate the object: $R(\theta)$

Translate the object back: $T(px, py)$

put in matrix form: $T(px, py)\, R(\theta)\, T(-px, -py) * P$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & P_x \\ 0 & 1 & P_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -P_x \\ 0 & 1 & -P_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Q] Rotate an object about 90° angle with respect to an arbitrary point (2,2) & the object is situated at point (5,1).

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 90° & -\sin 90° & 0 \\ \sin 90° & \cos 90° & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}$$