GENERAL EQUATION OF SECOND DEGREE

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General Equation of Second Degree

Describe various conditions of general equation of second degree is,

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0.$$

Which will represent the followings,

1. A pair of straight lines if the determinant, $\Delta = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} = 0.$

   Two parallel lines if $\Delta = 0, h^2 = \text{ab}$.

   Two perpendicular lines if $\Delta = 0, \text{a} + \text{b} = 0$.
2. A circle if $a = b, h = 0$.
3. A parabola if $\Delta \neq 0, h^2 = \text{ab}$
4. An ellipse if $\Delta \neq 0, h^2 - \text{ab} < 0$.
5. A hyperbola if $\Delta \neq 0, h^2 - \text{ab} > 0$.
6. A rectangular hyperbola if $a + b = 0, h^2 - ab > 0, \Delta \neq 0$.

Mathematical Problem:

Problem-01: Test the nature of the equation $3x^2 - 8xy - 3y^2 + 10x - 13y + 8 = 0$.

Solution: Given that,

$$3x^2 - 8xy - 3y^2 + 10x - 13y + 8 = 0 \text{ ... ... (i)}$$

Also the general equation of second degree is,

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \text{ ... ... (ii)}$$

Comparing (i) and (ii) we have,

$$a = 3, h = -4, b = -3, g = 5, f = -\frac{13}{2}, c = 8.$$

Now, $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

$= 3 \times (-3) \times 8 + 2 \times \left(-\frac{13}{2}\right) \times 5 \times (-4) - 3 \times (-\frac{13}{2})^2 - (-3) \times 25 - 8 \times 16$

$= -72 + 260 - \frac{507}{4} + 75 - 128$

$= \frac{33}{4}$

Since, $\Delta = \frac{33}{4} \neq 0$ so the given equation represents a conic.

Again, $h^2 - \text{ab} = 16 + 9 = 25 > 0$

And, $a + b = 3 - 3 = 0$

Since, $a + b = 0, h^2 - ab > 0, \Delta = 0$. so the given equation represents a rectangular hyperbola.

Problem-02: Test the nature of the equation $9\text{x}^2 - 24\text{xy} + 16\text{y}^2 - 18\text{x} - 101\text{y} + 19 = 0$.

Solution: Given that,

$$9x^2 - 24xy + 16y^2 - 18x - 101y + 19 = 0 \text{ ... ... (i)}$$

Also the general equation of second degree is,

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \text{ ... ... (ii)}$$

Comparing (i) and (ii) we have,

$$a = 9, h = -12, b = 16, g = -9, f = -\frac{101}{2}, c = 19.$$

Now, $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

$= 9 \times (-12) \times 19 + 2 \times \left(-\frac{101}{2}\right) \times (-9) \times (-12) - 9 \times \left(-\frac{101}{2}\right)^2 - 16 \times (-9)^2 - 19 \times (-12)^2$

$= -2052 - 10908 - \frac{91809}{4} - 1296 - 2736$

$= -\frac{159777}{4}$

Since, $\Delta = -\frac{159777}{4} \neq 0$ so the given equation represents a conic.

Again, $h^2 - \text{ab} = (-12)^2 - 9 \times 16 = 144 - 144 = 0$

Since, $h^2 - ab = 0, \Delta \neq 0.$ so the given equation represents a hyperbola. (**As desired)**

Problem-03: Test nature of the equation $8x^2 + 4xy + 5y^2 - 24x - 24y = 0$.

Solution:

Given general equation of second degree is

$$8x^2 + 4xy + 5y^2 - 24x - 24y = 0 \cdots\cdots\cdots\cdots\cdots\cdots(i)$$

Comparing this above equation with the standard equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ we get

$$a = 8, h = 2, b = 5, g = -12, f = -12 \ \& \ c = 0$$

Now,

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} 8 & 2 & -12 \\ 2 & 5 & -12 \\ -12 & -12 & 0 \end{vmatrix} = 8(0-144) - 2(0-144) - 12(-24+60)$$

$= 8(-144) - 2(-144) - 12(-24+60)$

$= -1152 + 288 - 432$

$= -1296 \neq 0$

And

$h^2 - ab = 2^2 - 40 = 4 - 40 = -36 < 0$

Since $\Delta \neq 0$ and $h^2 - ab < 0$. So the equation represents an ellipse.

Problem-04: Test nature of the equation $x^2 + 2xy + y^2 + 2x - 1 = 0$ to the standard form.

Solution:

Given general equation of second degree is

$$x^2 + 2xy + y^2 + 2x - 1 = 0 \cdots\cdots\cdots\cdots\cdots\cdots(i)$$

Comparing this above equation with the standard equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ we get

$$a = 1, h = 1, b = 1, g = 1, f = 0 \ \& \ c = -1$$

Now,

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = 1 \neq 0$$

And

$$h^2 - ab = 1 - 1 = 0$$

Since $\Delta \neq 0$ and $h^2 - ab = 0$. So the equation represents parabola.

H.W

Test the nature of the following equations and find its centre.

a. $2x^2 - 3xy + y^2 - 5x + 4y + 6 = 0$ $\quad$ $Ans: Hyperbola; (2,1).$

b. $4x^2 + 9y^2 - 8x + 36y - 31 = 0$ $\quad$ $Ans: Ellipse$

c. $2x^2 - 3y^2 + 8x + 30y - 27 = 0.$ $\quad$ $Ans: Hyperbola; (-2,5).$

d. $x^2 - xy - 2y^2 - x - 4y - 2 = 0.$

$Ans: Pair\ of\ straight\ lines; \left(0, \frac{7}{9}\right).$

e. $x^2 + 2xy + y^2 + 2x - 1 = 0.$ $\quad$ Ans: Parabola.

Pair of straight lines

Locus of a Point:

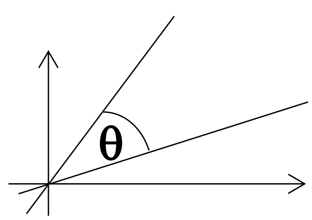
A locus of a point is a path in which it moves in a plane or in a space by following the certain rules/conditions.

Pair of straight lines:

A pair of straight lines is the locus of a point whose coordinates satisfy a second-degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$. A collection of combined two straight lines is called a pair of straight lines.

Homogeneous equation:

An equation in which degree of each term in it is equal is called Homogeneous equation. Such as $ax^2 + 2hxy + by^2 = 0$ is a homogeneous equation of degree or order 2 because degree of its each term is two. It is noted that homogeneous equation always represents straight lines passing through the origin.



- Angle between the lines represented by the equation $ax^2+2hxy+by^2=0$ is calculated by formula $\tan\theta=\dfrac{2\sqrt{h^2-ab}}{a+b}$
  - Lines be perpendicular if $a+b=0$
  - Lines be parallel/coincident if $h^2=ab$
    Since two lines passes through the point $(0,0)$, so lines must be coincident.
  - Lines represented by homogeneous equation is real if $h^2>ab$.
  - Lines represented by homogeneous equation are imaginary if $h^2<ab$. But passes through the point $(0,0)$.
- The equation of the bisectors of an angle produced by the pair of straight line represented by the equation $ax^2+2hxy+by^2=0$ is $\dfrac{x^2-y^2}{a-b}=\dfrac{xy}{h}$.

□□ Find the lines represented by the equation $3x^2-16xy+5y^2=0$.

Solution:

1st Process:

Given homogeneous equation is as follows

$$3x^2-16xy+5y^2=0$$

We expressed the given equation as

$$3x^2-16xy+5y^2=0$$
$$3x^2-15xy-xy+5y^2=0$$
$$3x(x-5y)-y(x-5y)=0$$
$$(x-5y)(3x-y)=0$$

Therefore $x-5y=0$ and $3x-y=0$

These are the straight lines passing through the origin.

2nd Process:

Given homogeneous equation is as follows

$$3x^2-16xy+5y^2=0$$

We expressed the given equation as

$$3x^2-16xy+5y^2=0$$
$$3x^2-16y.x+5y^2=0$$
$$x=\frac{16y\pm\sqrt{(-16y)^2-4.3.5y^2}}{2.3}$$
$$x=\frac{16y\pm\sqrt{256y^2-60y^2}}{6}$$
$$x=\frac{16y\pm\sqrt{196y^2}}{6}$$
$$x=\frac{16y\pm14y}{6}$$

$$\therefore\ x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Taking positive sign we get $x = \frac{16y+14y}{6} = \frac{30y}{6} = 5y$

Therefore $x = 5y \Rightarrow x - 5y = 0$

And taking negative sign we get $x = \frac{16y-14y}{6} = \frac{2y}{6} = \frac{y}{3}$

Therefore $x = \frac{y}{3} \Rightarrow 3x = y \ \therefore 3x - y = 0$

Therefore $x - 5y = 0$ and $3x - y = 0$

These are the straight lines passing through the origin.

H.W:

- Find the lines represented by the equation $3x^2 + 8xy - 3y^2 = 0$.
- Find the lines represented by the equation $2x^2 + 5xy + 3y^2 = 0$.
- Find the lines represented by the equation $8x^2 - 42xy - 11y^2 = 0$.
- Find the lines represented by the equation $5x^2 - 12xy + 3y^2 = 0$.
- Find the lines represented by the equation $3x^2 - 16xy + 5y^2 = 0$.
- Find the lines represented by the equation $33x^2 - 71xy - 14y^2 = 0$.

□□ Find the angle between the lines represented by the equation $3x^2 - 16xy + 5y^2 = 0$.

Solution:

Given homogeneous equation is as follows

$$3x^2 - 16xy + 5y^2 = 0$$

Comparing the given equation with the general homogeneous equation $ax^2 + 2hxy + by^2 = 0$ we have $a = 3, h = -8 \ \ and \ b = 5$.

Let an angle between the lines is θ.

Then we have $\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$

$$\tan\theta = \frac{2\sqrt{(-8)^2 - 3.5}}{3+5}$$

$$\tan\theta = \frac{2\sqrt{64-15}}{8}$$

$$\tan\theta = \frac{2\sqrt{49}}{8} = \frac{2.7}{8} = \frac{14}{8}$$

$$\therefore \ \theta = \tan^{-1}\left(\frac{14}{8}\right) = 60.26^0$$

Therefore, the angle between the lines is 60.26^0.

H.W:

- Find the angle between the lines represented by the equation $3x^2 + 8xy - 3y^2 = 0$.
- Find the angle between the lines represented by the equation $2x^2 + 5xy + 3y^2 = 0$.
- Find the lines represented by the equation $8x^2 - 42xy - 11y^2 = 0$.
- Find the lines represented by the equation $5x^2 - 12xy + 3y^2 = 0$.

□□ Find the equation of the bisectors of an angle produced by the pair of straight line represented by the equation $3x^2 - 16xy + 5y^2 = 0$ at origin.

Solution:

Given homogeneous equation is as follows

$$3x^2 - 16xy + 5y^2 = 0$$

Comparing the given equation with the general homogeneous equation $ax^2 + 2hxy + by^2 = 0$ we have $a = 3, h = -8 \ \ and \ b = 5$.

We know that,

The equation of the bisectors of an angle produced by the pair of straight line represented by the equation $ax^2 + 2hxy + by^2 = 0$ is $\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$.

So the required equation is $\frac{x^2 - y^2}{3 - 5} = \frac{xy}{-8}$

$$\frac{x^2 - y^2}{-2} = \frac{xy}{-8}$$

$$\frac{x^2 - y^2}{1} = \frac{xy}{4}$$

$$4(x^2 - y^2) = xy$$

$$4(x^2 - y^2) = xy$$

(Ans).

H.W:

- Find the equation of the bisectors of an angle produced by the pair of straight line represented by the equation $3x^2 + 8xy - 3y^2 = 0$ at origin.
- Find the equation of the bisectors of an angle produced by the pair of straight line represented by the equation $2x^2 + 5xy + 3y^2 = 0$ at origin.
- Find the equation of the bisectors of an angle produced by the pair of straight line represented by the equation $8x^2 - 42xy - 11y^2 = 0$ at origin.

Non-homogeneous equation:

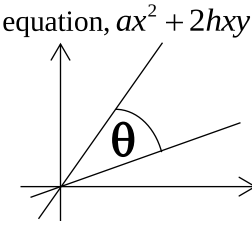
An equation in which degree of each term in it is not equal is called Non-homogeneous equation. Such as $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is a non- homogeneous equation of degree or order 2.

Note:

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, represents straight lines if

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

❖ Angle between the lines represented by the equation, $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is calculated by formula $\tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b}$



- Lines be perpendicular if $a+b=0$
- Lines be parallel if $h^2=ab$
- Lines represented by Non-homogeneous equation is real if $h^2>ab$.
- Lines represented by Non-homogeneous equation are imaginary if $h^2<ab$. But passes through a real point (a,b).

❖ The equation of the bisectors of an angle produced by the pair of straight line represented by the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ is $\frac{(x-\alpha)^2-(y-\beta)^2}{a-b}=\frac{(x-\alpha)(y-\beta)}{h}$, where (α,β) is the intersection point of those lines.

🗀 Show that $6x^2-5xy-6y^2+14x+5y+4=0$ represents pair of straight lines.

Solution:

Given equation is,

$$6x^2-5xy-6y^2+14x+5y+4=0 \cdots\cdots\cdots\cdots(i)$$

Comparing this above equation with the standard equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ we get

$$a=6\ ,h=-\frac{5}{2},b=-6,g=7,\ f=\frac{5}{2}\ \&c=4$$

Now,

$$\Delta=\begin{vmatrix} 6 & -\frac{5}{2} & 7 \\ -\frac{5}{2} & -6 & \frac{5}{2} \\ 7 & \frac{5}{2} & 4 \end{vmatrix}=6(-24-\frac{25}{4})-\left(-\frac{5}{2}\right)\left(-10-\frac{35}{2}\right)+7\left(-\frac{25}{4}+42\right)$$

$$=6(-24-\frac{25}{4})-\left(-\frac{5}{2}\right)\left(-10-\frac{35}{2}\right)+7\left(-\frac{25}{4}+42\right)$$

$$=6(-24-\frac{25}{4})+\frac{5}{2}\left(-10-\frac{35}{2}\right)+7\left(-\frac{25}{4}+42\right)$$

$$= (-144 - \frac{150}{4}) + \left(-25 - \frac{175}{4}\right) + \left(-\frac{175}{4} + 294\right)$$

$$= \frac{-576-150}{4} + \left(\frac{-100-175}{4}\right) + \left(\frac{-175+1176}{4}\right)$$

$$= \frac{-726}{4} + \left(\frac{-275}{4}\right) + \left(\frac{1001}{4}\right)$$

$$= -\frac{1001}{4} + \frac{1001}{4} = 0$$

Since $\Delta = 0$ so the given equation represents a pair of straight lines.

H.W:

1. Prove that $2y^2 - xy - x^2 + 2x + y - 1 = 0$ represents pair of straight lines.
2. Prove that $2y^2 + 3xy + 5y - 6x + 2 = 0$ represents pair of straight lines.
3. Prove that $3y^2 - 8xy - 3y^2 - 29x + 3y - 18 = 0$ represents pair of straight lines.
4. Prove that $x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$ represents pair of straight lines.
5. Prove that $2x^2 - 7xy + 3y^2 + x + 7y - 6 = 0$ represents pair of straight lines.

🗀 For what value of λ the equation $12x^2 + 36xy + \lambda y^2 + 6x + 6y + 3 = 0$ represents a pair of straight lines.

Solution:

Given equation is,

$$12x^2 + 36xy + \lambda y^2 + 6x + 6y + 3 = 0 \quad \cdots\cdots\cdots\cdots(i)$$

Comparing this above equation with the standard equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ we get

$$a = 12\ , h = 18, b = \lambda, g = 3, f = 3\ \&\ c = 3$$

Here the given equation represents a pair of straight lines if $\Delta = 0$.

Now,

$$\Delta = 0$$

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

$$\begin{vmatrix} 12 & 18 & 3 \\ 18 & \lambda & 3 \\ 3 & 3 & 3 \end{vmatrix} = 0$$

$$3.3\begin{vmatrix} 4 & 18 & 1 \\ 6 & \lambda & 1 \\ 1 & 3 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} 4 & 18 & 1 \\ 6 & \lambda & 1 \\ 1 & 3 & 1 \end{vmatrix} = 0$$

$$4(\lambda - 3) - 18(6 - 1) + 1(18 - \lambda) = 0$$

$$(4\lambda-12)-(108-18)+(18-\lambda)=0$$
$$(4\lambda-12)-(90)+(18-\lambda)=0$$
$$4\lambda-12-90+18-\lambda=0$$
$$3\lambda-84=0$$
$$3\lambda=84$$
$\lambda=28$, This is the required value of λ. **(Ans)**

H.W:

1. For what value of μ the equation $x^2-\mu xy+2y^2+3x-5y+2=0$ represents a pair of straight lines.
2. For what value of λ the equation $\lambda x^2+4xy+y^2-4x-2y-3=0$ represents a pair of straight lines.
3. For what value of μ the equation $2x^2+xy-y^2-2x-5y+\mu=0$ represents a pair of straight lines.
4. For what value of η the equation $\eta xy-8x+9y-12=0$ represents a pair of straight lines.

🗀 Find the equation of the straight lines represented by the equation
$$x^2+6xy+9y^2+4x+12y-5=0.$$

Solution:

Given equation is,
$$x^2+6xy+9y^2+4x+12y-5=0$$
Arrange the above equation as a quadratic equation in x we get
$$x^2+6xy+9y^2+4x+12y-5=0$$
$$x^2+(6y+4)x+9y^2+12y-5=0$$
$$\therefore\ x=\frac{-(6y+4)\pm\sqrt{(6y+4)^2-4.1.(9y^2+12y-5)}}{2.1}$$

$$\therefore\ x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(6y+4)\pm\sqrt{(6y+4)^2-4(9y^2+12y-5)}}{2}$$
$$x=\frac{-(6y+4)\pm\sqrt{36y^2+48y+16-(36y^2+48y-20)}}{2}$$
$$x=\frac{-(6y+4)\pm\sqrt{36y^2+48y+16-36y^2-48y+20}}{2}$$
$$x=\frac{-(6y+4)\pm\sqrt{16+20}}{2}$$
$$x=\frac{-(6y+4)\pm\sqrt{36}}{2}$$
$$x=\frac{-(6y+4)\pm6}{2}$$
Taking positive we get $x=\dfrac{-(6y+4)+6}{2}$

$$x=\frac{-6y-4+6}{2}$$
$$x=\frac{-6y+2}{2}$$
$$2x=-6y+2$$
$$x=-3y+1$$
$$x+3y-1=0$$

Taking negative we get $x=\dfrac{-(6y+4)-6}{2}$

$$x=\frac{-6y-4-6}{2}$$
$$x=\frac{-6y-10}{2}$$
$$2x=-6y-10$$
$$x=-3y-5$$
$$x+3y+5=0$$

Therefore, required equations of the straight lines $x+3y-1=0$ and $x+3y+5=0$. **(As desired)**

H.W:

Find the equation of the straight lines represented by the following equations

1. $2x^2-7xy+3y^2+x+7y-6=0$
2. $x^2-3xy+2y^2+3x-5y+2=0$
3. $2y^2-xy-x^2+2x+y-1=0$
4. $2y^2+3xy+5y-6x+2=0$
5. $3y^2-8xy-3y^2-29x+3y-18=0$
6. $x^2+6xy+9y^2+4x+12y-5=0$

🗀 Find the point of intersection of the straight lines represented by the equation

$$6x^2-5xy-6y^2+14x+5y+4=0.$$

Solution:

Given equation is,

$$6x^2-5xy-6y^2+14x+5y+4=0$$

Suppose $f(x,y)=6x^2-5xy-6y^2+14x+5y+4$

Now, Differentiating the function $f(x,y)$ with respect to x and y partially and equating with zero, we get

$$\frac{\partial f}{\partial x}=12x-5y+14$$
$$\Rightarrow 12x-5y+14=0 \cdots\cdots\cdots\cdots\cdots (i)$$

And

$$\frac{\partial f}{\partial x}=-5x-12y+5$$
$$\Rightarrow -5x-12y+5=0$$
$$\Rightarrow\ 5x+12y-5=0 \cdots\cdots\cdots\cdots\cdots (ii)$$

Solving equation (i) and (ii) we get the point of intersection of lines represented by the given equation.
Using cross multiplication method on equation (i) and (ii)

$$\frac{x}{25-168}=\frac{y}{70+60}=\frac{1}{144+25}$$

$$\frac{x}{-143}=\frac{y}{130}=\frac{1}{169}$$

$$x=-\frac{143}{169}=-\frac{11}{13}\ \&\ y=\frac{130}{169}=\frac{10}{13}$$

Therefore, the coordinates of point of intersection is $(x, y)=\left(-\frac{11}{13},\frac{10}{13}\right)$.

H.W:
Find the point of intersection of the straight lines represented by the following's equations

1. $2x^2-7xy+3y^2+x+7y-6=0$
2. $x^2-3xy+2y^2+3x-5y+2=0$
3. $2y^2-xy-x^2+2x+y-1=0$
4. $2y^2+3xy+5y-6x+2=0$
5. $3y^2-8xy-3y^2-29x+3y-18=0$
6. $x^2+6xy+9y^2+4x+12y-5=0$

🗀 Find the angle between the straight lines represented by the equation

$$6x^2-5xy-6y^2+14x+5y+4=0.$$

Solution:
Given equation is,

$$6x^2-5xy-6y^2+14x+5y+4=0$$

Comparing this above equation with the standard equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ we get

$$a=6\ ,h=-\frac{5}{2},b=-6,g=7,\ f=\frac{5}{2}\ \&c=4$$

Assume that θ be the angle between the straight lines then we have the followings

$$\tan\theta=\frac{2\sqrt{h^2-ab}}{a+b}$$

$$\tan\theta=\frac{2\sqrt{\frac{25}{4}+36}}{6-6}$$

$$\tan\theta=\frac{2\sqrt{\frac{25}{4}+36}}{0}$$

$$\tan\theta=\infty$$

$$\theta=\tan^{-1}(\infty)$$

$$\theta=\frac{\pi}{2}$$ **(As desired)**

H.W:
Find the angle between the straight lines represented by the followings equations

1. $2x^2-7xy+3y^2+x+7y-6=0$
2. $x^2-3xy+2y^2+3x-5y+2=0$
3. $2y^2-xy-x^2+2x+y-1=0$
4. $2y^2+3xy+5y-6x+2=0$
5. $3y^2-8xy-3y^2-29x+3y-18=0$
6. $x^2+6xy+9y^2+4x+12y-5=0$

🗀 Find the equation of the bisectors of the angle between the straight lines represented by the equation

$$6x^2-5xy-6y^2+14x+5y+4=0.$$

Solution:
Given equation is,

$$6x^2-5xy-6y^2+14x+5y+4=0$$

Comparing this above equation with the standard equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ we get

$$a=6\ ,h=-\frac{5}{2},b=-6,g=7,\ f=\frac{5}{2}\ \&c=4$$

Suppose $f\left(x,y\right)=6x^2-5xy-6y^2+14x+5y+4$

Now, Differentiating the function $f\left(x,y\right)$ with respect to x and y partially and equating with zero, we get

$$\frac{\partial f}{\partial x}=12x-5y+14$$
$$\Rightarrow 12x-5y+14=0\ \cdots\cdots\cdots\cdots\cdots\ (i)$$

And

$$\frac{\partial f}{\partial x}=-5x-12y+5$$
$$\Rightarrow -5x-12y+5=0$$
$$\Rightarrow\ 5x+12y-5=0\ \cdots\cdots\cdots\cdots\cdots\ (ii)$$

Solving equation (i) and (ii) we get the point of intersection of lines represented by the given equation.
Using cross multiplication method on equation (i) and (ii)

$$\frac{x}{25-168}=\frac{y}{70+60}=\frac{1}{144+25}$$
$$\frac{x}{-143}=\frac{y}{130}=\frac{1}{169}$$
$$x=-\frac{143}{169}=-\frac{11}{13}\ \ \&\ \ y=\frac{130}{169}=\frac{10}{13}$$

Therefore, the coordinates of point of intersection is $\left(x,y\right)=\left(-\frac{11}{13},\frac{10}{13}\right)$ $i.e\ \left(\alpha,\beta\right)=\left(-\frac{11}{13},\frac{10}{13}\right)$.

If $\left(\alpha,\beta\right)$ be the point of intersection of lines by given equations the equation of the bisectors is as follows

$$\frac{(x-\alpha)^2-(y-\beta)^2}{a-b}=\frac{(x-\alpha)(y-\beta)}{h}$$

$$\frac{\left(x+\frac{11}{13}\right)^2-\left(y-\frac{10}{13}\right)^2}{6+6}=\frac{\left(x+\frac{11}{13}\right)\left(y-\frac{10}{13}\right)}{-\frac{5}{2}}$$

$$\frac{\left(x+\frac{11}{13}\right)^2-\left(y-\frac{10}{13}\right)^2}{12}=\frac{2\left(x+\frac{11}{13}\right)\left(y-\frac{10}{13}\right)}{-5}$$

$$\frac{x^2+\frac{22x}{13}+\frac{121}{169}-y^2+\frac{20y}{13}-\frac{100}{169}}{12}=\frac{2\left(xy-\frac{10x}{13}+\frac{11y}{13}-\frac{110}{169}\right)}{-5}$$

$$-5\left(x^2+\frac{22x}{13}+\frac{121}{169}-y^2+\frac{20y}{13}-\frac{100}{169}\right)=24\left(xy-\frac{10x}{13}+\frac{11y}{13}-\frac{110}{169}\right)$$

$$\left(-5x^2-\frac{110x}{13}-\frac{605}{169}+5y^2+\frac{100y}{13}+\frac{500}{169}\right)=\left(24xy-\frac{240x}{13}+\frac{264y}{13}-\frac{2640}{169}\right)$$

$$-5x^2-\frac{110x}{13}-\frac{605}{169}+5y^2+\frac{100y}{13}+\frac{500}{169}=24xy-\frac{240x}{13}+\frac{264y}{13}-\frac{2640}{169}$$

$$-845x^2-1430x-605+845y^2+1300y+500=4056xy-3120x+3432y-2640$$

$$-845x^2-4056xy+845y^2-1430x+3120x+1300y-3432y-605+500+2640=0$$

$$845x^2+4056xy-845y^2-1690x+2132y-2535=0 \quad \textbf{(As desired)}$$

H.W:

Find the equation of the bisectors of the angle between the straight lines represented by the following equations

1. $2x^2-7xy+3y^2+x+7y-6=0$
2. $x^2-3xy+2y^2+3x-5y+2=0$
3. $2y^2-xy-x^2+2x+y-1=0$
4. $2y^2+3xy+5y-6x+2=0$
5. $3y^2-8xy-3y^2-29x+3y-18=0$
6. $x^2+6xy+9y^2+4x+12y-5=0$