Lesson 3
Principles of Prestressed Concrete

CE 414: Prestressed Concrete

Principles of Prestressed Concrete

There are 03(three) principles or concepts of Prestressed Concrete

- **First Concept:** Prestressing to transform concrete into an Elastic Material.
- **Second Concept:** Prestressing for Combination of High Strength Steel with Concrete.
- **Third Concept:** Prestressing to Achieve load Balancing.

Ref. book: Design of Prestressed Concrete by T.Y. Lin & Ned H. Burns, Chapter: 01

First Concept

From this standpoint concrete is visualized as being subject to two systems of forces: internal prestress and external load, with the tensile stresses due to the external load counteracted by the compressive stresses due to the prestress. Similarly, the cracking of concrete due to load is prevented or delayed by the precompression produced by the tendons. So long as there are no cracks, the stresses, strains, and deflections of the concrete due to the two systems of forces can be considered separately and superimposed if necessary.

Ref. book: Design of Prestressed Concrete by T.Y. Lin & Ned H. Burns, Chapter: 01

First Concept

In its simplest form, let us consider a simple rectangular beam prestressed by a tendon through its centroidal axis (Fig. 1-13) and loaded by external loads. The tensile prestress force F in the tendon produces an equal compressive force F in the concrete, which also acts at the centroid of the tendon. In this case the force is at the centroid of the cross section. Due to the prestress F, a uniform compressive stress of

$$f = \frac{F}{A} \tag{1-1}$$

will be produced across the section that has an area A. If M is the external moment at a section due to the load on and the weight of the beam, then the stress at any point across that section due to M is

$$f = \frac{My}{I} \tag{1-2}$$

First Concept

Ref. book: Design of Prestressed Concrete by T.Y. Lin & Ned H. Burns, Chapter: 01

where y is the distance from the centroidal axis and I is the moment of inertia of the section. Thus the resulting stress distribution is given by

$$f = \frac{F}{A} \pm \frac{My}{I} \qquad (1\text{-}3)$$

as shown in Fig. 1-13.

First Concept

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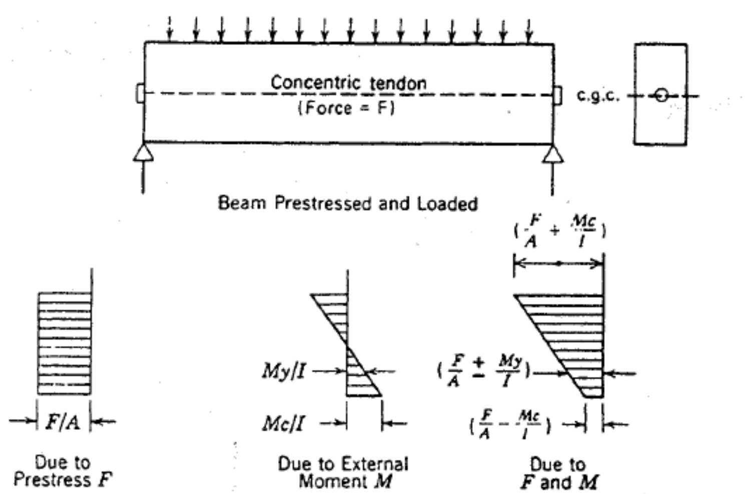

Fig. 1-13. Stress distribution across a concentrically prestressed-concrete section.

First Concept

Ref. book: Design of Prestressed Concrete by T.Y. Lin & Ned H. Burns, Chapter: 01

The solution is slightly more complicated when the tendon is placed eccentrically with respect to the centroid of the concrete section, Fig. 1-14. Here the resultant compressive force F in the concrete acts at the centroid of the tendon which is at a distance e from the c.g.c. as shown in Fig. 1-14. Due to an eccentric prestress, the concrete is subject to a moment as well as a direct load. The moment produced by the prestress is Fe, and the stresses due to this moment are

$$f = \frac{Fey}{I} \quad (1\text{-}4)$$

Thus, the resulting stress distribution is given by

$$f = \frac{F}{A} \pm \frac{Fey}{I} \pm \frac{My}{I} \quad (1\text{-}5)$$

as shown in the figure.

First Concept

Ref. book: Design of Prestressed Concrete by T.Y. Lin & Ned H. Burns, Chapter: 01

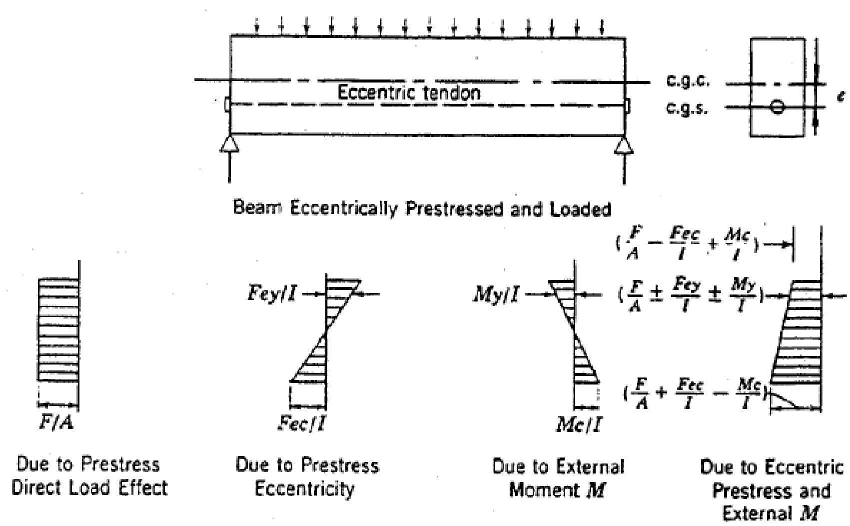

Fig. 1-14. Stress distribution across an eccentrically prestressed-concrete section.

Ref. book: Design of Prestressed Concrete by T.Y. Lin & Ned H. Burns, Chapter: 01

First Concept

EXAMPLE: 1.1

A prestressed-concrete rectangular beam 20 in. by 30 in. has a simple span of 24 ft and is loaded by a uniform load of 3 k/ft including its own weight, Fig. 1-15. The prestressing tendon is located as shown and produces an effective prestress of 360 k. Compute fiber stresses in the concrete at the midspan section (span = 7.31 m, load = 43.8 kN/m and $F = 1601$ kN).

Solution Using formula 1-5, we have $F = 360$ k, $A = 20 \times 30 = 600$ in.^{2} (neglecting any hole due to the tendon), $e = 6$ in., $I = bd^3/12 = 20 \times 30^3/12 = 45{,}000$ in.^{4}; $y = 15$ in. for extreme fibers.

$$M = 3 \times 24^2/8 = 216 \text{ k-ft } (293 \text{ kN-m})$$

Ref. book: Design of Prestressed Concrete by T.Y. Lin & Ned H. Burns, Chapter: 01

First Concept

EXAMPLE: 1.1

Therefore, assuming compressive stress negative, we have

$$f = \frac{F}{A} \pm \frac{Fey}{I} \pm \frac{My}{I}$$

$$= \frac{-360{,}000}{600} \pm \frac{360{,}000 \times 6 \times 15}{45{,}000} \pm \frac{216 \times 12{,}000 \times 15}{45{,}000}$$

$$= -600 \pm 720 \pm 864$$

$$= -600 + 720 - 864 = -744 \text{ psi } (-5.13 \text{ N/mm}^2) \text{ for top fiber}$$

$$= -600 - 720 + 864 = -456 \text{ psi } (-3.14 \text{ N/mm}^2) \text{ for bottom fiber}$$

The resulting stress distribution is shown in Fig. 1-15.

Ref. book: Design of Prestressed Concrete by T.Y. Lin & Ned H. Burns, Chapter: 01

First Concept

EXAMPLE: 1.1

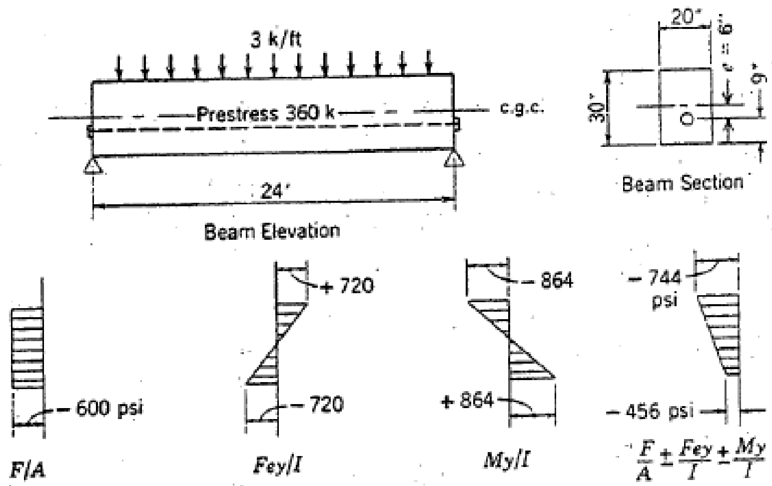

Fig. 1-15. Example 1-1.

Ref. book: Design of Prestressed Concrete by T.Y. Lin & Ned H. Burns, Chapter: 01

First Concept

EXAMPLE: 1.2

A prestressed-concrete rectangular beam 20 in. by 30 in. has a simple span of 24 ft and is loaded by a uniform load of 3 k/ft including its own weight, Fig. 1-18. The prestressing tendon is located as shown and produces an effective prestress of 360 k. Compute fiber stresses in the concrete at the midspan section (span = 7.31 m, load = 43.8 kN/m and F = 1601 kN).

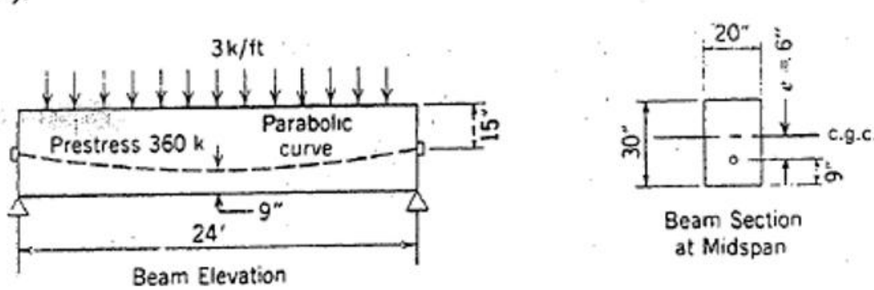

Fig. 1-18.

First Concept

Ref. book: Design of Prestressed Concrete by T.Y. Lin & Ned H. Burns, Chapter: 01

EXAMPLE: 1.2

Solution Using formula 1-5, we have $F=360$ k, $A=20\times30=600$ in.^{2} (neglecting any hole due to the tendon), $e=6$ in., $I=bd^3/12=20\times30^3/12=45{,}000$ in.^{4}; $y=15$ in. for extreme fibers.

$$M=3\times24^2/8=216 \text{ k-ft } (293 \text{ kN}-\text{m})$$

Therefore, assuming compressive stress negative, we have

$$f=\frac{F}{A}\pm\frac{Fey}{I}\pm\frac{My}{I}$$
$$=\frac{-360{,}000}{600}\pm\frac{360{,}000\times6\times15}{45{,}000}\pm\frac{216\times12{,}000\times15}{45{,}000}$$
$$=-600\pm720\pm864$$
$$=-600+720-864=-744 \text{ psi } (-5.13 \text{ N/mm}^2) \text{ for top fiber}$$
$$=-600-720+864=-456 \text{ psi } (-3.14 \text{ N/mm}^2) \text{ for bottom fiber}$$

First Concept

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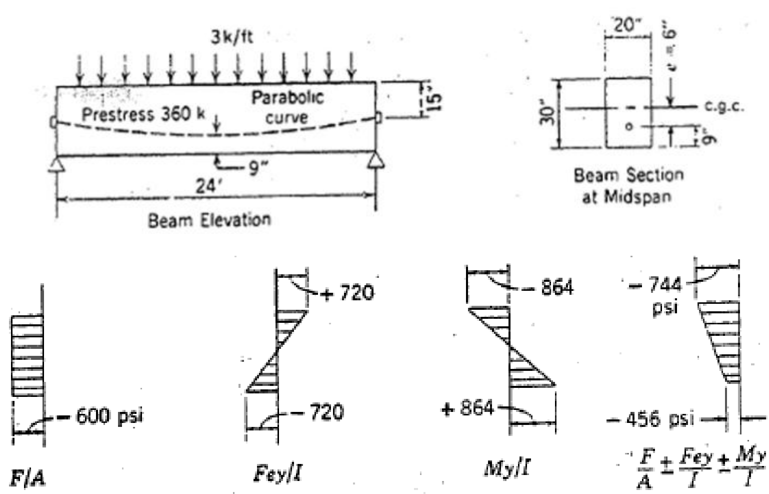
EXAMPLE: 1.2



Stress Distribution Diagram

Second Concept

Ref. book: Design of Prestressed Concrete by T.Y. Lin & Ned H. Burns, Chapter: 01

Second Concept—Prestressing for Combination of High-Strength Steel with Concrete. This concept is to consider prestressed concrete as a combination of steel and concrete, similar to reinforced concrete, with steel taking tension and concrete taking compression so that the two materials form a resisting couple against the external moment, Fig. 1-19. This is often an easy concept for engineers familiar with reinforced concrete where the steel supplies a tensile force and the concrete supplies a compressive force, the two forces forming a couple with a lever arm between them. Few engineers realize, however, that similar behavior exists in prestressed concrete.

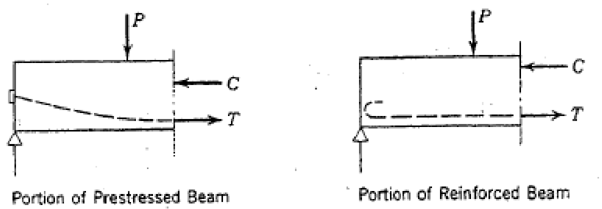


Fig. 1-19. Internal resisting moment in prestressed- and reinforced-concrete beams.

Second Concept

Ref. book: Design of Prestressed Concrete by T.Y. Lin & Ned H. Burns, Chapter: 01

EXAMPLE: 1.3

A prestressed-concrete rectangular beam 20 in. by 30 in. has a simple span of 24 ft and is loaded by a uniform load of 3 k/ft including its own weight, Fig. 1-18. The prestressing tendon is located as shown and produces an effective prestress of 360 k. Compute fiber stresses in the concrete at the midspan section (span = 7.31 m, load = 43.8 kN/m and F = 1601 kN).

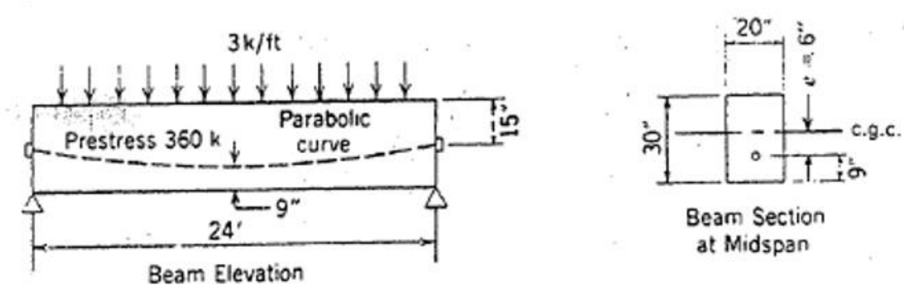


Fig. 1-18.

Second Concept

Ref. book: Design of Prestressed Concrete by T.Y. Lin & Ned H. Burns, Chapter: 01

EXAMPLE: 1.3

Solution Take one half of the beam as a freebody, thus exposing the internal couple, Fig. 1-21. The external moment at the section is

$$M = \frac{wL^2}{8}$$

$$= \frac{3 \times 24^2}{8}$$

$$= 216 \text{ k-ft } (293 \text{ kN-m})$$

The internal couple is furnished by the forces $C = T = 360$ k, which must act with a lever arm of

$$\frac{216}{360} \times 12 = 7.2 \text{ in. } (183 \text{ mm})$$

Second Concept

Ref. book: Design of Prestressed Concrete by T.Y. Lin & Ned H. Burns, Chapter: 01

EXAMPLE: 1.3

Since T acts at 9 in. from the bottom, C must be acting at 16.2 in. from it. Thus the center of the compressive force C is located.

So far we have been dealing only with statics, the validity of which is not subject to any question. Now, if desired, the stress distribution in the concrete can be obtained by the usual elastic theory, since the center of the compressive force is already known. For $C = 360{,}000$ lb (1,601 kN) acting with an eccentricity of $16.2 - 15 = 1.2$ in. (30.48 mm),

$$f = \frac{F}{A} \pm \frac{Mc}{I}$$

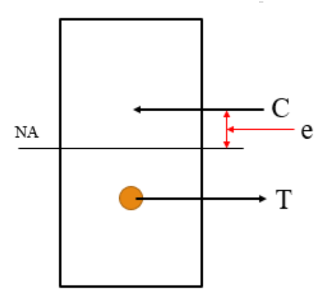
Here, M= F e

$$= \frac{-360{,}000}{600} \pm \frac{360{,}000 \times 1.2 \times 15}{45{,}000}$$

$$= -600 \mp 144$$

$$= -744 \text{ psi } (-5.13 \text{ N/mm}^2) \text{ for top fiber}$$

$$= -456 \text{ psi } (-3.14 \text{ N/mm}^2) \text{ for bottom fiber}$$



Third Concept

Ref. book: Design of Prestressed Concrete by T.Y. Lin & Ned H. Burns, Chapter: 01

Third Concept—Prestressing to Achieve Load Balancing. This concept is to visualize prestressing primarily as an attempt to balance the loads on a member. This concept was essentially developed by the author, although undoubtedly also utilized by other engineers to a lesser degree.

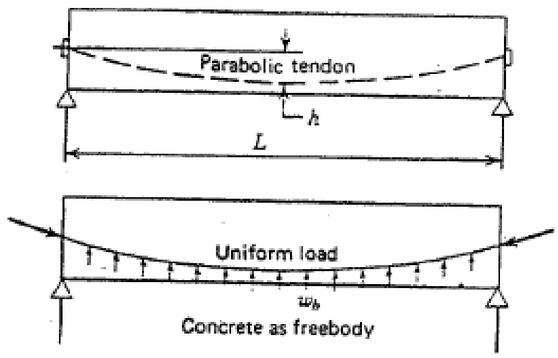


Fig. 1-22. Prestressed beam with parabolic tendon.

Third Concept

Ref. book: Design of Prestressed Concrete by T.Y. Lin & Ned H. Burns, Chapter: 01

In the overall design of a prestressed concrete structure, the effect of prestressing is viewed as the balancing of gravity loads so that members under bending such as slabs, beams, and girders will not be subjected to flexural stresses under a given loading condition. This enables the transformation of a flexural member into a member under direct stress and thus greatly simplifies both the design and analysis of otherwise complicated structures.

The application of this concept requires taking the concrete as a freebody, and replacing the tendons with forces acting on the concrete along the span.

Take, for example, a simple beam prestressed with a parabolic tendon (Fig. 1-22) if

F = prestressing force
L = length of span
h = sag of parabola

Third Concept

Ref. book: Design of Prestressed Concrete by T.Y. Lin & Ned H. Burns, Chapter: 01

F = prestressing force
L = length of span
h = sag of parabola

The upward uniform load is given by

$$w_b = \frac{8Fh}{L^2}$$

Thus, for a given downward uniform load w, the transverse load on the beam is balanced, and the beam is subjected only to the axial force F, which produces uniform stresses in concrete, $f = F/A$. The change in stresses from this balanced condition can easily be computed by the ordinary formulas in mechanics, $f = Mc/I$. The moment in this case is the unbalanced moment due to $(w - w_b)$, the unbalanced load.

Third Concept

Ref. book: Design of Prestressed Concrete by T.Y. Lin & Ned H. Burns, Chapter: 01

EXAMPLE: 1.4

A prestressed-concrete rectangular beam 20 in. by 30 in. has a simple span of 24 ft and is loaded by a uniform load of 3 k/ft including its own weight, Fig. 1-18. The prestressing tendon is located as shown and produces an effective prestress of 360 k. Compute fiber stresses in the concrete at the midspan section (span = 7.31 m, load = 43.8 kN/m and F = 1601 kN).

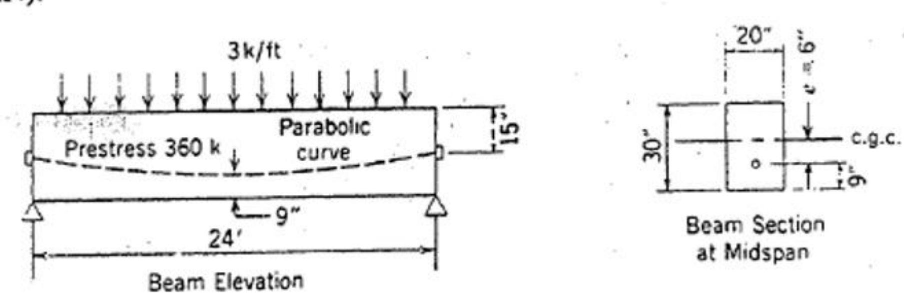


Fig. 1-18.

Ref. book: Design of Prestressed Concrete by T.Y. Lin & Ned H. Burns, Chapter: 01

Third Concept

EXAMPLE: 1.4

Solution The upward uniform force from the tendon on the concrete is

$$w_b = \frac{8Fh}{L^2}$$

$$= \frac{8 \times 360 \times (6/12)}{24^2}$$

$$= 2.5 \text{ k/ft } (36.5 \text{ kN/m})$$

Hence the net downward (unbalanced) load on the concrete beam is $(3-2.5) = 0.5$ k/ft (7.3 kN/m), and the moment at midspan due to that load is

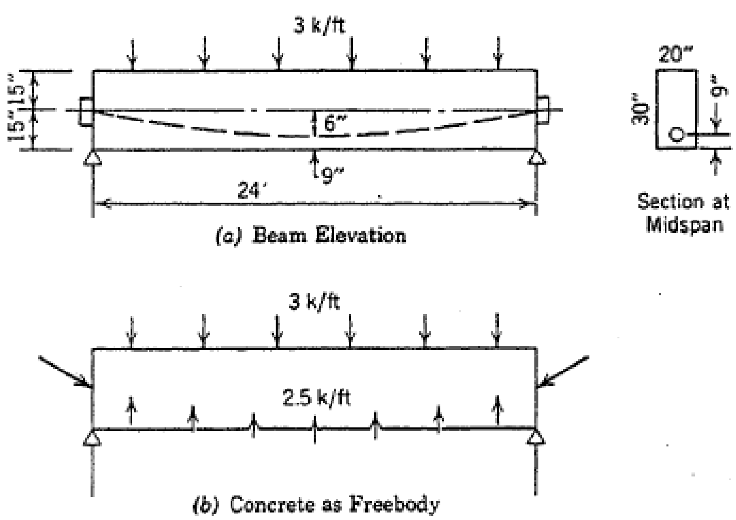
$$M = \frac{wL^2}{8} = \frac{0.5 \times 24^2}{8}$$

$$= 36 \text{ k-ft } (48.8 \text{ kN-m})$$

Ref. book: Design of Prestressed Concrete by T.Y. Lin & Ned H. Burns, Chapter: 01

Third Concept

EXAMPLE: 1.4



(a) Beam Elevation

Section at Midspan

(b) Concrete as Freebody

Third Concept

Ref. book: Design of Prestressed Concrete by T.Y. Lin & Ned H. Burns, Chapter: 01

EXAMPLE: 1.4

$$f = \frac{F}{A} \pm \frac{Mc}{I}$$

Here, M= 36 kip-ft

$$= \frac{-360{,}000}{600} \pm \frac{(36x12x1000)x15}{45{,}000}$$

$$= -600 \mp 144$$

$$= -744 \text{ psi } (-5.13 \text{ N/mm}^2) \text{ for top fiber}$$

$$= -456 \text{ psi } (-3.14 \text{ N/mm}^2) \text{ for bottom fiber}$$

Third Concept

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EXAMPLE: 1.4

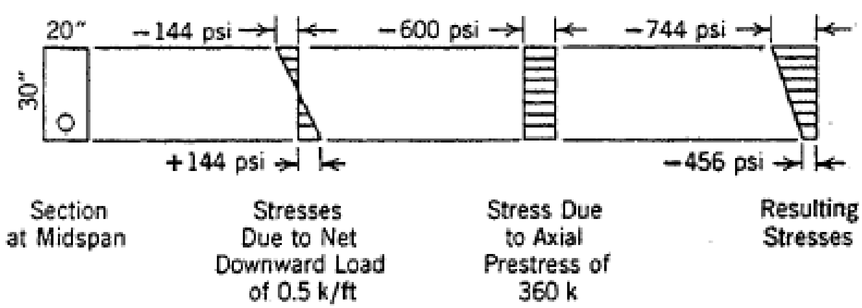

Fig. 1-25. Example 1-4.