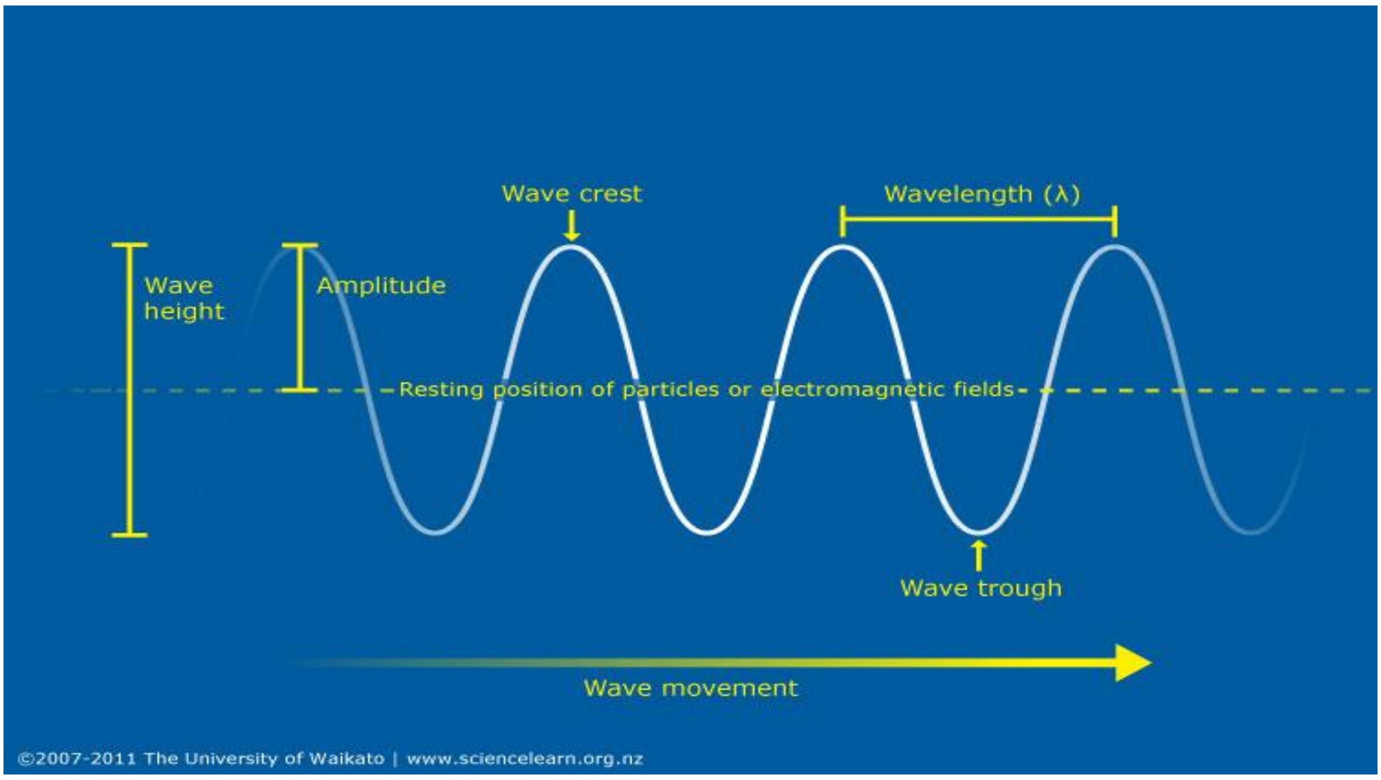
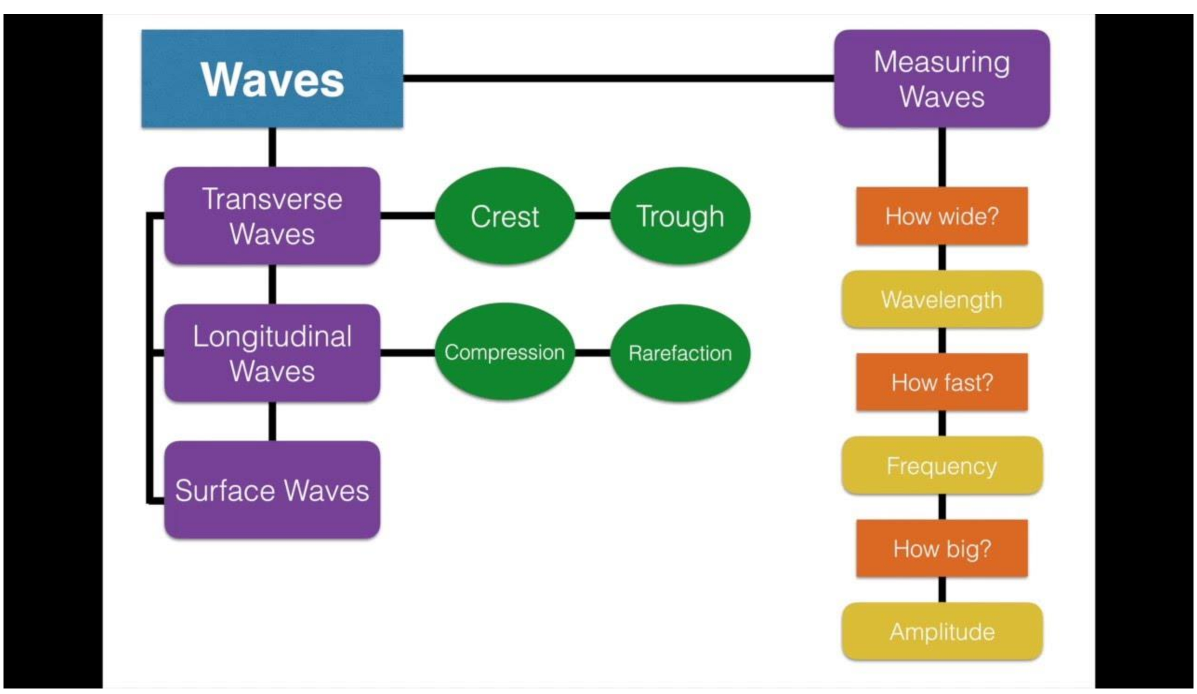
Wave:

Wave is an oscillation accompanied by a transfer of energy. A wave is a disturbance that transfers energy through matter or space. There are two main types of waves.



Mechanical waves propagate through a medium, and the substance of this medium is deformed. Restoring forces then reverse the deformation. For example, sound waves propagate via air molecules colliding with their neighbors. When the molecules collide, they also bounce away from each other (a restoring force). This keeps the molecules from continuing to travel in the direction of the wave.

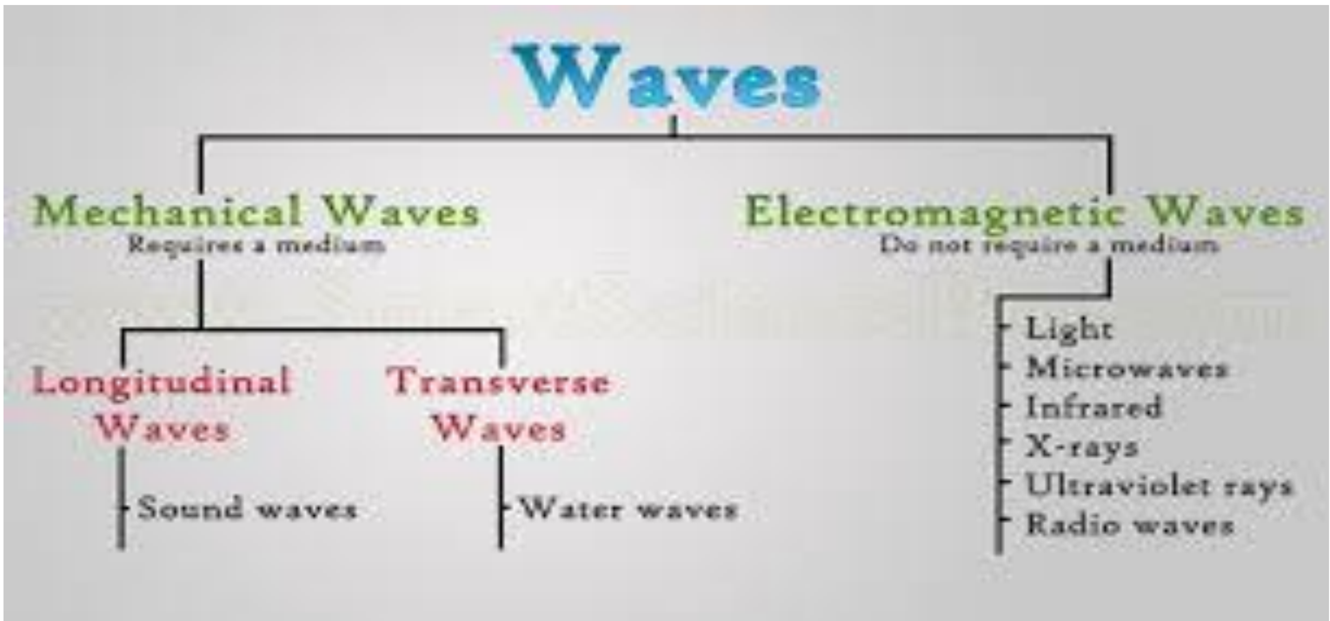


Electromagnetic waves, do not require a medium. Instead, they consist of periodic oscillations of electrical and magnetic fields originally generated by charged particles, and can therefore travel through a vacuum. These types vary in wavelength, and include radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays and gamma rays.

Progressive or traveling wave:

A plane progressive wave is one which travels onward through the medium in a given direction without attenuation. i.e.,with its constant amplitude.

A progressive wave/ traveling wave may be either transverse or longitudinal. In either case, there exists a regular phase difference between any two successive particles of the medium.



Equation of progressive waves:

Let a progressive wave originating at a point O travels with a velocity v along positive X-axis as shown in fig-1. So, the displacement of the particle O at x = 0 is given by

$y = a \sin \omega t$ ……….. (1)

where a = amplitude, ω = angular velocity of the particle = 2Πn = 2Π/T and ωt = phase angle of the particle or simply phase.

Since the successive particles to the right of O receive and repeat its movements after definite interval of time.We know, the distance travelled by a wave in one complete vibration is called wavelength λ and the phase difference between two particles situated at a distance λ is 2Π.

Now, let the phase difference of the particle at P, a distance x from O be δ. Then the equation of motion of the particle at P is

$y = a \sin (\omega t - \delta)$………………(2)

Since in travelling the distance λ the phase difference is 2Π, so the phase difference for distance x is δ = 2Πx/λ, i.e.

Phase difference, $\delta = \frac{2\pi}{\lambda} \times \text{path difference}$

Again,

$$\omega = \frac{2\pi}{T} = 2\pi n = \frac{2\pi v}{\lambda} \qquad [\because n = \frac{1}{T} \text{ and } v = n\lambda]$$

Putting the values of δ and ω in equation (2), we get

$$y = a\sin\left(\frac{2\pi vt}{\lambda} - \frac{2\pi x}{\lambda}\right)$$

$$=> y = a\sin\frac{2\pi}{\lambda}(vt - x) \;\; \ldots\ldots.. (3)$$

Equation (3) represents a progressive wave or simple harmonic wave. It gives the displacement of any particle at time t.

Similarly, for a particle at a distance x in the negative direction (i.e., to the left of O), the equation for displacement is

$$y = a\sin\frac{2\pi}{\lambda}(vt + x) \;\; \ldots\ldots..(4)$$

Equation (4) represents a progressive wave or simple harmonic wave travelling in the negative X-axis.

Prob-1: The equation of a progressive wave is y = 5 sin (200πt-1.57x), here all quantities are expressed in S.I. unit. Find the amplitude, frequency, time period and velocity of the wave.

Solution: We have, y = 5 sin (200πt-1.57x).....(1)

and we know, $y = A\sin\frac{2\pi}{\lambda}(vt - x) = A\sin(\frac{2\pi}{\lambda}vt - \frac{2\pi}{\lambda}x)$ ....(2).

Comparing eq. (1) and (2), we get,

A = 5m, 2πvt/λ = 200πt or, 2v/ λ = 200 or, 2n = 200 or, Frequency, n = 100 Hz.

2πx/λ = 1.57 x or, 2π/λ = 1.57 or, Wavelength, λ = 2π/1.57 = 4 m.

Velocity, v = n λ = 100 X 4 = 400 Hz, Time period, T = 1/n = 1/100 = 0.01 s.

Prob-2: The amplitude of a wave is 0.2 m. Find the displacement of the point at a distance x = λ/6 from the source at time t = T/3.

Solution: We know,

$$y = a\sin\frac{2\pi}{\lambda}(vt - x) = 0.2\sin\frac{2\pi}{\lambda}\left(v \times \frac{T}{3} - \frac{\lambda}{6}\right) = 0.2\sin(\frac{2\pi vT}{3\lambda} - \frac{\pi}{3})$$

$$= 0.2\sin(\frac{2\pi nT}{3} - \frac{\pi}{3})$$

$$= 0.2\sin(\frac{2\pi}{3} - \frac{\pi}{3})$$

$$= 0.2\sin\frac{\pi}{3}$$

= 0.173 m (Ans)

Differential equation of wave motion/progressive wave/simple harmonic wave:

The general equation of a simple harmonic wave or progressive wave is,

$$y = a \sin \frac{2\pi}{\lambda}(vt - x) \quad \ldots\ldots.. (1)$$

Differentiating equation (1) with respect to time,

$$\frac{dy}{dt} = \frac{2\pi av}{\lambda} \cos \frac{2\pi}{\lambda}(vt - x) \ldots\ldots\ldots\ldots\ldots..(2)$$

Differentiating equation (2) with respect to time,

$$\frac{d^2y}{dt^2} = \frac{4\pi^2 av^2}{\lambda^2} \sin \frac{2\pi}{\lambda}(vt - x) \ldots\ldots\ldots\ldots\ldots..(3)$$

To find the value of compression, we differentiate eqn. (1) with respect to x,

$$\frac{dy}{dx} = -\frac{2\pi a}{\lambda} \cos \frac{2\pi}{\lambda}(vt - x) \ldots\ldots\ldots\ldots\ldots..(4)$$

To find the rate of change of compression with respect to distance, we differentiate eqn. (4) with respect to x,

$$\frac{d^2y}{dx^2} = -\frac{4\pi^2 a}{\lambda^2} \sin \frac{2\pi}{\lambda}(vt - x) \ldots\ldots\ldots\ldots\ldots..(5)$$

From eqns. (2) and (4),

$$\frac{dy}{dt} = -v \frac{dy}{dx} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(6)$$

From eqns. (3) and (5),

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Equation (7) represents the differential equation of wave motion/ progressive wave/ simple harmonic wave.

The general differential equation of wave motion can be written as,

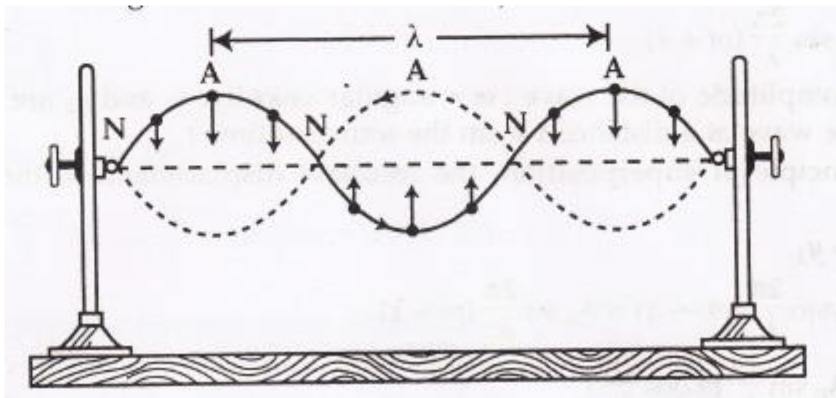
$$\frac{d^2y}{dt^2} = K \frac{d^2y}{dx^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

$$\text{Where } K = v^2 \implies v = \sqrt{K}$$

Thus, knowing the value of K, the value of the wave velocity can be calculated.

Stationary Waves

Definition: The resultant wave produced by the superposition of two progressive waves, having same wavelength and amplitude, traveling in opposite directions is called stationary wave. The stationary wave has no forward motion but remains fixed in peace.
In stationary waves there are certain points where the amplitude is zero. These points are nodes and there are some points where the amplitude is maximum. These points are called antinodes. In fig-1 points A are antinodes and N is nodes.



Characteristics of stationary waves: There are some characteristics of stationary waves which are mentioned below:

1. Stationary waves are produced when two identical progressive waves traveling along the same straight line but opposite direction are superposed.
2. Crests and trough or compression and rarefaction do not progress forward through the medium, but simply appear and disappear at the same place alternately.
3. The point where amplitude is zero is called nodes and where it is maximum is called antinodes.
4. All the particles, except those at the nodes, execute simple harmonic motion.
5. The distance between two adjacent nodes and antinodes is equal to half the wavelength.
6. The distance between three adjacent nodes or antinodes or between two loops is equal to the wavelength of the wave.
7. There is no propagation of energy in stationary wave.
8. Stationary waves are producing both by transverse and longitudinal wave.

Equation of a stationary wave

Let a progressive wave move along positive X-axis with velocity v. The equation of this wave is,

$$y_1 = A_0 \sin\frac{2\pi}{\lambda}(vt - x)$$

And the equation of progressive wave moving along negative X-axis with the same velocity v is

$$y_2 = A_0 \sin\frac{2\pi}{\lambda}(vt + x)$$

Here A_o =amplitude of the wave; ω= angular velocity; y_1 and y_2 are the displacements of a particle of the wave at a distance x from the source at time t.

By the principal of superposition, the resultant displacement of the particle at x at a time t is

$$y = y_1 + y_2 = \frac{A_0 \sin 2\pi}{\lambda}(vt - x) + \frac{A_0 \sin 2\pi}{\lambda}(vt + x)$$

$$= \frac{2A_0 \sin 2\pi}{\lambda} vt. \frac{\cos 2\pi}{\lambda} x$$

$$= 2A_0 \cos\frac{2\pi}{\lambda} x \sin\frac{2\pi}{\lambda} vt$$

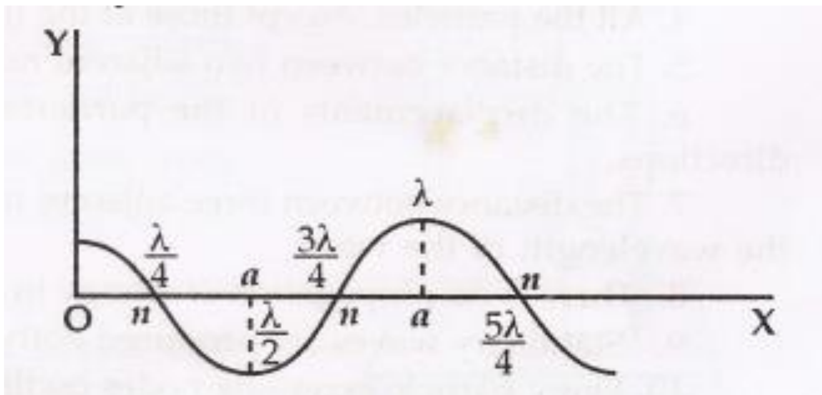
$$= A\sin\frac{2\pi}{\lambda} vt$$

$$= A\sin 2\pi nt$$

$\therefore y = A\sin\omega t$ ----------(1) . Here $A = 2A_0 \cos\frac{2\pi}{\lambda} x$ = displacement of the particle of wave at x.

Eq. (1) represents a simple harmonic vibration of same wavelength λ as the superposing wave.

The amplitude $A = 2A_0 \cos\frac{2\pi}{\lambda}x$ is not a constant. For different values of x, the amplitude has different values as shown in fig-2. It should be mentioned that this simple harmonic vibration does not represent a progressive wave since its phase does not contain term (vt-x). So eq. (1) represents a stationary wave.



Nodes: The amplitude of the stationary wave is $A = 2A_0 \cos\frac{2\pi}{\lambda}x$ It depends on the position of the particle. The point where A= 0, i. e; amplitude zero, nodes will formed.

Now, condition for $A = 2A_0 \cos\frac{2\pi}{\lambda}x = 0$ is

$$\frac{\cos 2\pi}{\lambda}x = 0$$

$$or\ ,\ \frac{2\pi}{\lambda}x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

or, $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$

These are points of nodes. Distance between two successive nodes $\frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2}$

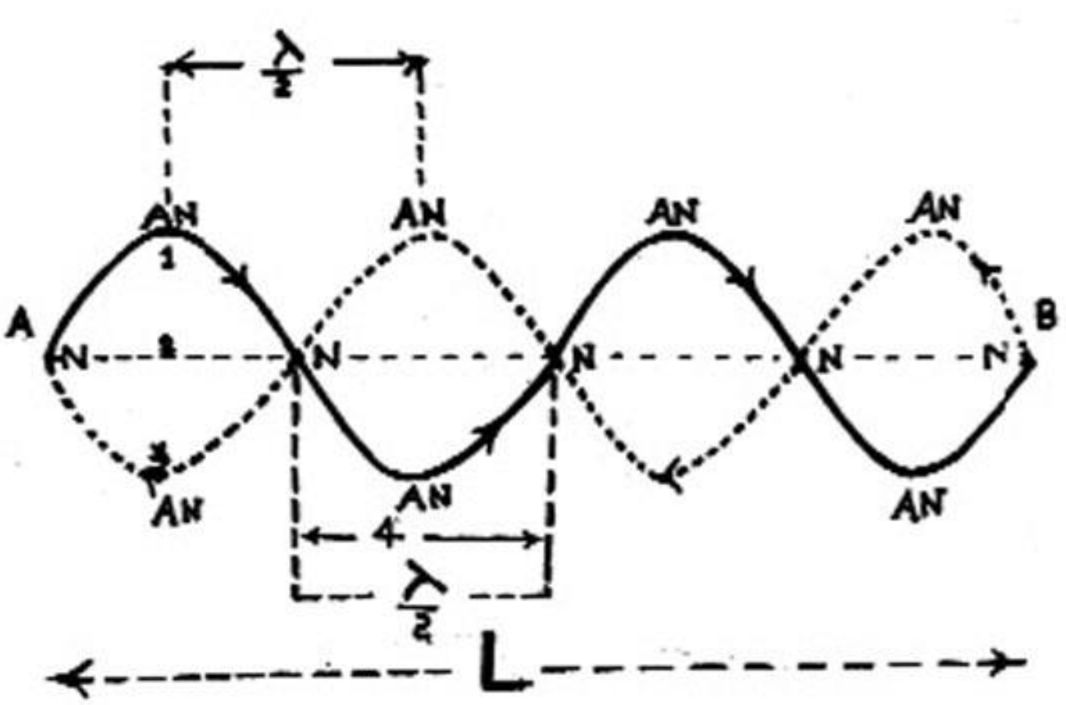


Fig. 6.5 Positions of nodes (N) and antinodes (AN) in a standing wave
1..and 3. Complete P.E. 2. Complete K.E. 4. Loop

Antinodes: The point where amplitude A is maximum, antinodes is formed. So, condition for maximum amplitude is

$$A = 2A_0 \cos\frac{2\pi}{\lambda}x = \pm 2A_0$$

$$\frac{\cos 2\pi}{\lambda}x = \pm 1$$

$$\frac{2\pi}{\lambda}x = 0, \pi, 2\pi, 3\pi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots etc$$

$$x = \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2} \ldots\ldots\ldots\ldots\ldots\ldots \frac{n\lambda}{2} \quad (Where\ n = 0, 1, 2, 3 \ldots\ldots\ldots)$$

∴ Distance between two successive antinodes = $[\frac{2\lambda}{2} - \frac{\lambda}{2}] = \frac{\lambda}{2}$

The distance between a nodes and adjacent antinodes is $\frac{\lambda}{4}$. In fig.1 points 'N' and 'A' represent nodes and antinodes. Between two nodes there exist an antinode.