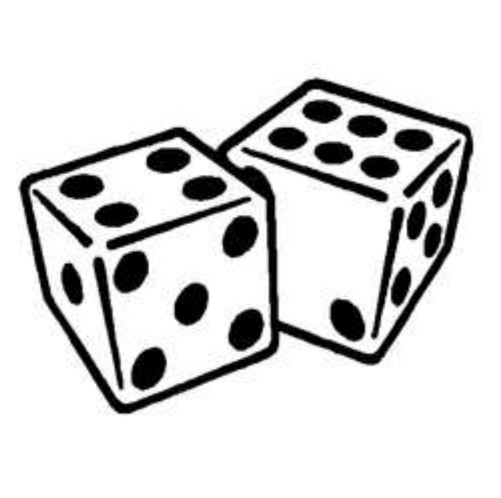
BAYES RULE

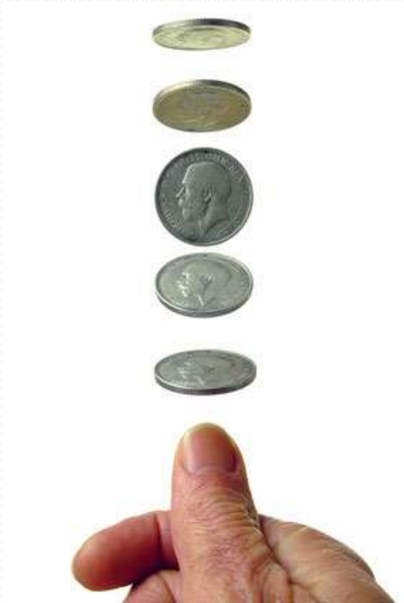
Probability

- **Probability** is the measure of the likelihood that an event will occur. Probability is quantified as a number between 0 and 1 (where 0 indicates impossibility and 1 indicates certainty).



Example

- A simple example is the toss of a fair (unbiased) coin. Since the two outcomes are equally probable, the probability of "heads" equals the probability of "tails", so the probability is 1/2 (or 50%) chance of either "heads" or "tails".



Conditional Probability

- a **conditional probability** measures the probability of an event given that (by assumption, presumption, assertion or evidence) another event has occurred. If the event of interest is *A* and the event *B* is known or assumed to have occurred, "the conditional probability of *A* given *B*", or "the probability of *A* under the condition *B*", is usually written as $P(A|B)$

When to Apply Bayes' Theorem

- Part of the challenge in applying Bayes' theorem involves recognizing the types of problems that warrant its use. You should consider Bayes' theorem when the following conditions exist.
- Within the sample space, there exists an event B, for which $P(B) > 0$.
- The analytical goal is to compute a conditional probability of the form: $P(A_k \mid B)$.
- You know at least one of the two sets of probabilities described below.
  - $P(A_k \cap B)$ for each A_k
  - $P(A_k)$ and $P(B \mid A_k)$ for each A_k

BAYES RULE

- The Bayes Theorem was developed and named for Thomas Bayes(1702-1761)

- Show the Relation between one conditional probability and its inverse.

- Provide a mathematical rule for revising an estimate or forecast in light of experience and observation.

Continue…

In the 18th Century , Thomas Bayes,

- Ponder this question:

“Does God really exist?”

- Being interested in the mathematics, he attempt to develop a formula to arrive at the probability that God does exist based on the evidence that was available to him on earth.

Later, **Laplace** refined **Bayes’ work** and gave it the name “Bayes’ Theorem”.

Definition

- In probability theory and statistics, Bayes' theorem (alternatively Bayes' law or Bayes' rule) describes the probability of an event, based on conditions that might be related to the event.

$$P(A/B) = \frac{P(B/A)P(A)}{P(B)}$$

Continue…

- Bayes' Theorem is a method of revising a probability, given that additional information is obtained. For two event:

$$P(A/B) = \frac{P(B/A)P(A)}{P(B)}$$

Explanation…

- Where A and B are events:

- P(A) and P(B) are the probabilities of A and B without regard to each other.
- P(A | B), a conditional probability, is the probability of observing event A given that B is true.
- P(B | A) is the probability of observing event B given that A is true.

Bayesian inference

- **Bayesian inference** is a method of statistical inference in which Bayes' theorem is used to update the probability for a hypothesis as evidence. It Involves:

Prior Probability:

The initial Probability based on the present level of information.

Posterior Probability:

A revised Probability based on additional information.

Example of Bayes Rule

- Marie is getting married tomorrow, at an outdoor ceremony in the desert. In recent years, it has rained only 5 days each year. Unfortunately, the weatherman has predicted rain for tomorrow. When it actually rains, the weatherman correctly forecasts rain 90% of the time. When it doesn't rain, he incorrectly forecasts rain 10% of the time. What is the probability that it will rain on the day of Marie's wedding?

Solution…

- The sample space is defined by two mutually-exclusive events - it rains or it does not rain. Additionally, a third event occurs when the weatherman predicts rain. Notation for these events appears below.

- Event A_1. It rains on Marie's wedding.
- Event A_2. It does not rain on Marie's wedding.
- Event B. The weatherman predicts rain.

- In terms of probabilities, we know the following:

- P(A_1) = 5/365 =0.0136985 [It rains 5 days out of the year.]
- P(A_2) = 360/365 = 0.9863014 [It does not rain 360 days out of the year.]
- P(B | A_1) = 0.9 [When it rains, the weatherman predicts rain 90% of the time.]
- P(B | A_2) = 0.1 [When it does not rain, the weatherman predicts rain 10% of the time.]

We want to know $P(A_1 | B)$, the probability it will rain on the day of Marie's wedding, given a forecast for rain by the weatherman. The answer can be determined from Bayes' theorem, as shown below.

$$P(A_1 | B) = \frac{P(A_1)\,P(B | A_1)}{P(A_1)\,P(B | A_1) + P(A_2)\,P(B | A_2)}$$

$$P(A_1 | B) = (0.014)(0.9) / [\,(0.014)(0.9) + (0.986)(0.1)\,]$$

$$P(A_1 | B) = 0.111$$

THANK YOU !!!