CE 414
Prestressed Concrete

Lesson 5
Analysis of Section for Flexure

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REFERENCE

5

ANALYSIS OF SECTIONS FOR FLEXURE

Book Chapter

- Prestressed Concrete- by T.Y. Lin

STRESSES IN CONCRETE (CONCENTRIC TENDON)

Stresses in Concrete Due to Prestress

Some of the basic principles of stress computation for prestressed concrete have already been mentioned in section 1-2. They will be discussed in greater detail here. First of all, let us consider the effect of prestress. According to present

practice, stresses in concrete due to prestress are always computed by the elastic theory. Consider the prestress F existing at the time under discussion, whether it be the initial or the final value. If F is applied at the centroid of the concrete section, and if the section under consideration is sufficiently far from the point of application of the prestress, then, by St. Venant's principle, the unit stress in concrete is uniform across that section and is given by the usual formula,

$$f = \frac{F}{A}$$

where A is the area of that concrete section.

STRESSES IN CONCRETE (CONCENTRIC TENDON)

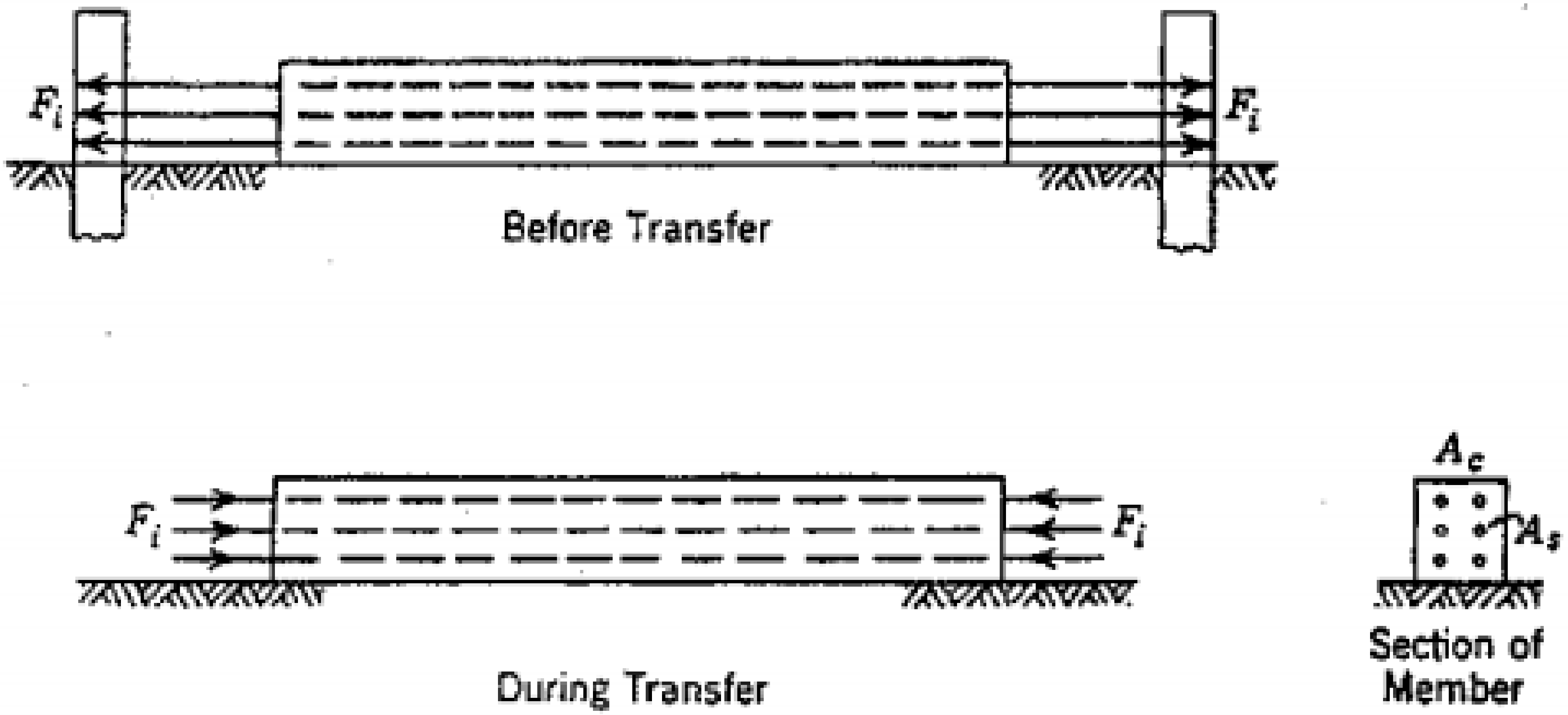

Fig. 5-1. Transfer of concentric prestress in a pretensioned member.

STRESSES IN CONCRETE (CONCENTRIC TENDON)

For a pretensioned member, when the prestress in the steel is transferred from the bulkheads to the concrete, Fig. 5-1, the force that was resisted by the bulkheads is now transferred to both the steel and the concrete in the member. The release of the resistance from the bulkheads is equivalent to the application of an opposite force F_i to the member. Using the transformed section method, and with A_c = net sectional area of concrete, the compressive stress produced in the concrete is

$$f_c = \frac{F_i}{A_c + nA_s} = \frac{F_i}{A_t} \tag{5-1}$$

while that induced in the steel is

$$\Delta f_s = nf_c = \frac{nF_i}{A_c + nA_s} = \frac{nF_i}{A_t} \tag{5-2}$$

which represents the immediate reduction of the prestress in the steel as a result of the transfer.

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STRESSES IN CONCRETE (CONCENTRIC TENDON)

shortening of concrete and approximated by

$$\Delta f_s = \frac{nF_i}{A_c} \quad \text{or} \quad \frac{nF_i}{A_g} \tag{5-3}$$

which differs a little from formula 5-2 but is close enough for all practical purposes, since the total amount of reduction is only about 2 or 3% and the value of n cannot be accurately known anyway. The high-strength steel used for prestressing requires smaller area for the tension steel than would be used in reinforced concrete, thus there is not a large difference between A_c and A_g.

STRESSES IN CONCRETE (CONCENTRIC TENDON)

Example 5.1 (from T.Y.Lin)

A pretensioned member, similar to that shown in Fig. 5-1, has a section of 8 in. by 12 in. (203 mm by 305 mm). It is concentrically prestressed with 0.8 sq in. (516 mm^2) of high-tensile steel wire, which is anchored to the bulkheads of a unit stress of 150,000 psi (1034 N/mm^2). Assuming that $n=6$, compute the stresses in the concrete and steel immediately after transfer.

Solution

1. An exact theoretical solution. Using the elastic theory, we have

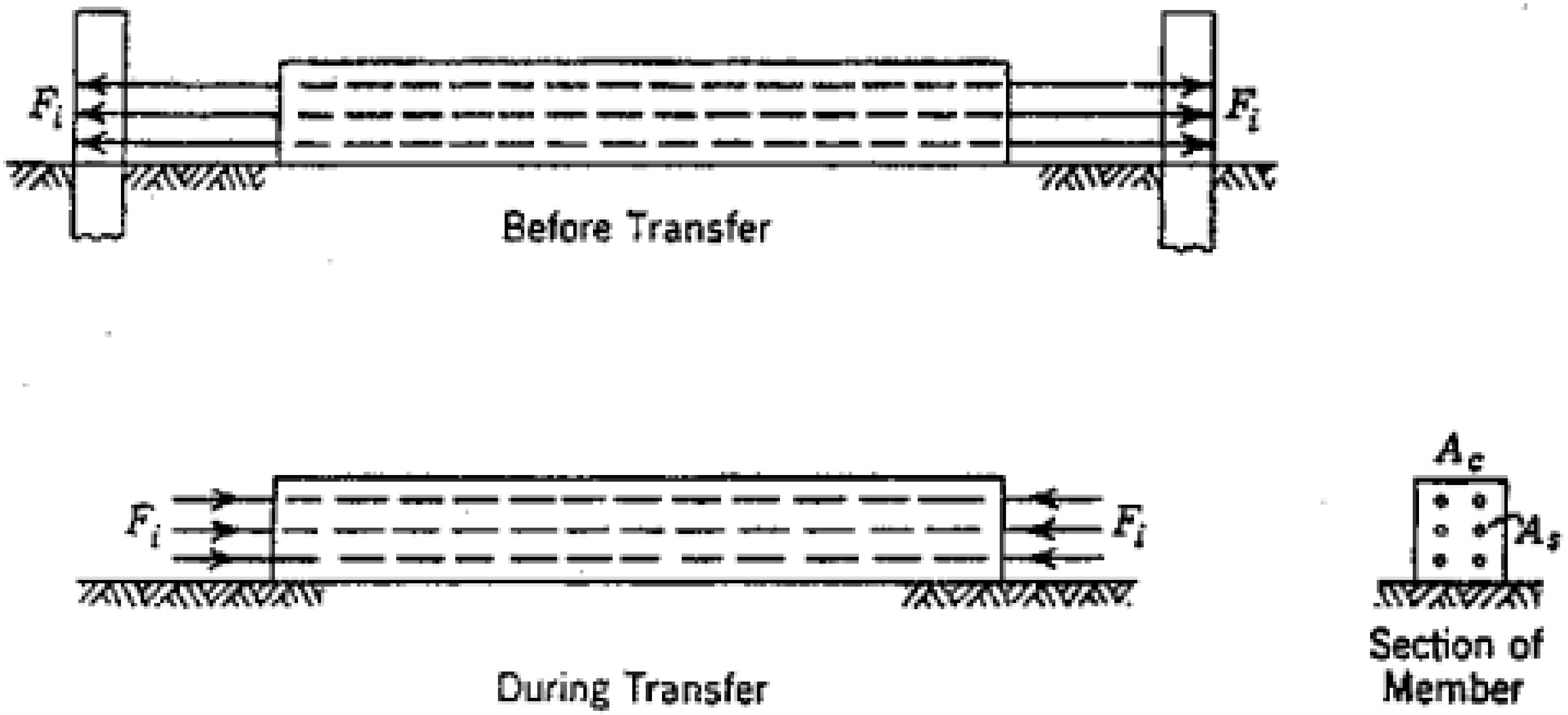
$$f_c = \frac{F_i}{A_c + nA_s} = \frac{F_i}{A_g + (n-1)A_s}$$

$$= \frac{0.8 \times 150{,}000}{12 \times 8 + 5 \times 0.8} = 1200 \text{ psi } (8.3 \text{ N/mm}^2)$$

$$nf_c = 6 \times 1200 = 7200 \text{ psi } (49.6 \text{ N/mm}^2)$$

Stress in steel after transfer $= 150{,}000 - 7200 = 142{,}800$ psi (985 N/mm^2).

STRESSES IN CONCRETE (CONCENTRIC TENDON)



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STRESSES IN CONCRETE (CONCENTRIC TENDON)

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2. An approximate solution. The loss of prestress in steel due to elastic shortening of

Note that, in this second solution, the approximations introduced are: (1) using gross area of concrete instead of net area, (2) using the initial stress in steel instead of the reduced stress. But the answers are very nearly the same for both solutions. The second method is more convenient and is usually followed.

STRESSES IN CONCRETE (CONCENTRIC TENDON)

Example 5.1 **(from T.Y.Lin)**

2. An approximate solution. The loss of prestress in steel due to elastic shortening of concrete is estimated by

$$= n\frac{F_i}{A_g}$$

Here, Fi = As x S
= 0.8 x 150,000 = 120,000 lb

$$= 6\frac{120,000}{8\times 12} = 7500 \text{ psi } (51.7 \text{ N/mm}^2)$$

Stress in steel after loss $= 150,000 - 7500 = 142,500$ psi (983 N/mm²). Stress in concrete is

$$f_c = \frac{142,500\times 0.8}{96} = 1190 \text{ psi } (8.2 \text{ N/mm}^2)$$

STRESSES IN CONCRETE (ECCENTRIC TENDON)

Next, suppose that the prestress F is applied to the concrete section with an eccentricity e, Fig. 5-2; then it is possible to resolve the prestress into two components: a concentric load F through the centroid, and a moment Fe. By the usual elastic theory, the fiber stress at any point due to moment Fe is given by the formula

$$f = \frac{My}{I} = \frac{Fey}{I} \tag{5-4}$$

Then the resultant fiber stress due to the eccentric prestress is given by

$$f = \frac{F}{A} \pm \frac{Fey}{I} \tag{5-5}$$

STRESSES IN CONCRETE (ECCENTRIC TENDON)

The question again arises as to what section should be considered when computing the values of e and I, whether the gross or the net concrete section or the transformed section, and what prestress F to be used in the formula, the initial or the reduced value. Consider a pretensioned member, Fig. 5-3. The steel has already been bonded to the concrete; the release of the force from the bulkhead is equivalent to the application of an eccentric force to the composite member; hence the force should be the total F_i, and I should be the moment of

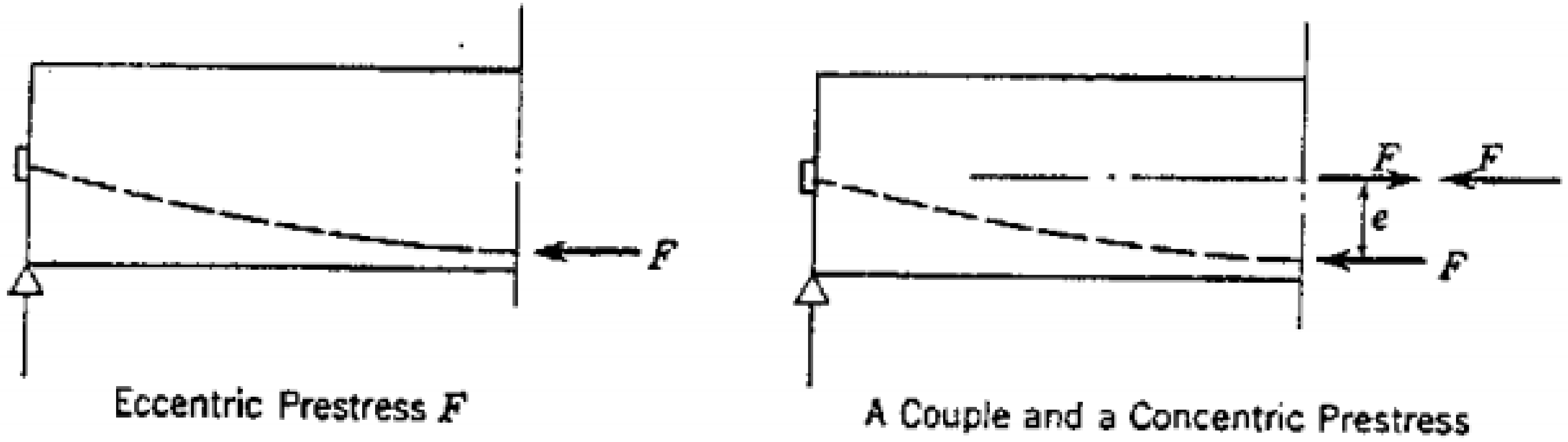

Fig. 5-2. Eccentric prestress on a section

STRESSES IN CONCRETE (ECCENTRIC TENDON)

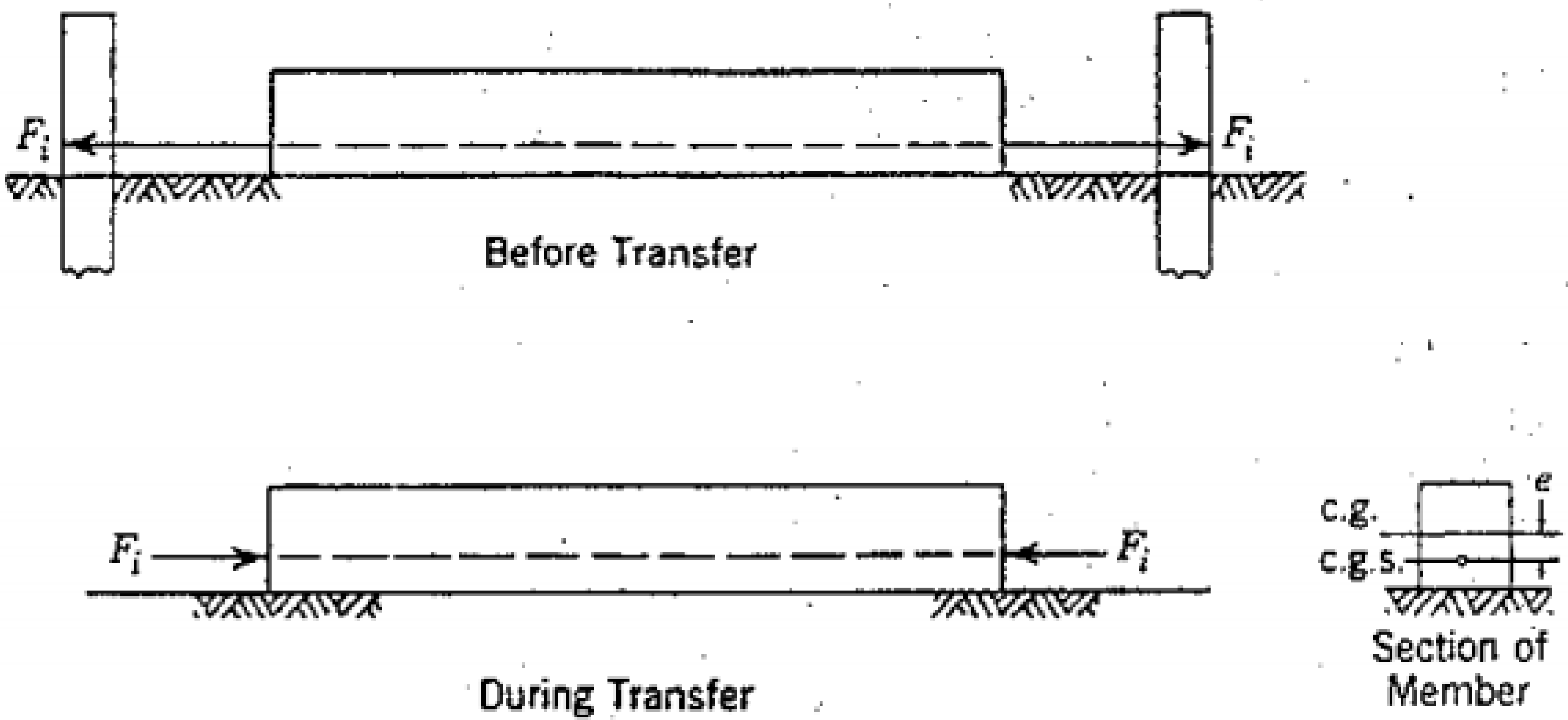

Fig. 5-3. Transfer of eccentric prestress in a pretensioned member.

inertia of the transformed section, and *e* should be measured from the centroidal axis of that transformed section. However, in practice, this procedure is seldom followed. Instead, the gross or net concrete section is considered, and either the initial or the reduced prestress is applied. The error is negligible in most cases.

STRESSES IN CONCRETE (ECCENTRIC TENDON)

Example 5.2 (from T.Y.Lin)

A pretensioned member similar to that shown in Fig. 5-3 has a section of 8 in. by 12 in. (203 mm by 305 mm) deep. It is eccentrically prestressed with 0.8 sq in. (516 mm^2) of high-tensile steel wire which is anchored to the bulkheads at a unit stress of 150,000 psi (1034 N/mm^2). The c.g.s. is 4 in. (101.6 mm) above the bottom fiber. Assuming that $n=6$, compute the stresses in the concrete immediately after transfer due to the prestress only.

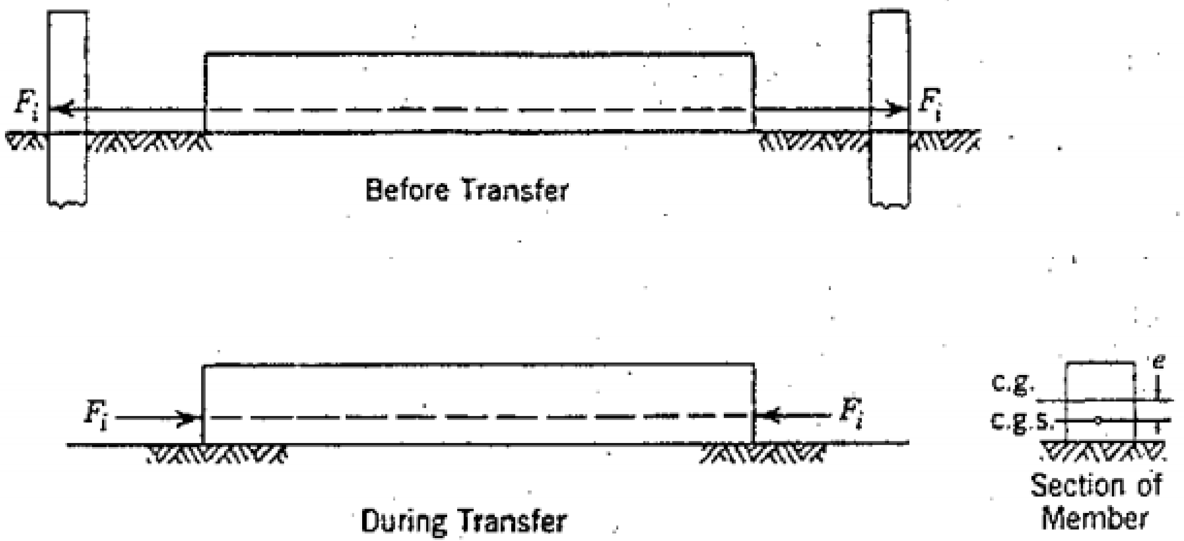

Fig. 5-3. Transfer of eccentric prestress in a pretensioned member.

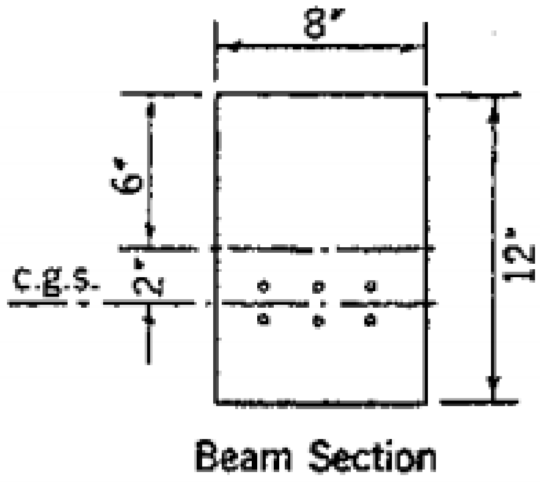

Fig. 5-4. Example 5-2.

STRESSES IN CONCRETE (ECCENTRIC TENDON)

Example 5.2 **(from T.Y.Lin)**

Solution

2. An approximate solution. The loss of prestress can be approximately computed, as in example 5-1, to be 7500 psi in the steel. Hence the reduced prestress is 142,500 psi or 114,000 lb.

The loss of prestress due to elastic shortening of concrete

$$= n\frac{F_i}{A_g}$$

Here, Fi = As x S
= 0.8 x 150,000 = 120,000 lb

$$= 6\frac{120{,}000}{8\times 12} = 7500 \text{ psi } (51.7 \text{ N/mm}^2)$$

Stress in steel after loss $= 150{,}000 - 7500 = 142{,}500$ psi (983 N/mm^2).

STRESSES IN CONCRETE (ECCENTRIC TENDON)

Example 5.2 (from T.Y.Lin)

Extreme fiber stresses in the concrete can be computed to be

$$f_c = \frac{F}{A} \pm \frac{Fey}{I}$$

$$= \frac{-114{,}000}{96} \pm \frac{114{,}000 \times 2 \times 6}{(8 \times 12^3)/12}$$

$$= -1187 \pm 1187$$

$$= 0 \text{ in the top fiber}$$

$$= -2374 \text{ psi } (-16.37 \text{ N/mm}^2) \text{ in the bottom fiber}$$

STRESSES IN CONCRETE (ECCENTRIC TENDON)

Example 5.3 (from T.Y.Lin)

A posttensioned beam has a midspan cross section with a duct of 2 in. by 3 in. (50.8 mm by 76.2 mm) to house the wires, as shown in Fig. 5-6. It is prestressed with 0.8 sq in. (516 mm^2) of steel to an initial stress of 150,000 psi (1034 N/mm^2). Immediately after transfer the stress is reduced by 5% owing to anchorage loss and elastic shortening of concrete. Compute the stresses in the concrete at transfer.

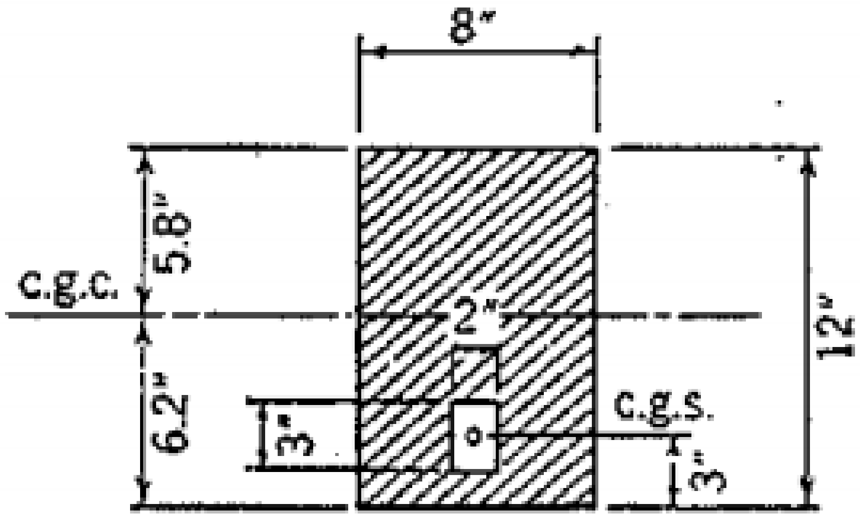

Fig. 5-6. Example 5-3.

STRESSES IN CONCRETE (ECCENTRIC TENDON)

Example 5.3 (from T.Y.Lin)

Total prestress in steel = 150,000 × 0.8 × 95% = 114,000 lb (507 kN)

Solution

2. Using the gross section of concrete. An approximate solution using the gross concrete section would give results not so close in this case (11% difference):

Extreme fiber stresses in the concrete can be computed to be

$$f_c = \frac{F}{A} \pm \frac{Fey}{I}$$

$$f_c = \frac{-114{,}000}{96} \pm \frac{114{,}000 \times 3 \times 6}{(8 \times 12^3)/12}$$

$$= -1270 + 1940 = +1783$$

$$= +596 \text{ psi } (+4.11 \text{ N/mm}^2) \text{ for top fiber}$$

$$= -2970 \text{ psi } (-20.48 \text{ N/mm}^2) \text{ for bottom fiber}$$


Any
questions

STRESSES IN CONCRETE (DUE TO LOADS)

Stresses in Concrete Due to Loads

Stresses in concrete produced by external bending moment, whether due to the beam's own weight or to any externally applied loads, are computed by the usual elastic theory.

$$f = \frac{My}{I} \tag{5-6}$$

For a pretensioned beam, steel is always bonded to the concrete before any external moment is applied. Hence the section resisting external moment is the combined section. In other words, the values of y and I should be computed on the basis of a transformed section, considering both steel and concrete. For approximation, however, either the gross or the net section of concrete alone can be used in the calculations; the magnitude of error so involved can be estimated and should not be serious except in special cases.

STRESSES IN CONCRETE (DUE TO LOADS)

Example 5.4 (from T.Y.Lin)

A posttensioned bonded concrete beam, Fig. 5-8, has a prestress of 350 kips (1,557 kN) in the steel immediately after prestressing, which eventually reduces to 300 kips (1334 kN) due to losses. The beam carries two live loads of 10 kips (44.48 kN) each in addition to its own weight of 300 plf (4.377 kN/m). Compute the extreme fiber stresses at midspan, (*a*) under the initial condition with full prestress and no live load, and (*b*) under the final condition, after the losses have taken place, and with full live load.

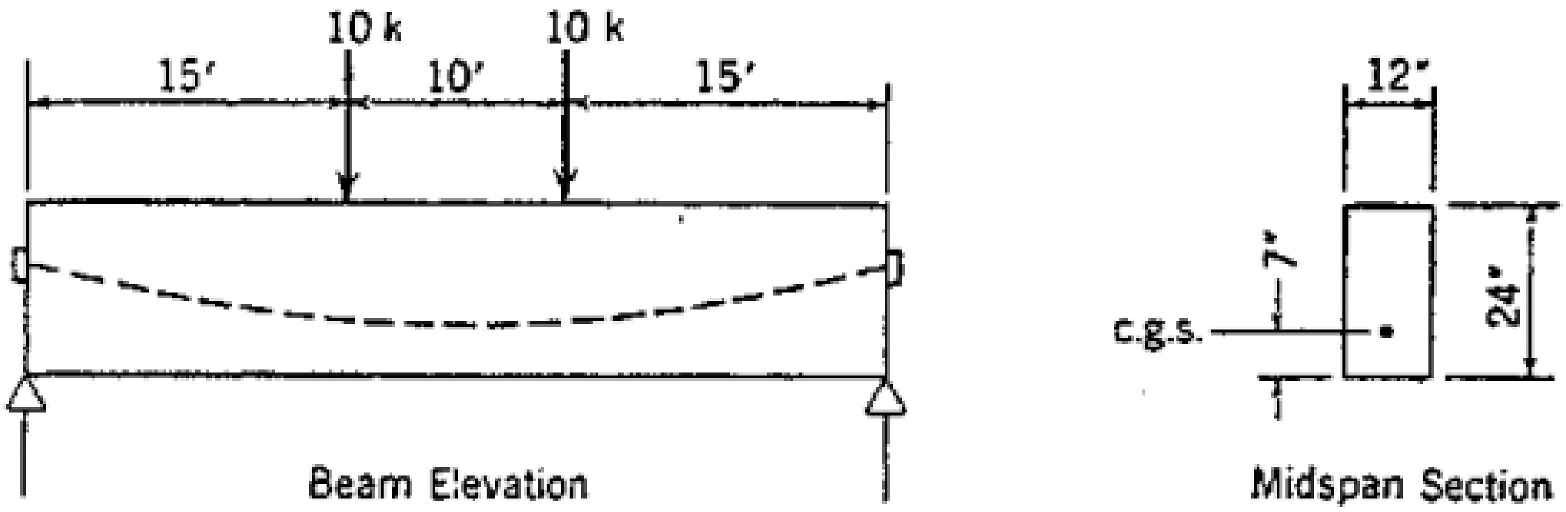

Fig. 5-8. Example 5-4.

STRESSES IN CONCRETE (DUE TO LOADS)

Example 5.4 (from T.Y.Lin)

Solution To be theoretically exact, the net concrete section should be used up to the time of grouting, after which the transformed section should be considered. This is not deemed necessary, and an approximate but sufficiently exact solution is given below, using the gross section of concrete at all times that is,

$$I = 12 \times 24^3/12 = 13{,}800 \text{ in.}^4 \ (5744 \times 10^6 \text{ mm}^4)$$

1. *Initial condition*: Dead-load moment at midspan, assuming that the beam is simply supported after prestressing:

$$M = \frac{wL^2}{8} = \frac{300 \times 40^2}{8} = 60{,}000 \text{ ft-lb } (81{,}360 \text{ N}-\text{m})$$

$$f = \frac{F}{A} \pm \frac{Fey}{I} \pm \frac{My}{I}$$

$$= \frac{-350{,}000}{288} \pm \frac{350{,}000 \times 5 \times 12}{13{,}800} \pm \frac{60{,}000 \times 12 \times 12}{13{,}800}$$

$$= -1215 + 1520 - 625 = -320 \text{ psi } (-2.21 \text{ N/mm}^2), \text{ top fiber}$$

$$= -1215 - 1520 + 625 = -2110 \text{ psi } (-14.55 \text{ N/mm}^2), \text{ bottom fiber}$$

STRESSES IN CONCRETE (DUE TO LOADS)

Example 5.4 (from T.Y.Lin)

2. *Final condition.* Live-load moment at midspan = 150,000 ft-lb (203,400 N-m); therefore, total external moment = 210,000 ft-lb (284,760 N-m), while the prestress is reduced to 300,000 lb (1,334 kN); hence,

$$f = \frac{-300{,}000}{288} \pm \frac{300{,}000 \times 5 \times 12}{13{,}800} \pm \frac{210{,}000 \times 12 \times 12}{13{,}800}$$

$$= -1040 + 1300 - 2190 = -1930 \text{ psi } (-13.31 \text{ N/mm}^2), \text{ top fiber}$$

$$= -1040 - 1300 + 2190 = -150 \text{ psi } (-1.03 \text{ N/mm}^2), \text{ bottom fiber}$$

STRESSES IN CONCRETE (DUE TO LOADS)

Example 5.5 (from T.Y.Lin)

For the same problem as in example 5-4, compute the concrete stresses under the final loading conditions by locating the center of pressure *C* for the concrete section.

A posttensioned bonded concrete beam, Fig. 5-8, has a prestress of 350 kips (1,557 kN) in the steel immediately after prestressing, which eventually reduces to 300 kips (1334 kN) due to losses. The beam carries two live loads of 10 kips (44.48 kN) each in addition to its own weight of 300 plf (4.377 kN/m). Compute the extreme fiber stresses at midspan, (*a*) under the initial condition with full prestress and no live load, and (*b*) under the final condition, after the losses have taken place, and with full live load.

It is internal resisting couple concept

STRESSES IN CONCRETE (DUE TO LOADS)

Example 5.5 (from T.Y.Lin)

Solution Referring to Fig. 5-10, a is computed by

$$a = (210 \times 12)/300 = 8.4 \text{ in. } (213 \text{ mm})$$

Hence e for C is $8.4 - 5 = 3.4$ in. Since $C = F = 300{,}000$ lb (1,334 kN).

$$f = \frac{C}{A} \pm \frac{Cey}{I}$$

$$= \frac{-300{,}000}{288} \pm \frac{300{,}000 \times 3.4 \times 12}{13{,}800}$$

$$= -1040 - 890 = -1930 \text{ psi } (-13.31 \text{ N/mm}^2), \text{ top fiber}$$

$$= -1040 + 890 = -150 \text{ psi } (-1.03 \text{ N/mm}^2), \text{ bottom fiber}$$

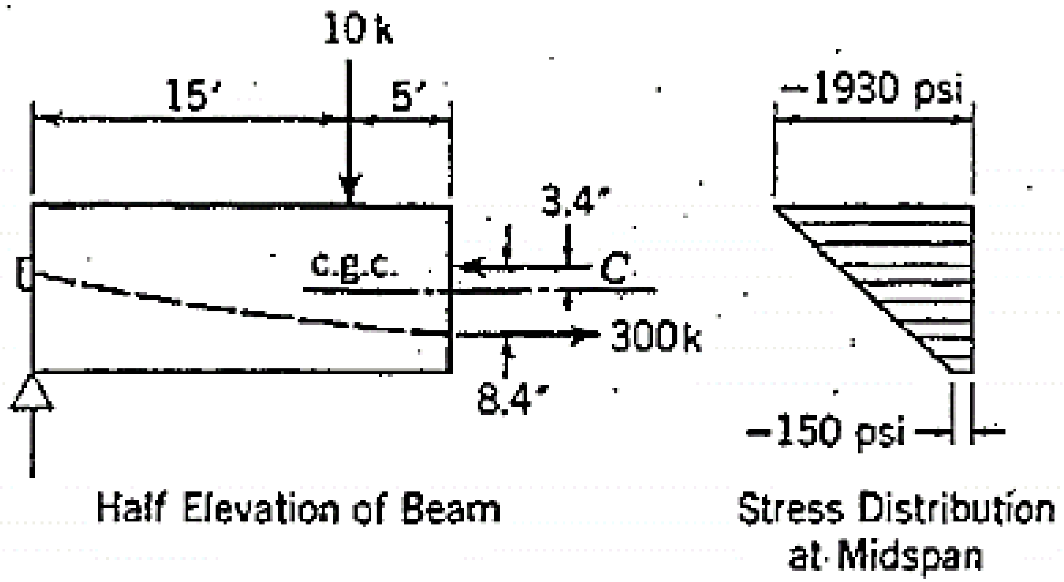

Fig. 5-10. Example 5-5.

STRESSES IN STEEL (DUE TO LOADS)

In order to get a clear understanding of the behavior of a prestressed-concrete beam, it will be interesting to first study the variation of steel stress as the load increases. For the midspan section of a simple beam, the variation of steel stress with load on the beam is shown in Fig. 5-11. Along the *X*-axis is plotted the load on the beam, and along the *Y*-axis is plotted the stress in the steel. As prestress is applied to the steel, the stress in the steel changes from *A* to *B*, where *B* is at the level of f_0, which is the initial prestress in the steel after losses due to anchorage and elastic shortening have taken place.

STRESSES IN STEEL (DUE TO LOADS)

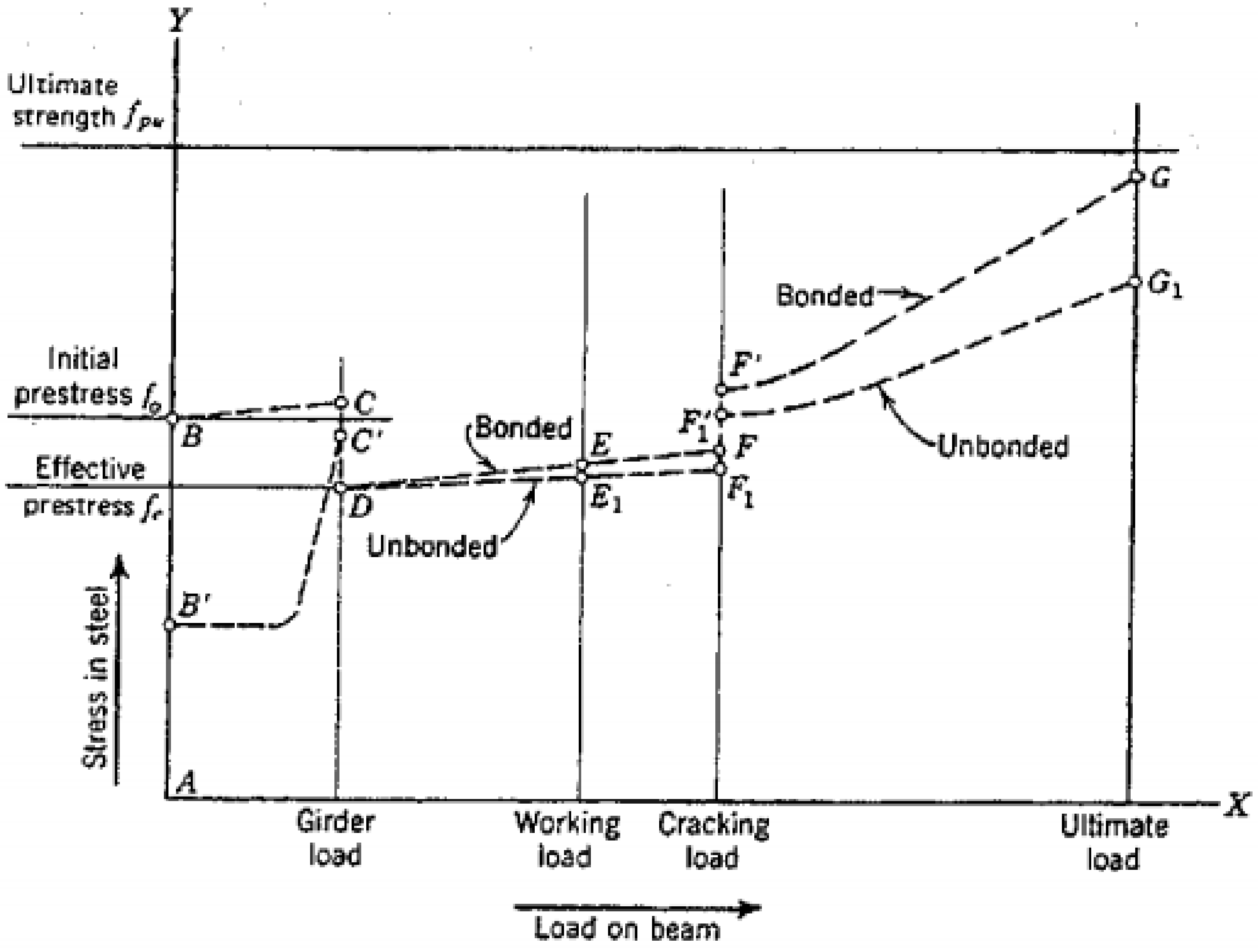

Fig. 5-11. Variation of steel stress with load.

STRESSES IN STEEL (DUE TO LOADS)

Immediately after transfer, no load will yet be carried by the beam if it is supported on its falsework and if it is not cambered upward by the prestress. As the falsework is removed, the beam carries its own weight and deflects downward slightly, thus changing the stress in the steel, increasing it from B to C. When the dead weight of the beam is relatively light, then it can be bowed upward during the course of the transfer of prestress. The beam may actually begin to carry load when the average prestress in the steel is somewhere at B'. There may be a sudden breakaway of the beam's soffit from the falsework so that the weight of the beam is at once transferred to be carried by the beam itself, or the weight may be transferred gradually, depending on the actual conditions of support. But, in any event, the stress in steel will increase from B' up to point C'. The stress at C' is slightly lower than f_0 by virtue of the loss of prestress in the steel as caused by the upward bending of the beam. Consider now that the losses of prestress take place so that the stress in the steel drops from C or C' to some point D, representing the effective prestress f_e for the beam. Actually, the losses will not take place all at once but will continue for some length of time. However, for convenience in discussion, let us assume that all the losses take place before the application of superimposed dead and live loads.

STRESSES IN STEEL (DUE TO LOADS)

At the section of maximum moment, the stress in an unbonded tendon will increase more slowly than that in a bonded tendon. This is because any strain in an unbonded tendon will be distributed throughout its entire length. Hence, as the load is increased to the working or the cracking load, the steel stress will increase from D to E_1, F_1, and F_1', below E, F, and F', respectively, Fig. 5-11. To compute the average strain for the cable, it is necessary to determine the total lengthening of the tendon due to moments in the beam. This can be done by integrating the strain along the entire length. Let M be the moment at any point of an unbonded beam; the unit strain in concrete at any point is given by

$$\delta = \frac{f}{E} = \frac{My}{E_c I}$$

STRESSES IN STEEL (DUE TO LOADS)

The total strain along the cable is then

$$\Delta = \int \delta\, dx = \int \frac{My}{E_c I} dx$$

The average strain is

$$\frac{\Delta}{L} = \int \frac{My}{LE_c I} dx$$

The average stress is

$$f_s = E_s \frac{\Delta}{L} = \int \frac{MyE_s}{LE_c I} dx = \frac{n}{L} \int \frac{My}{I} dx \tag{5-10}$$

If y and I are constant and M is an integrable form of x, the solution of this integral is simple. Otherwise, it will be easier to use a graphical or an approximate integration.

STRESSES IN STEEL (DUE TO LOADS)

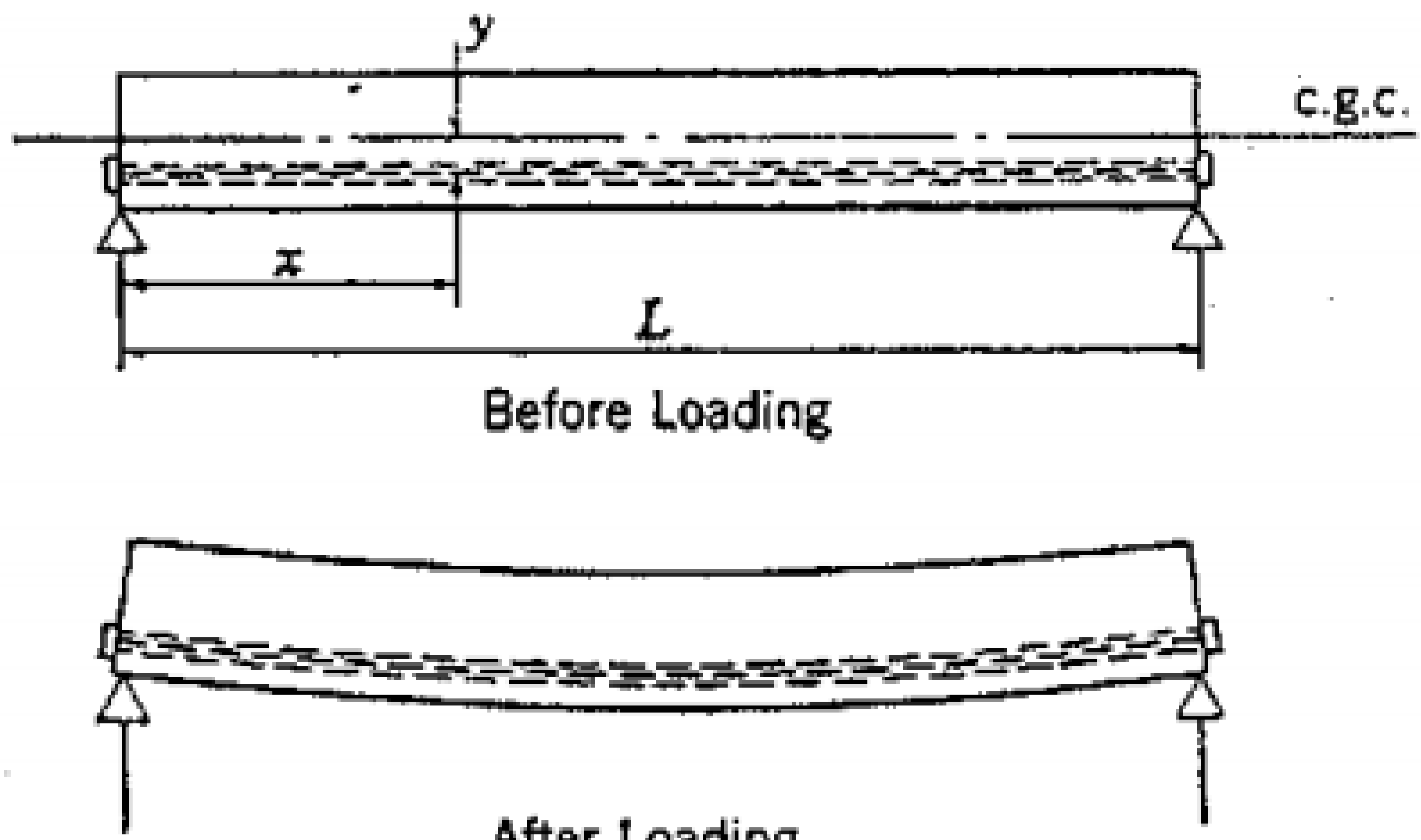

Fig. 5-12. Change of cable length in an unbonded beam.

STRESSES IN STEEL (DUE TO LOADS)

After cracks have developed in the unbonded beam, stress in the steel increases more rapidly with the load, but again it does not increase as fast as that at the maximum moment section in a similar bonded beam. In an unbonded beam, it is generally not possible to develop the ultimate strength of the steel at the rupture of the beam. Thus the stress curve is shown going up from F_1' to G_1, with G_1 below G by an appreciable amount. It is evident that the ultimate load for an unbonded beam is less than that for a corresponding bonded one, although there may be very little difference between the cracking loads for the two beams. There is a tendency for the unbonded beams to develop large cracks before rupture. These large cracks tend to concentrate strains at some localized sections in the concrete, thus lowering ultimate strength. Therefore, the strength of unbonded beams may be appreciably increased by the addition of nonprestressed bonded reinforcements, which tend to spread the cracks and to limit their size, as well as to contribute toward the tensile force in the ultimate resisting couple. The ACI Code specifies minimum amounts of such supplemental bonded reinforcement.

STRESSES IN STEEL (DUE TO LOADS)

Example 5.6 (from T.Y.Lin)

A posttensioned simple beam on a span of 40 ft is shown in Fig. 5-13. It carries a superimposed load of 750 plf in addition to its own weight of 300 plf. The initial prestress in the steel is 138,000 psi, reducing to 120,000 psi after deducting all losses and assuming no bending of the beam. The parabolic cable has an area of 2.5 sq in., $n = 6$. Compute the stress in the steel at midspan, assuming: (1) the steel is bonded by grouting: (2) the steel is unbonded and entirely free to slip. (Span = 12.2 m, superimposed load = 10.94 kN/m, self-weight = 4377 kN/m, initial prestress = 951.5 N/mm^2, effective prestress = 827.4 N/mm^2, and cable area = 1613 mm^2.)

Solution

1. Moment at midspan due to dead and live loads is

$$\frac{wL^2}{8} = \frac{(300+750)40^2}{8}$$

$$= +210{,}000 \text{ ft-lb } (+284{,}760 \text{ N-m})$$

STRESSES IN STEEL (DUE TO LOADS)

<u>Example 5.6</u> (from T.Y.Lin)

Moment at midspan due to prestress is

$$2.5 \times 120{,}000 \times \tfrac{5}{12} = -125{,}000 \text{ ft-lb } (-169{,}500 \text{ N-m})$$

Net moment at midspan is $210{,}000 - 125{,}000 = 85{,}000$ ft-lb ($115{,}260$ N$-$m). Stress in concrete at the level of steel due to bending, using I of gross concrete section, is

$$= \frac{My}{I} = \frac{85{,}000 \times 12 \times 5}{13{,}800} = 370 \text{ psi } (2.55 \text{ N/mm}^2)$$

Stress in steel is thus increased by

$$f_s = nf_c = 6 \times 370 = 2220 \text{ psi } (15.31 \text{ N/mm}^2)$$

Resultant stress in steel $= 122{,}220$ psi (842.7 N/mm^2) at midspan.

STRESSES IN STEEL (DUE TO LOADS)

Example 5.6 (from T.Y.Lin)

Solution

2. If the cable is unbonded and free to slip, the average strain or stress must be obtained for the whole length of cable as given by formula 5-10,

$$f_s = \frac{n}{L} \int \frac{My}{I} dx$$

Using y_0 and M_0 for those at midspan and measuring x from the midspan, we can express y and M in terms of x, thus,

$$M = M_0\left[1 - \left(\frac{x}{L/2}\right)^2\right]$$

$$y = y_0\left[1 - \left(\frac{x}{L/2}\right)^2\right]$$

STRESSES IN STEEL (DUE TO LOADS)

Example 5.6 (from T.Y.Lin)

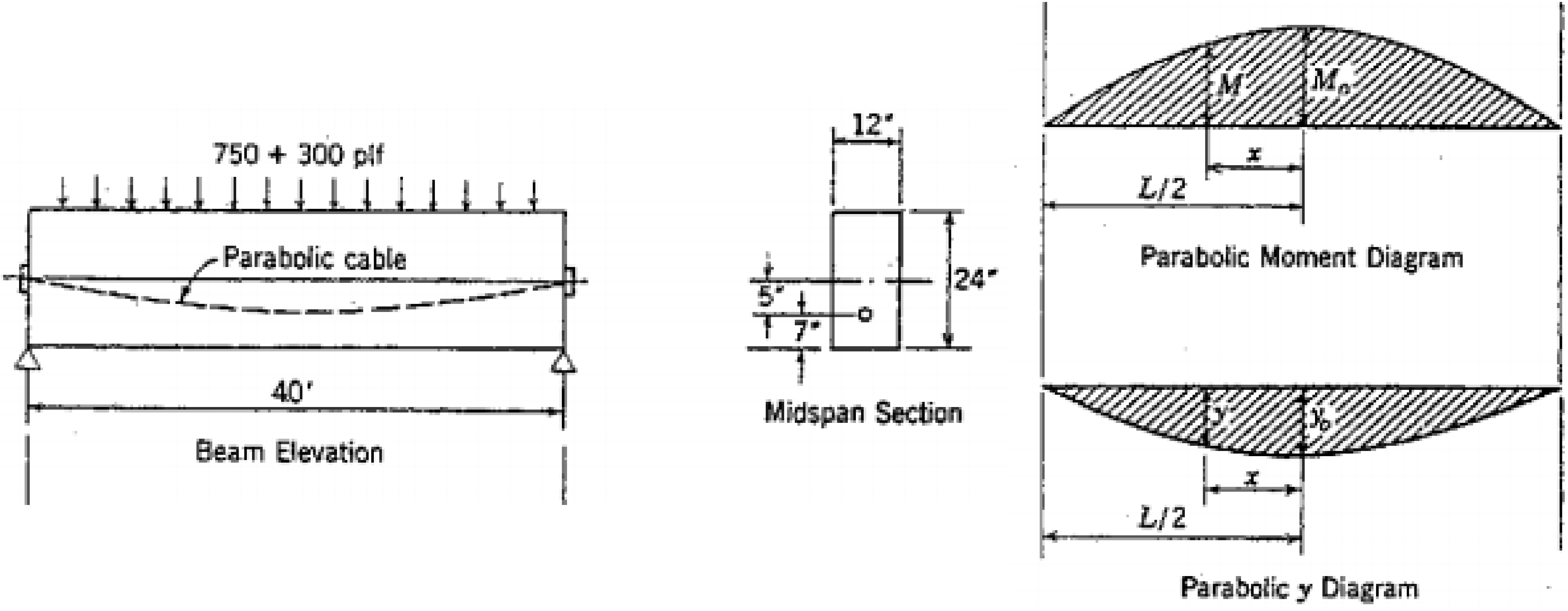

Fig. 5-13. Example 5-6.

STRESSES IN STEEL (DUE TO LOADS)

Example 5.6 (from T.Y.Lin)

$$f_s = \frac{n}{LI}\int_{-L/2}^{+L/2} M_0 y_0 \left[1-\left(\frac{x}{L/2}\right)^2\right]^2 dx$$

$$= \frac{nM_0y_0}{LI}\left[x - \frac{2}{3}\frac{x^3}{(L/2)^2} + \frac{x^5}{5(L/2)^4}\right]_{-L/2}^{+L/2}$$

$$= \frac{8}{15}\left(\frac{nM_0y_0}{I}\right)$$

which is $\frac{8}{15}$ of the stress for midspan of the bonded beam, or $\frac{8}{15}(2220) = 1180$ psi (8.14 N/mm^2).

Resultant stress in steel is $120{,}000 + 1180 = 121{,}180$ psi (835.5 N/mm^2) throughout the entire cable. In this calculation, the I of the gross concrete section is used and the effect of the increase in the steel stress on the concrete stresses is also neglected. But these are errors of the second order. Since the change in steel stress is relatively small, exact computations are seldom required in an actual design problem.

CRACKING MOMENT

Cracking Moment

The moment producing first hair cracks in a prestressed concrete beam is computed by the elastic theory, assuming that cracking starts when the tensile stress in the extreme fiber of concrete reaches its modulus of rupture. Questions have been raised as to the correctness of this method. First, some engineers believed that concrete under prestress became a complex substance whose behavior could not be predicted by the elastic theory with any accuracy.[1] Then it was further questioned whether the usual bending test for modulus of rupture could give values to represent the tensile strength of concrete in a prestressed beam. However, most available test data seem to indicate that the elastic theory is sufficiently accurate up to the point of cracking, and the method is currently used. The ACI Code value for modulus of rupture, f_r, is $7.5\sqrt{f_c'}$ with units for both f_r and f_c' as psi.

CRACKING MOMENT

Attention must be paid to the fact that the modulus of rupture is only a measure of the beginning of hair cracks which are often invisible to the naked eye. A tensile stress higher than the modulus is necessary to produce visible cracks. On the other hand, if the concrete has been previously cracked by overloading, shrinkage, or other causes, cracks may reappear at the slightest tensile stress. If the beam is made of concrete blocks, the cracking strength will depend on the tensile strength of the joining material.

Referring to formula 5-7, if f_r is the modulus of rupture, it is seen that, when

$$-\frac{F}{A}-\frac{Fec}{I}+\frac{Mc}{I}=f_r$$

cracks are supposed to start. Transposing terms, we have the value of cracking moment given by

$$M=Fe+\frac{FI}{Ac}+\frac{f_r I}{c} \tag{5-11}$$

where $f_r I/c$ gives the resisting moment due to modulus of rupture of concrete, Fe the resisting moment due to the eccentricity of prestress, and FI/Ac that due to the direct compression of the prestress.

CRACKING MOMENT

Formula 5-11 can be derived from another approach. When the center of pressure in the concrete is at the top kern point, there will be zero stress in the bottom fiber. The resisting moment is given by the prestress F times its lever arm measured to the top kern point, (see Appendix A for definition of kern points k_t and k_b), Fig. 5-14, thus,

$$M_1 = F\left(e + \frac{r^2}{c}\right)$$

Additional moment resisted by the concrete up to its modulus of rupture is $M_2 = f_r I/c$. Hence the total moment at cracking is given by

$$M = M_1 + M_2 = F\left(e + \frac{r^2}{c}\right) + \frac{f_r I}{c} \qquad (5\text{-}12)$$

which can be seen to be identical with formula 5-11.

In order to be theoretically correct when applying the above two formulas, care must be exercised in choosing the proper section for the computation of I, r, e, and c. For computing the term $f_r I/c$, the transformed section should be used for bonded beams, while the net concrete section should be used for unbonded beams (proper modification being made for the value of prestress due

CRACKING MOMENT

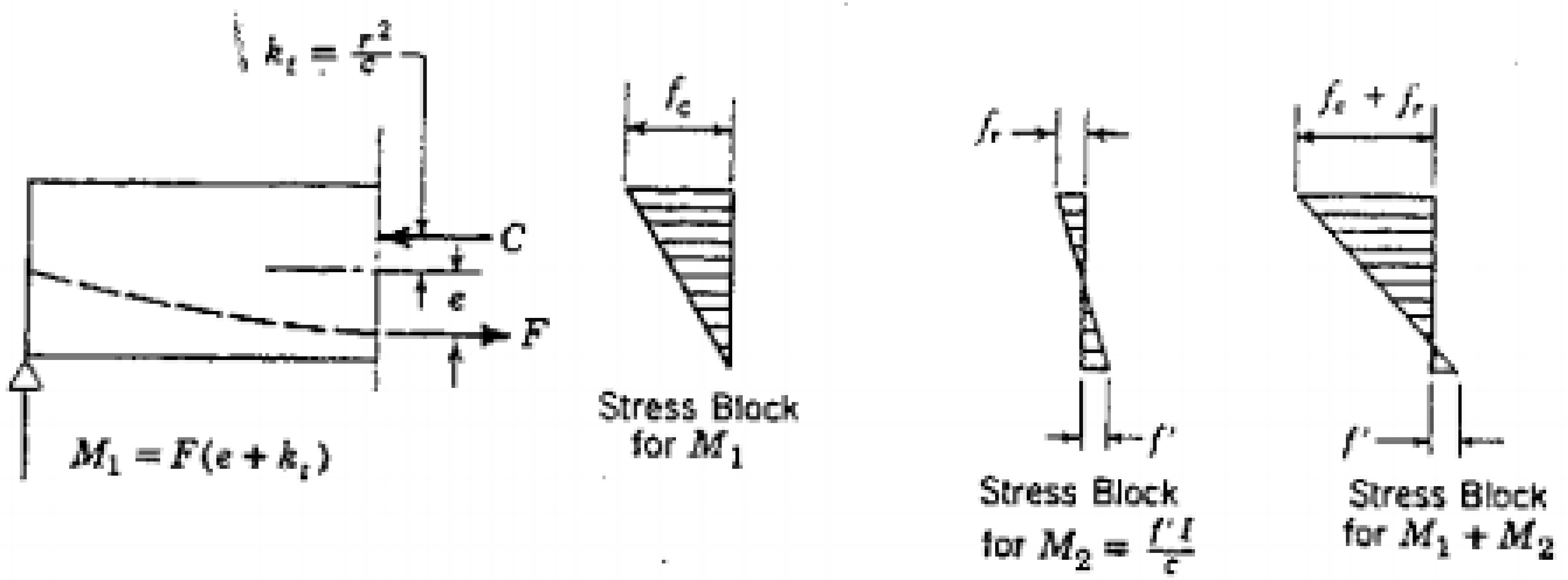

Fig. 5-14. Cracking moment.

CRACKING MOMENT

to bending of the beam as explained in section 4-8). For the term $F\left(e+\frac{r^2}{c}\right)$, either the gross or the net section should be considered, depending on the computation of the effective prestress F. For a practical problem, these refinements are often unnecessary, and it will be easier to use one section for all the computations. In order to simplify the computations, the gross section of the concrete is most often used. If the area of holes is an important portion of the gross area, then net area may be used. If the percentage of steel is high, the transformed area may be preferred. The engineer must use discretion in choosing a method of solution consistent with the degree of accuracy required for his particular problem.

CRACKING MOMENT

Example 5.7 **(from T.Y.Lin)**

For the problem given in example 5-6, compute the total dead and live uniform load that can be carried by the beam, (1) for zero tensile stress in the bottom fibers, (2) for cracking in the bottom fibers at a modulus of rupture of 600 psi (4.14 N/mm^2), and assuming concrete to take tension up to that value.

Solution

1. Considering the critical midspan section and using the gross concrete section for all computations, k_t is readily computed to be at 4 in. (101.6 mm) above the middepth, Fig. 5-15. To obtain zero stress in the bottom fibers, the center of pressure must be located at the top kern point. Hence the resisting moment is given by the prestress multiplied by the lever arm, thus

$$F(e+k_t)=300(5+4)/12=225 \text{ k-ft } (305.1 \text{ kN-m})$$

CRACKING MOMENT

<u>Example 5.7</u> **(from T.Y.Lin)**

Solution .

2. Additional moment carried by the section up to beginning of cracks is

$$\frac{f_r I}{c} = \frac{600 \times 13{,}800}{12}$$

$$= 690{,}000 \text{ in.-lb}$$

$$= 57.6 \text{ k-ft } (78.1 \text{ kN} - \text{m})$$

Total moment at cracking is $225 + 57.6 = 282.6$ k-ft (383.2 kN-m), which can also be obtained directly by applying formula 5-11 or 5-12.

CRACKING MOMENT

Example 5.7 **(from T.Y.Lin)**

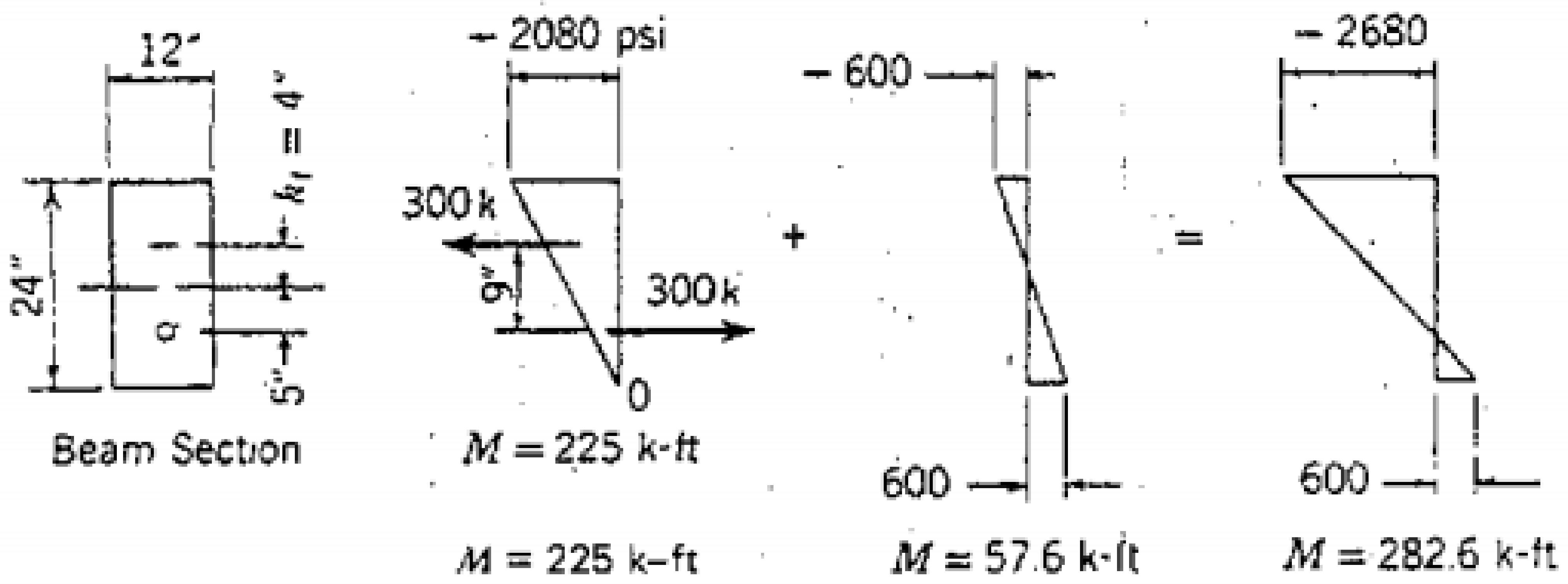

Fig. 5-15. Example 5-7.

CRACKING MOMENT

Example 5.7 **(from T.Y.Lin)**

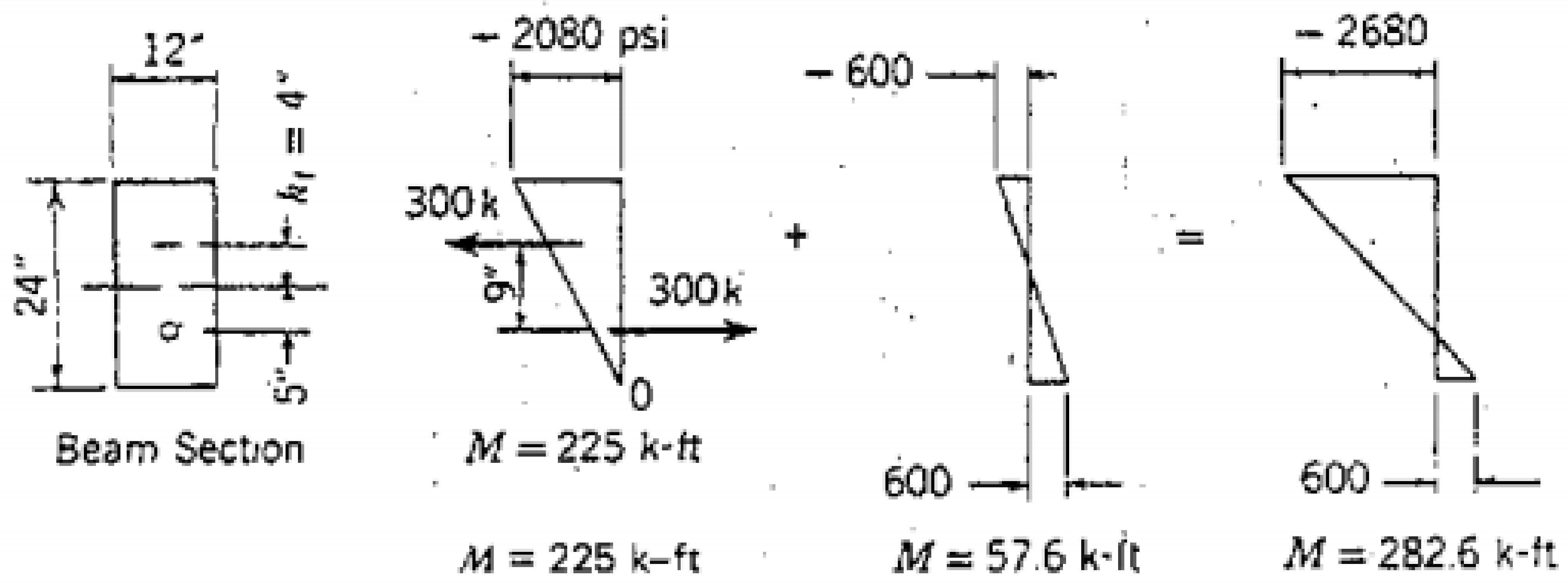

Fig. 5-15. Example 5-7.

ULTIMATE MOMENT (BONDED TENDON)

Ultimate Moment—Bonded Tendons

Exact analysis for the ultimate strength of a prestressed-concrete section under flexure is a complicated theoretical problem, because both steel and concrete are generally stressed beyond their elastic range. The following section develops such an analysis technique for bonded beams. However, for the purpose of practical design, where an accuracy of 5-10% is considered sufficient, relatively simple procedures can be developed.

Many tests have been run, and many papers written, on the ultimate flexural strength of prestressed concrete sections. Worthy of special mention are the group of papers on this thesis[2] presented before the First International Congress on Prestressed Concrete held in London, October 1953, and another summary paper presented at the Third Congress of the International Federation for Prestressing.[3] In the United States, laboratory investigations carried out at the University of Illinois and the Portland Cement Association gave the results of extensive tests, together with definite recommendations.[4,5,6] Although formulas for ultimate strength proposed by various authors seem to differ greatly on the surface, they generally yield values within a few per cent of one another. Hence it can be concluded that the ultimate strength of prestressed concrete under flexure can be predicted with sufficient accuracy.

ULTIMATE MOMENT (BONDED TENDON)

A simple method for determining ultimate flexural strength following the ACI Code is presented herewith, based on the results of the aforementioned tests as well as others. This method is limited to the following conditions.

1. The failure is primarily a flexural failure, without shear, bond, or anchorage failure which might decrease the strength of the section.
2. The beams are bonded. Unbonded beams possess different ultimate strength and are discussed later.
3. The beams are statically determinate. Although the discussions apply equally well to individual sections of continuous beams, the ultimate strength of continuous beams as a whole is explained by the plastic hinge theory to be discussed in Chapter 10.
4. The load considered is the ultimate load obtained as the result of a short static test. Impact, fatigue, or long-time loadings are not considered.

ULTIMATE MOMENT (BONDED TENDON)

Of the methods proposed for determining the ultimate flexural strength of prestressed-concrete sections, some are purely empirical and others highly theoretical. The empirical methods are generally simple but are limited only to the conditions which were encountered in the tests. The theoretical ones are intended for research studies and hence unnecessarily complicated for the designer. For the purpose of design, a rational approach is presented in the following, consistent with test results, but neglecting refinements so that reasonably correct values can be obtained with the minimum amount of effort. The method is based on the simple principle of a resisting couple in a prestressed beam, as that in any other beam. At the ultimate load, the couple is made of two forces, T' and C', acting with a lever arm a'. The steel supplies the tensile force T', and the concrete, the compressive force C'.


Any
questions