LECTURE-3

PROJECTILE MOTION

Two dimensional motion

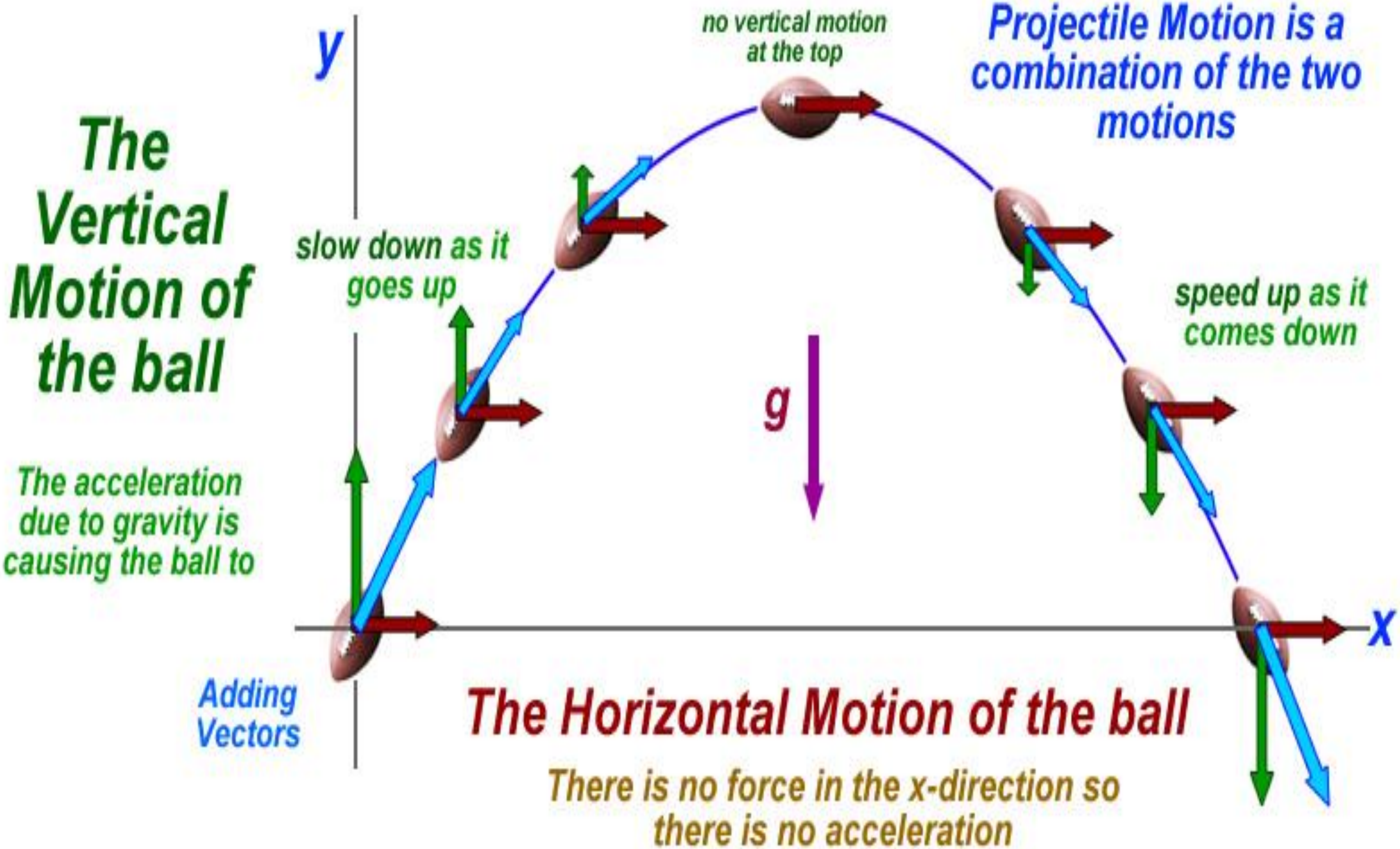
Projectile motion

When an object is thrown obliquely into space, it is called projectile and its motion is called projectile motion. A projectile moves through the space under the influence of gravitational force. Two co-ordinate must be used to describe the projectile motion, since it moves horizontally as well as vertically. The motion of a football, cricket ball, missile etc. is examples of projectile motion.

Everyday Examples of Projectile Motion

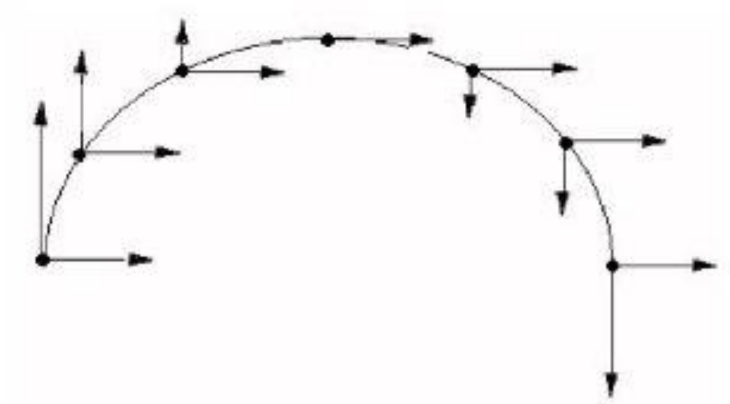
1. Baseball being thrown
2. Water fountains
3. Fireworks Displays
4. Soccer ball being kicked
5. Ballistics Testing

Projectile Motion - A Vector Perspective



Combining the Components

Together, these components produce what is called a **trajectory** or path. This path is **parabolic** in nature.



Component	Magnitude	Direction
Horizontal	Constant	Constant
Vertical	Changes	Changes

Some definitions relating to projectile motion

Velocity of projection: The initial velocity at which an object is thrown upward is called velocity of projection.

Angle of projection: The angle between the velocity of projection and the horizontal plane is called angle of projection.

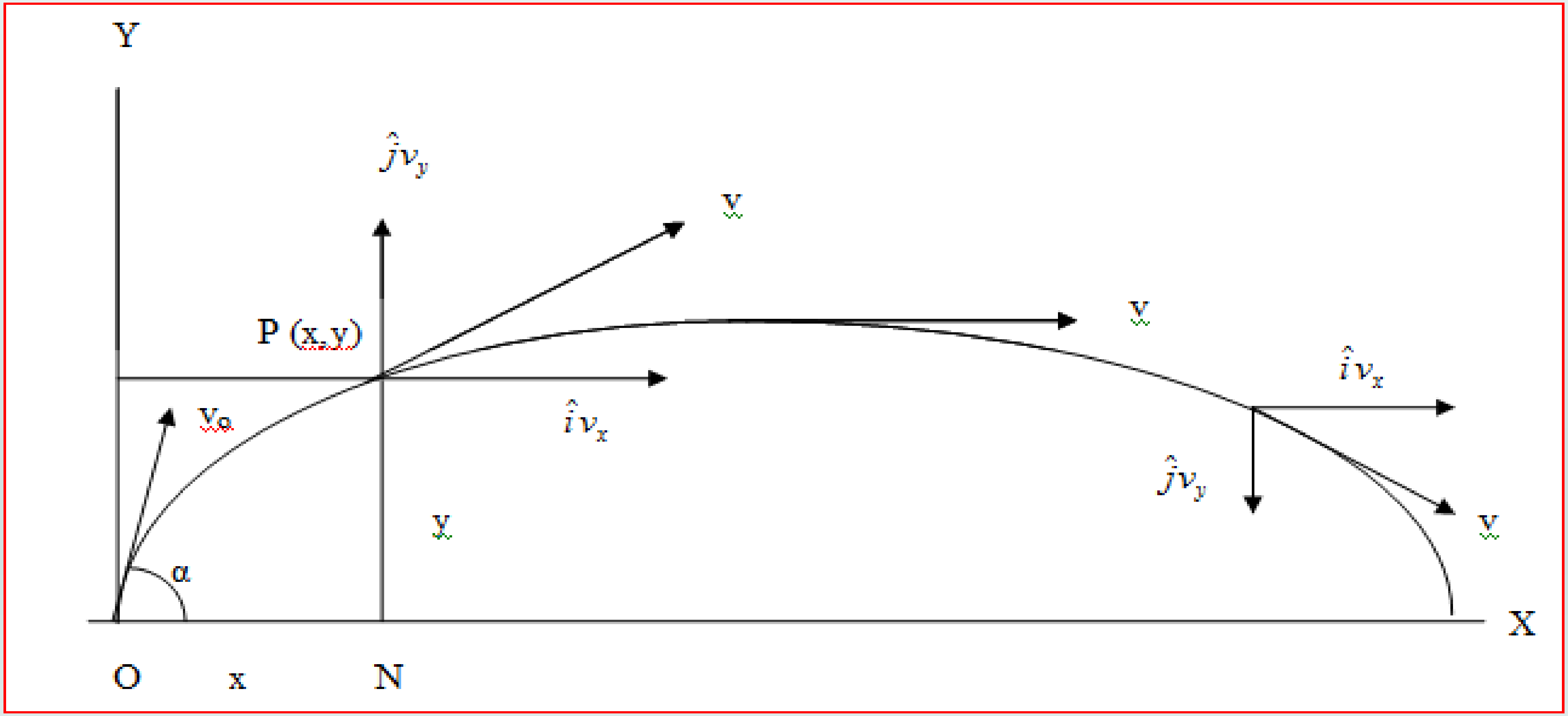
Time of flight: The time taken from the point of projection and return to the ground is called time of flight.

Range: The distance between the point of projection and the point at which it falls on a plane is called the range.

Trajectory: The path of the projectile under the action of gravity

Derivation of equation of motion of a projectile

Let a projectile begins its flight from a point O with initial velocity v_o and making an angle α with the horizontal direction as shown in fig-1 . Taking O as origin let the horizontal and vertical directions be considered along X and Y-axes. So, at t=0, the horizontal component of initial velocity,



DERIVATION OF EQUATION OF MOTION OF A PROJECTILE

$v_{x0} = v_0 \cos\alpha$ and vertical component, $v_{y0} = v_0 \sin\alpha$

Now from the equation of motion,

$v_x = v_{x0} + a_x t$, we get $v_x = v_{x0} = v_0 \cos\alpha \quad [\because a_x = 0]$

DERIVATION OF EQUATION OF MOTION OF A PROJECTILE

Let at t=t, the projectile reaches the point P, whose co-ordinate is (x,y) and where its velocity is v. So, the displacement of the projectile parallel to the ground i.e. along X-axis is, $x = ON = v_0 \cos\alpha \times t$

Or, $t = \dfrac{x}{v_0 \cos\alpha}$ ....(1)

DERIVATION OF EQUATION OF MOTION OF A PROJECTILE

Now, the vertical component of the velocity is, $v_y = v_{y0} + a_y t$ or, $v_y = v_{y0} = v_0 \sin\alpha - gt$ $[\because a_y = -g]$

So, vertical displacement at t=t, $y = PN = y_0 + v_{y0}t + \frac{1}{2}a_y t^2 \Rightarrow y = 0 + v_0 \sin\alpha t - \frac{1}{2}gt^2$

$$\Rightarrow y = v_0 \sin\alpha t - \frac{1}{2}gt^2 \quad \text{..................}(2)$$

Derivation of equation of motion of a projectile

Putting the value of t from equation (1) into equation (2), we have

$$y = v_0 \sin\alpha \times \frac{x}{v_0 \cos\alpha} - \frac{1}{2} g \left(\frac{x}{v_0 \cos\alpha}\right)^2 = x \tan\alpha - \frac{1}{2} g \frac{x^2}{v_0^2 \cos^2\alpha} = x \tan\alpha - \frac{g x^2}{2 v_0^2 \cos^2\alpha} \;\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

In equation (3) α, g and v_0 are constants. So, taking $\tan\alpha = b$ and $\frac{g}{2v_0^2 \cos^2\alpha} = c$ as constants eq. (3) can be written as, $y = bx - cx$ ....(4). This is an equation of a parabola. Hence the path of motion (called trajectory) of a projectile is parabolic.

MAXIMUM HEIGHT

Now, we will derive some important expressions relating projectile motion.

(i) Maximum height of the path of a projectile:

We know from equation of motion, $v^2 = v_0^2 + 2as$

Since the vertical component of the projectile is along Y-axis, so above equation reduces to,

$$v_y^2 = v_{y0}^2 + 2a_y y \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

MAXIMUM HEIGHT

Let the maximum height reached by the projectile be H when final velocity v= 0 i.e., $v_y = 0$. So, we get from equation (1), $0 = v_{y0}^2 - 2gH = v_0^2 \sin^2 \alpha - 2gH \Rightarrow H = \dfrac{v_0^2 \sin^2 \alpha}{2g}$ ---(2) $[\because a_y = -g \text{ and } v_{y0} = v_0 \sin \alpha]$

When $\alpha = 90^0$, the height will be maximum. So, $H = \dfrac{v_0^2}{2g}$ ....(3).

By knowing v_0 and g, H can be determined.

TIME TO REACH MAXIMUM HEIGHT

(ii) Time to reach maximum height: We know from equation of motion, $v = v_0 + at$.

Since the vertical component of the projectile is along Y-axis, so above equation reduces to,

$v_y = v_{y0} + a_y t$ ....(4)

Now, the vertical component of initial velocity $v_{y0} = v_0 \sin\alpha$ and at maximum height final velocity v = 0. So,

from eq. (4) we get, $0 = v_0 \sin\alpha - gt \Rightarrow t = \dfrac{v_0 \sin\alpha}{g}$ ---------(5) $[\because a_y = -g]$

By knowing v_0, g and α, t can be determined.

TIME OF FLIGHT

(iii) Time of flight: Let the time of flight be T. Now, we know the time of ascend to the maximum height = time of descent to the ground. So, $T = t + t = 2t = 2 \times \frac{v_0 \sin \alpha}{g} = \frac{2v_0 \sin \alpha}{g}$ ....(6) [By using eq. (5)]

By knowing v_0, g and α, T can be determined.

HORIZONTAL RANGE

(iv) Horizontal range: The linear distance from the oint of projection to the end of the flight is called the horizontal range. This is represented by R.

R= horizontal component of the initial velocity X time of flight

$$R = v_0 \cos\alpha \times T \Rightarrow R = v_0 \cos\alpha \times \frac{2v_0 \sin\alpha}{g} \Rightarrow R = \frac{v_0^2 \times 2\sin\alpha\cos\alpha}{g} \Rightarrow R = \frac{v_0^2 \sin 2\alpha}{g} \quad \text{.........}(7)$$

By knowing v_0, g and α, R can be determined.

MAXIMUM HORIZONTAL RANGE

(v) Maximum horizontal range: From eq. (7), it is evident that R will be maximum when $\sin 2\alpha = 1$

$\Rightarrow \sin 2\alpha = \sin 90^0 \Rightarrow 2\alpha = 90^0 \Rightarrow \alpha = 45^0$. In that case, $R_{max} = \frac{v_0^2}{g}$ ....(8)

That is, if an object is thrown at an angle 45^0 with the horizontal direction, the horizontal range will be maximum.

PROBLEMS RELATING PROJECTILE MOTION

Problems relating projectile

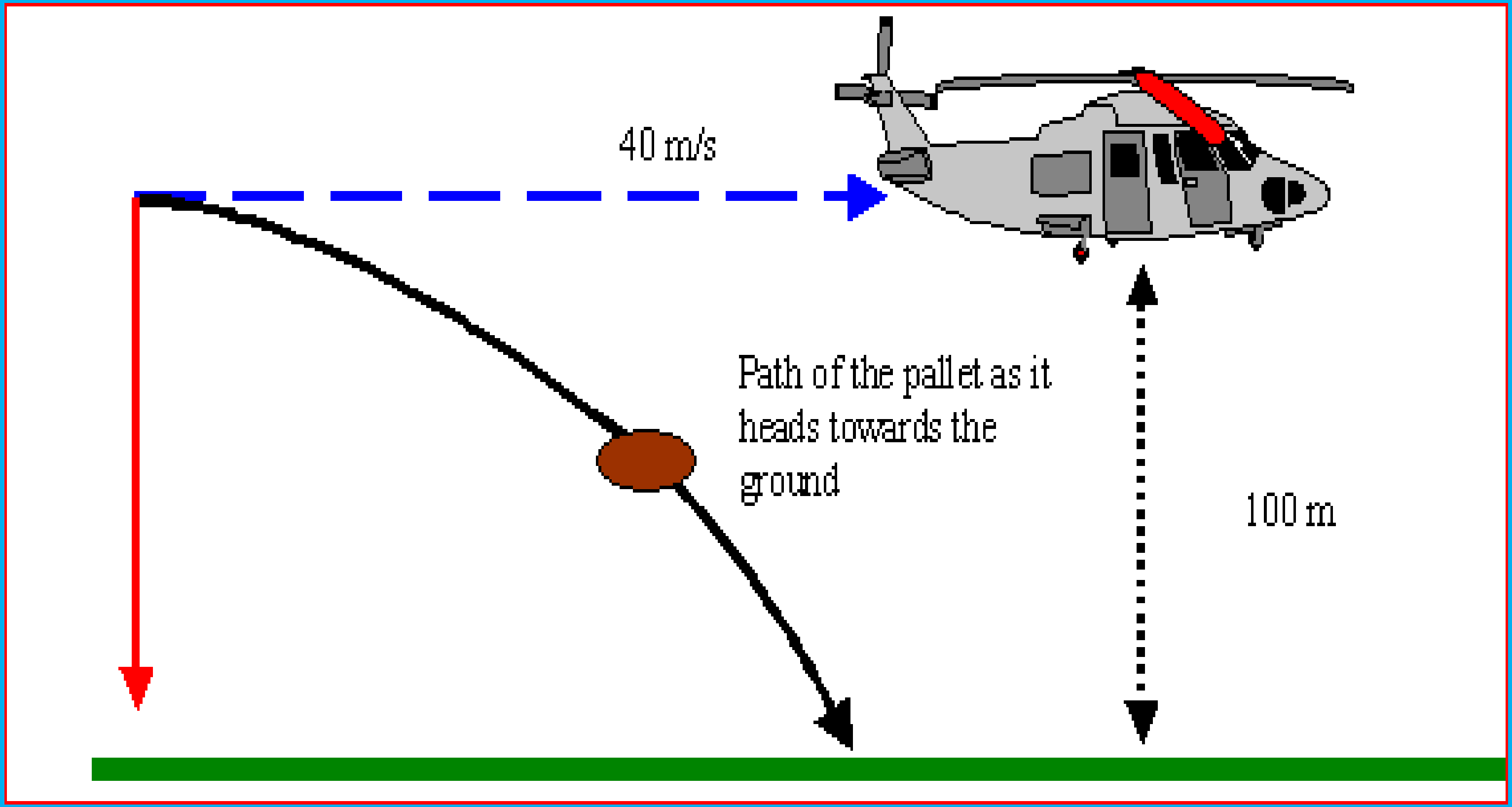
Prob-1: A bouncing ball leaves the ground with a velocity of 4.36 m/s at an angle of 81 degrees above the horizontal. a) How long did it take the ball to land? b) How high did the ball bounce? c) What was the ball's range?

Solution: a) We know, $T = \dfrac{2v_0 \sin \alpha}{g} = \dfrac{2 \times 4.36 \times \sin 81}{9.81} = 0.878\text{ s.}$

b) $H = \dfrac{v_0^2 \sin^2 \alpha}{2g} = \dfrac{(4.36)^2 (\sin 81)^2}{2 \times 9.81} = 0.95 m/s$ c) $R = \dfrac{v_0^2 \sin 2\alpha}{g} = \dfrac{(4.36)^2 \sin(2 \times 81)}{9.81} = 0.60 m$

PROBLEMS RELATING PROJECTILE MOTION

Prob-2: Look at the diagram below. A pallet is dropped from a helicopter to the ground. We will ignore the air resistance.



PROBLEMS RELATING PROJECTILE MOTION

a) What is the horizontal velocity? B) Can you show that the vertical velocity is 44.7 m/s towards the ground? Note that the horizontal velocity is ignored, c) What is the resultant velocity of the pallet just before it hits the ground?

Solution: a) We know, $v_x = v_0 = 40m/s$ b) We have, $y = \frac{1}{2}gt^2 \Rightarrow t^2 = \frac{2y}{g} \Rightarrow t = \sqrt{\frac{2\times100}{9.8}} = 4.52s$

And again $v_y = v_0 + gt = 0 + 9.80\times4.52 = 44.3m/s \approx 44.7m/s(shown)$

c) $\therefore v = \sqrt{v_x^2 + v_y^2} = \sqrt{40^2 + 44.3^2} = 59.69m/s.$

PROBLEMS RELATING PROJECTILE MOTION

Prob-3: The horizontal range of a projectile is 96m and its initial velocity is 66 m/s. What is the angle of projection?

Solution: We know,

$$R = \frac{v_0^2 \sin 2\alpha}{g} \Rightarrow \sin 2\alpha = \frac{Rg}{v_0^2} \Rightarrow 2\alpha = \sin^{-1}(\frac{Rg}{v_0^2}) \Rightarrow 2\alpha = \sin^{-1}(\frac{96 \times 9.8)}{(66)^2} \Rightarrow 2\alpha = 12.473$$

$$\Rightarrow \alpha = \frac{12.473}{2} \Rightarrow \alpha = 6.24^0$$

Self assessment: An object is thrown at velocity 40 m/s making an angle 600 with the horizontal plane. Find the maximum height and the horizontal range.

PROBLEMS RELATING PROJECTILE MOTION

A soccer player kicks a ball at an angle of 37^0 from the horizontal with an initial speed of 20 m/s. (A right angle, one of whose angles is 37^0 has sides in the ratio 3 : 4: 5 or 6 : 8 : 10). Assume that the ball moves in a vertical plane.

(a) ***Find out the time t_1 at which the ball reaches the highest point of its trajectory***

(b) ***How high does the ball go?***

(c) ***What is the horizontal range of the ball?***

(d) ***How long is it in the air?***

(e) ***What is the velocity of the ball as it strikes the ground?***

Solution:

a) $t_1 = \frac{v_0 Sin\theta_0}{g} = \frac{20\times\times sin37^0}{9.8} = 1.22\ sec.$

b) $H = \frac{{v_0}^2 Sin^2\theta_0}{2g} = \frac{20^2\times(sin37^0)^2}{2\times9.8} = 709.8m.$

a) $R = \frac{{v_0}^2 sin2\theta_0}{g} = \frac{20^2\times(sin2\times37^0)}{9.8} = 39.24\ meters$

a) $T = 2t_1 = 2 \text{ X } 1.22 = 2.44\text{sec}$

a) $v_x = v_0 cos\theta_0 = 20\ \times cos37 = 15.97\ m/s$

$v_y = v_0 sin\theta_0 - \ gt = 20\ \times sin37 - 9.8 \times 1.22 = 0.08\ m/s$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{15.97^2 + 0.08^2} = 15.97\ m/s$