CE 414: Prestressed Concrete
Lecture 5
Prestress loss (contd.)

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- Loss due to bending
- Wobble and curvature coefficient
- Theory of friction loss in prestress
- Frictional loss calculation

4-9 Loss or Gain Due to Bending of Member

Loss of prestress due to a uniform shortening of the member under axial stress was discussed in section 4-3. When a member bends, further changes in the prestress may occur: there may be either a loss or a gain in prestress, depending on the direction of bending and the location of the tendon. If there are several tendons and they are placed at different levels, the change of prestress in them will differ. Then it may be convenient to consider only the centroid of all the tendons (the c.g.s. line) to get an average value of the change in prestress.

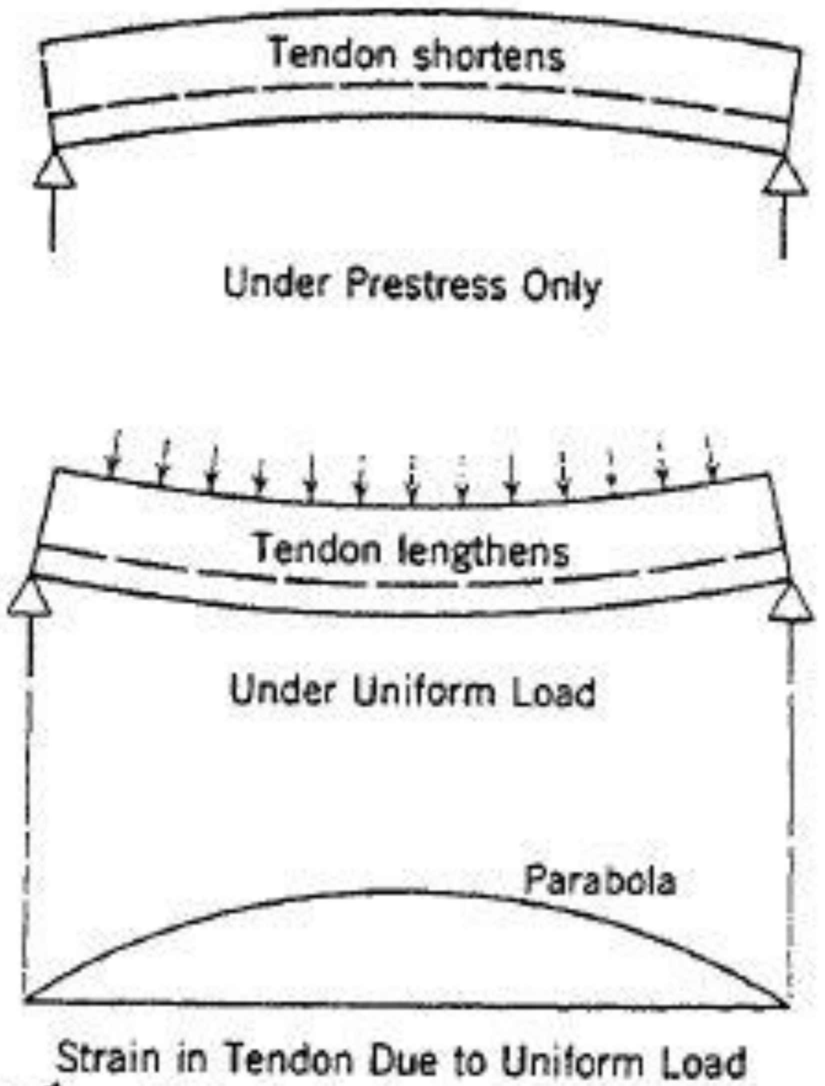

Fig. 4-3. Variation of strain in bonded tendon.

EXAMPLE 4-3

A concrete beam 8 in. by 18 in. deep is prestressed with an unbonded tendon through the lower third point, Fig. 4-5, with a total initial prestress of 144,000 lb. Compute the loss of

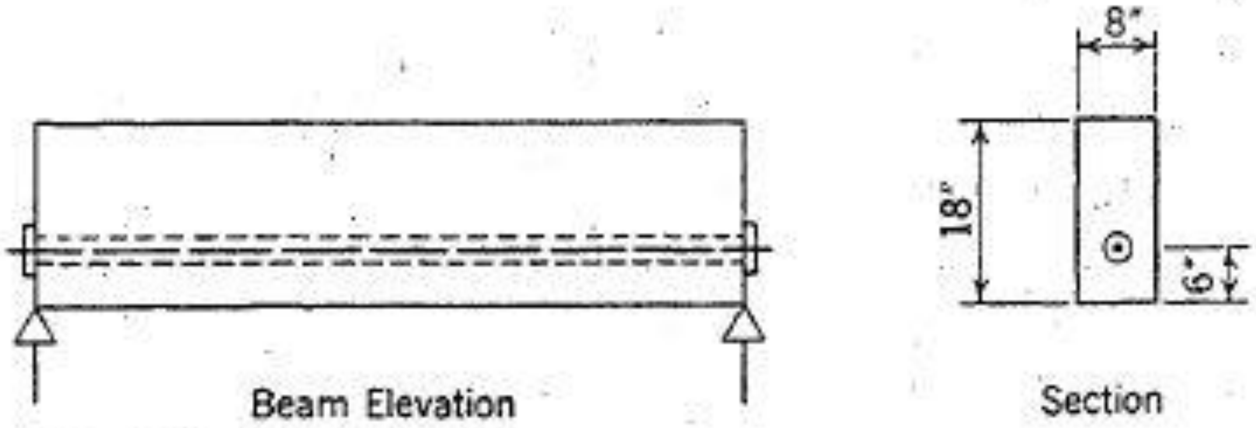

Fig. 4-5. Example 4-3.

prestress in the tendon due to the bowing up of the beam under prestress, neglecting the weight of the beam itself. $E_s = 30{,}000{,}000$ $E_c = 4{,}000{,}000$ psi. Beam is simply supported ($F_0 = 640.5$ KN, $E_s = 207$ KN/mm^2, and $E_c = 27.6$ KN/mm^2).

Solution Owing to the eccentric prestress, the beam is under a uniform bending moment of

$$144{,}000 \times 3 \text{ in.} = 432{,}000 \text{ in.-lb } (48{,}816 \text{ N-m})$$

The concrete fiber stress at the level of the cable due to this bending is

$$f = \frac{My}{I} = \frac{432{,}000 \times 3}{(8 \times 18^3)/12} = 333 \text{ psi } (2.30 \text{ N/mm}^2) \text{ compression}$$

(Note that stress due to the axial prestress of 144,000 lb is not included here; also, the gross area of the concrete is used for simplicity.)

Unit compressive strain along the level of the tendon is therefore

$$333/4{,}000{,}000 = 0.000083$$

Corresponding loss of prestress in steel is

$$0.000083 \times 30{,}000{,}000 = 2500 \text{ psi } (17.2 \text{ N/mm}^2)$$

More serious frictional loss occurs between the tendon and its surrounding material, whether concrete or sheathing, and whether lubricated or not. This frictional loss can be conveniently considered in two parts: the length effect and the curvature effect. The length effect is the amount of friction that would be encountered if the tendon is a straight one, that is, one that is not purposely bent or curved. Since in practice the duct for the tendon cannot be perfectly straight, some friction will exist between the tendon and its surrounding material even though the tendon is meant to be straight. This is sometimes described as the wobbling effect of the duct and is dependent on the length and stress of the tendon, the coefficient of friction between the contact materials, and the workmanship and method used in aligning and obtaining the duct. Some approximate values for coefficients used to compute these losses are given in the Commentary of the ACI Code, Table 4-7.

The loss of prestress due to curvature effect results from the intended curvature of the tendons in addition to the unintended wobble of the duct. This loss is again dependent on the coefficient of friction between the contact

Table 4-7 Friction Coefficients for Posttensioning Tendons*

Type of Tendon	Wobble Coefficient K per Foot	Curvature Coefficient μ
Tendons in flexible metal sheathing		
Wire tendons	0.0010–0.0015	0.15–0.25
7-wire strand	0.0005–0.0020	0.15–0.25
High strength bars	0.0001–0.0006	0.08–0.30
Tendons in rigid metal duct		
7-wire strand	0.0002	0.15–0.25
Pregreased tendons		
Wire tendons and 7-wire strand	0.0003–0.0020	0.05–0.15
Mastic-coated tendons		
Wire tendons and 7-wire strand	0.0010–0.0020	0.05–0.15

*ACI Code Commentary.

4-11 Frictional Loss, Theoretical Considerations

The basic theory of frictional loss of a cable around a curve is well known in physics. In its simple form, it can be derived as follows. Consider an infinitesimal length dx of a prestressing tendon whose centroid follows the arc of a circle of radius R, Fig. 4-7, then the change in angle of the tendon as it goes around that length dx is

$$d\alpha = \frac{dx}{R}$$

For this infinitesimal length dx, the stress in the tendon may be considered constant and equal to F; then the normal component of pressure produced by the stress F bending around an angle $d\alpha$ is given by

$$N = F\,d\alpha = \frac{F\,dx}{R}$$

The amount of frictional loss dF around the length dx is given by the pressure times a coefficient of friction μ, thus,

$$dF = -\mu N$$

$$= \frac{-\mu F dx}{R} = -\mu F d\alpha$$

Transposing, we have

$$\frac{dF}{F} = -\mu d\alpha$$

Integrating this on both sides, we have

$$\log_e F = -\mu\alpha$$

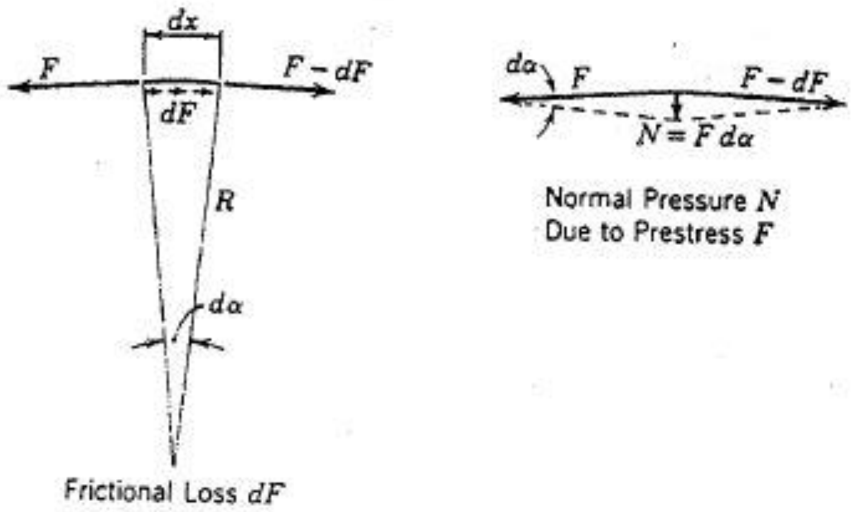

Fig. 4-7. Frictional loss along length dx.

Using the limits F_1 and F_2, we have the conventional friction formula

$$F_2 = F_1 e^{-\mu\alpha} = F_1 e^{-\mu L/R} \tag{4-15}$$

since $\alpha = L/R$ for a section of constant R.

For tendons with a succession of curves of varying radii, it is necessary to apply this formula to the different sections in order to obtain the total loss.

The above formula can also be applied to compute frictional loss due to wobble or length effect. Substituting the loss KL for $\mu\alpha$ in formula 4-15, we have

$$\log_e F = -KL \qquad F_2 = F_1 e^{-KL} \tag{4-16}$$

If it is intended to combine the length and curvature effect, we can simply write

$$\log_e F = -\mu\alpha - KL$$

For limits F_1 and F_2,

$$F_2 = F_1 e^{-\mu\alpha - KL} \tag{4-17}$$

Or, in terms of unit stresses,

$$f_2 = f_1 e^{-\mu\alpha - KL} \tag{4-17a}$$

The friction loss is obtained from this expression. Loss of steel stress is given as $FR = f_1 - f_2$, the steel stress at the jacking end is f_1, and the length to the point is L, Fig. 4-8(a). Thus, we find

$$FR = f_1 - f_2 = f_1 - f_1 e^{-\mu\alpha - KL} = f_1(1 - e^{-\mu\alpha - KL}) \tag{4-18}$$

exponential form. If the normal pressure is assumed to be constant, the total frictional loss around a curve with angle α and length L is, Fig. 4-8.

$$F_2 - F_1 = -\mu F_1 \alpha = -\frac{\mu F_1 L}{R} \tag{4-19}$$

For length or wobble effect, we can again substitute KL for $\mu\alpha$ thus,

$$F_2 - F_1 = -KLF_1 \tag{4-20}$$

To compute the total loss due to both curvature and length effect, the above two formulas can be combined, giving

$$F_2 - F_1 = -KLF_1 - \mu F_1 \alpha = -F_1(KL + \mu\alpha) \tag{4-21}$$

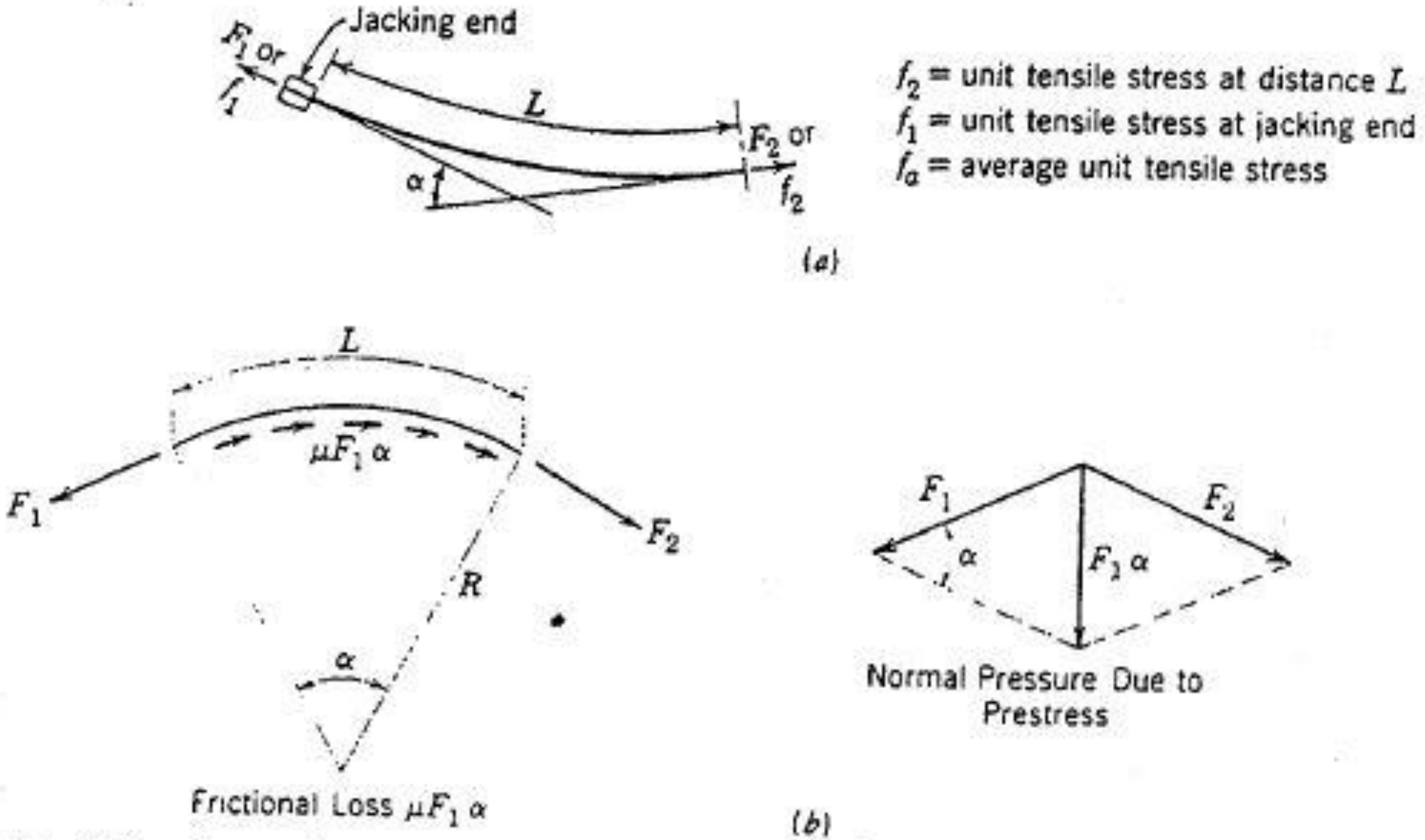

Fig. 4-8. Approximate frictional loss along circular curve.

Transposing terms, we have

$$\frac{F_2 - F_1}{F_1} = -KL - \mu\alpha = -\left(K + \frac{\mu}{R}\right)L \tag{4-22}$$

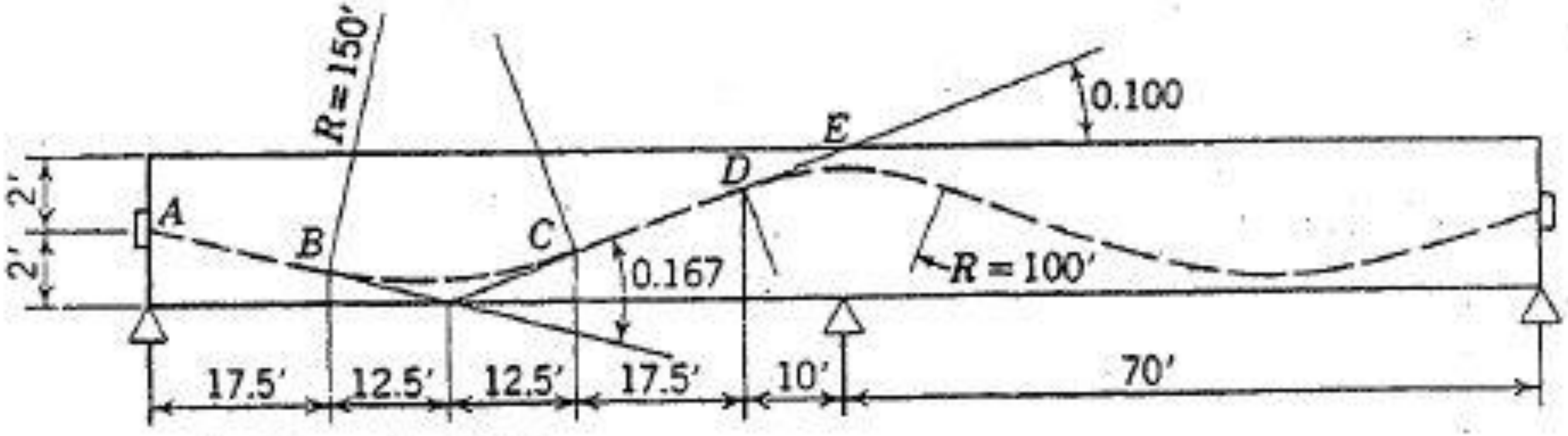

Fig. 4-10. Example 4-4.

EXAMPLE 4-4

A prestressed-concrete beam is continuous over two spans, Fig. 4-10, and its curved tendon is to be tensioned from both ends. Compute the percentage loss of prestress due to friction, from one end to the center of the beam (*A* to *E*). The coefficient of friction between the cable and the duct is taken as 0.4, and the average "wobble" or length effect is represented by $K = 0.0008$ per ft.

Solution

1. A simple approximate solution will first be presented. Using formula 4-22,

$$\frac{F_2 - F_1}{F_1} = -KL - \mu\alpha$$

$$= -0.0008 \times 70 - 0.4(0.167 + 0.100)$$

$$= -0.056 - 0.107$$

$$= -0.163$$

Solution

2. The above solution does not take into account the gradual reduction of stress from A toward E. A more exact solution would be to divide the tendon into four portions from A to E, and consider each portion after the loss has been deducted from the preceding portions. Thus, for stress at $A = F_1$,

$$AB\text{, length effect: } KL = 0.0008 \times 17.5 = 0.014$$

$$\text{Stress at } B = 1 - 0.014 = 0.986F_1$$

$$BC\text{, length effect: } KL = 0.0008 \times 25 = 0.020$$

$$\text{Curvature effect: } \mu\alpha = 0.4 \times 0.167 = 0.067$$

$$\text{Total: } 0.020 + 0.067 = 0.087$$

Using the reduced stress at B of 0.986, the loss is $0.087 \times 0.986 = 0.086$.

$$\text{Stress at } C = 0.986 - 0.086 = 0.900F_1$$

$$CD\text{, length effect: } KL = 0.0008 \times 17.5 = 0.014$$

Using the reduced stress of 0.900 at C, the loss is $0.014 \times 0.900 = 0.013$.

$$\text{Stress at } D = 0.900 - 0.013 = 0.887F_1$$

$$DE\text{, length effect: } KL = 0.0008 \times 10 = 0.008$$

$$\text{Curvature effect: } \mu\alpha = 0.4 \times 0.100 = 0.040$$

$$\text{Total: } 0.008 + 0.040 = 0.048$$

$$\text{Loss} = 0.048 \times 0.887 = 0.043$$

$$\text{Stress at } E = 0.887 - 0.043 = 0.844F_1$$

$$\text{Total loss from } A \text{ to } E = 1 - 0.844 = 0.156 = 15.6\%$$

This computation can be tabulated in order to simplify the work. It can be further noticed that this second method yields a loss only slightly less than the first approximate method.

Solution

3. A still more exact solution is to use the conventional friction formula 4-18, which takes into account not only the variation of stress from segment to segment but also that from point to point all along the cable. The solution is tabulated as shown.

Segment	L	KL	α	$\mu\alpha$	$KL+\mu\alpha$	$e^{-KL-\mu\alpha}$	Stress at End of Segment
AB	17.5	0.014	0	0	0.014	0.986	$0.986F_1$
BC	25	0.020	0.167	0.067	0.087	0.916	$0.903F_1$
CD	17.5	0.014	0	0	0.014	0.986	$0.890F_1$
DE	10	0.008	0.100	0.040	0.048	0.953	$0.848F_1$

The total frictional loss from A to E is given as

$$1-0.848=0.152=15.2\%$$

process of prestressing. For average steel and concrete properties, cured under average air conditions, the tabulated percentages may be taken as representative of the average losses.

	Pretensioning, %	Posttensioning, %
Elastic shortening and bending of concrete	4	1
Creep of concrete	6	5
Shrinkage of concrete	7	6
Steel relaxation	8	8
Total loss	25	20