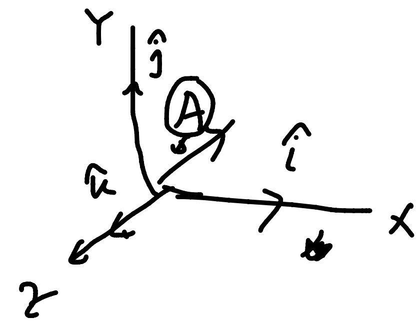
E1, 27.06.21

Sunday, June 27, 2021 3:06 PM

vector and scalar

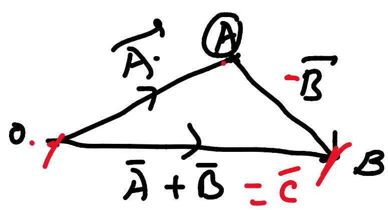


unit vector, $\boxed{\bar{A} = x\hat{i} + y\hat{j} + z\hat{k}}$

$$\hat{A} = \frac{\bar{A}}{|\bar{A}|}$$

Magnitude, $|\vec{A}| = \sqrt{x^2 + y^2 + z^2}$

Adding



$\overline{OB} = \bar{C}$

Subtracting

$\vec{OA} + \vec{AB} = \overline{OB}$

~~$\bar{A} - \bar{B}$~~

$\vec{A} + (-\vec{B}) = \bar{C}$

$\Rightarrow \bar{A} - \bar{B} = \bar{c}$

$r_1 = 2\hat{i} - \hat{j} + \hat{k}$

$r_2 = \hat{i} + 3\hat{j} - 2\hat{k}$

$r_3 = -2\hat{i} + \hat{j} - 3\hat{k}$

$r_4 = 3\hat{i} + 2\hat{j} + 5\hat{k}$

$r_4 = a r_1 + b r_2 + c r_3$

$\Rightarrow 3\hat{i} + 2\hat{j} + 5\hat{k}$

$= a(2\hat{i} - \hat{j} + \hat{k}) + b(\hat{i} + 3\hat{j} - 2\hat{k})$

$+ c(-2\hat{i} + \hat{j} - 3\hat{k})$

$\Rightarrow 3\hat{i} + 2\hat{j} + 5\hat{k} = (2a + b - 2c)\hat{i}$

$+ (-a + 3b + c)\hat{j}$

$+\,(a - 2b - 3c\quad)\hat{u}$

Now,

$3 = 2a + b - 2c$ — (i) $\Rightarrow$ b, c a, b, c

$2 = -\cancel{a} + 3b + c$ — (ii)

$5 = \cancel{a} - 2b - 3c$ — (iii) =

~~From~~ Now, (ii) + (iii) $\Rightarrow$

$$2 + 5 = b - 2c$$

$$\Rightarrow b - 2c = 7 \quad \text{— (iv)}$$

Again, (i) + (ii) $\times 2 \Rightarrow$

$$3 = \cancel{2a} + b - \cancel{2c}$$

$$4 = -\cancel{2a} + 6b + \cancel{2c}$$

$$(+)\quad 7 = 7b$$

$$\boxed{\therefore b = 1}$$

From (iv) $\Rightarrow$ $1 - 2c = 7$

$$\Rightarrow -2c = 6$$

$$\therefore \boxed{c = -3}$$

From (i) $\Rightarrow$ $b = 1,\ c = -3$

$$3 = 2a + 1 - 2\times(-3)$$

$$\Rightarrow 3 = 2a + 1 + 6$$

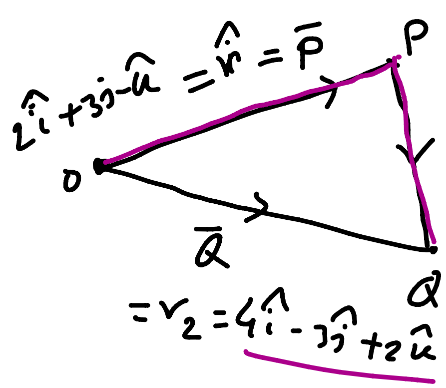
$\Rightarrow 3 - 7 = 2a$

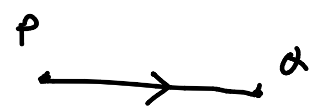
$\Rightarrow 2a = -4$

$\therefore a = -2.$

$\therefore a = -2, \quad b = 1, \quad c = -3$

Ans:





PQ

PQ

$\overrightarrow{OP} + \overrightarrow{PQ} = \overrightarrow{OQ}$

$\Rightarrow \overline{PQ} = \overline{OQ} - \overline{OP}$

$= r_2 - r_1$

$= 4\hat{i} - 3\hat{j} + 2\hat{k} - (2\hat{i} + 3\hat{j} - \hat{k})$

$= \square$ ✓ $= 2\hat{i} - 6\hat{j} + 3\hat{k}$

$\vec{A} =$

$\vec{B} =$

Product of vectors

- Scalar or Dot product (.)
- Vector product or Cross product (×)

Dot product   Cross product

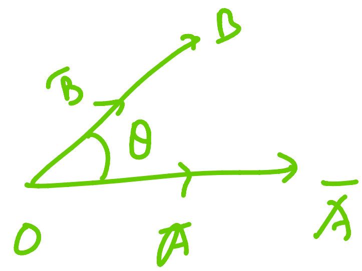
(·)   (×)

$\vec{A}\cdot\vec{B}\cdot\vec{C}$ $\vec{A}\times\vec{B}\times\vec{C}$

Scalar / Dot product:

$$\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$
$$\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

Defined by



$$\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta$$

$$\theta = \cos^{-1}\left[\frac{\vec{A}\cdot\vec{B}}{|\vec{A}|\,|\vec{B}|}\right]$$

$$|\vec{A}|\,|\vec{B}|\cdot\cos\theta = \vec{A}\cdot\vec{B}$$

$$\Rightarrow \cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}|\,|\vec{B}|}$$

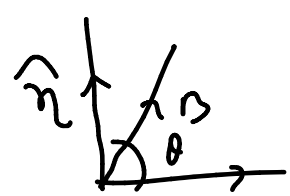
$$\therefore \theta = \cos^{-1}\left[\frac{\vec{A}\cdot\vec{B}}{|\vec{A}|\,|\vec{B}|}\right] \quad (1)$$

(i) Two vectors are perpendicular

$$\therefore \vec{A}\cdot\vec{B} = 0 \quad (ii)$$

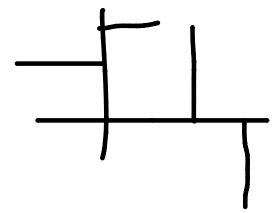
Vector product / cross product: $\vec{A}\times\vec{B}$

Defined by



$$\vec{A} \times \vec{B} = \underbrace{|\vec{A}|\,|\vec{B}|}\sin\theta \cdot \hat{\eta}$$

A



(i) Two vectors are parallel if

$$\vec{A} \times \vec{B} = 0 \quad \text{(v)}$$

$$\sin\theta = \frac{\vec{A} \times \vec{B}}{|\vec{A}|\,|\vec{B}|\,\hat{\eta}}$$

$$\hat{\eta} = \pm \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|} \quad \text{(iv)}$$

$$\therefore \theta = \sin^{-1}\left|\frac{\vec{A} \times \vec{B}}{|\vec{A}|\,|\vec{B}|\,\hat{\eta}}\right| \quad \text{(iii)}$$

① $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$
$\vec{B} = 3\hat{i} - 4\hat{j} + 2\hat{k}$

(i) a unit vector of perpendicular
(ii) angle ✓
(iii) angle of sine
(iv) when $\vec{A}\cdot\vec{B} = 0$ (perpend.
(v) parallel.

(i) A unit vector of perpendi.

$$\hat{\eta} = \pm \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|} \quad \text{—— ①}$$

Here,

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & -4 & 2 \end{vmatrix}$$

$\vec{A} \times \vec{B} =$

$$\begin{vmatrix} 2 & 3 \\ -4 & 2 \end{vmatrix}$$

$$\begin{vmatrix} 3 & -4 & 2 \end{vmatrix}$$

$$= \hat{i}(4+12) - \hat{j}(2-9) + \hat{k}(-4-6)$$

$$\underline{\bar{A} \times \bar{B}} = 16\hat{i} + 7\hat{j} - 10\hat{k}$$

$$|\bar{A} \times \bar{B}| = \sqrt{(16)^2 + (7)^2 + (-10)^2}$$

$$= \sqrt{405}$$

$\emptyset$ From ①

$$\hat{n} = \pm \frac{16\hat{i} + 7\hat{j} - 10\hat{k}}{\sqrt{405}}$$

$$= \pm \left(\frac{16}{\sqrt{405}}\hat{i} + \frac{7}{\sqrt{405}}\hat{j} - \frac{10}{\sqrt{405}}\hat{k} \right)$$

(ii)

$$\theta = \cos^{-1} \left| \frac{\bar{A} \cdot \bar{B}}{|\bar{A}||\bar{B}|} \right| \quad ①$$

$i \cdot i = 1$

$i \cdot j = 0$

$$\bar{A} \cdot \bar{B} = (1\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (3\hat{i} - 4\hat{j} + 2\hat{k})$$

$$= 3 + (-8) + 6$$

$$\bar{A} \cdot \bar{B} = 1$$

$\bar{A} \cdot \bar{B} = 1$

$|\bar{A}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$

$|\bar{B}| = \sqrt{3^2 + (-4)^2 + 2^2} = \sqrt{9 + 16 + 4} = \sqrt{29}$

From ①

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{14}\,\sqrt{29}}\right)$$

$= \square$

① Find the value of a, when $\bar{p} = 2\hat{i} + a\hat{j} - 3\hat{k}$

$\bar{Q} = 6\hat{i} - 3\hat{j} - 9\hat{k}$ are parallel and perpendicular

$\bar{p} \times \bar{Q} = 0$

$\bar{p} \times \bar{Q} = 0$

$\Rightarrow a = 0$

$a = 0$

$\bar{p} \cdot \bar{Q} = 0$