APRIORI Algorithm

Professor Anita Wasilewska

Lecture Notes

The Apriori Algorithm: Basics

The Apriori Algorithm is an influential algorithm for mining frequent itemsets for boolean association rules.

Key Concepts :

- Frequent Itemsets: The sets of item which has minimum support (denoted by L_i for i^{th}-Itemset).
- Apriori Property: Any subset of frequent itemset must be frequent.
- Join Operation: To find L_k , a set of candidate k-itemsets is generated by joining L_{k-1} with itself.

The Apriori Algorithm in a Nutshell

- Find the *frequent itemsets*: the sets of items that have minimum support
  - A subset of a frequent itemset must also be a frequent itemset
    - i.e., if {*AB*} is a frequent itemset, both {*A*} and {*B*} should be a frequent itemset
  - Iteratively find frequent itemsets with cardinality from 1 to k (k-itemset)
- Use the frequent itemsets to generate association rules.

The Apriori Algorithm : Pseudo code

- Join Step: C_k is generated by joining $L_{k-1}$with itself
- Prune Step: Any (k-1)-itemset that is not frequent cannot be a subset of a frequent k-itemset
- Pseudo-code:

C_k: Candidate itemset of size k
L_k : frequent itemset of size k

L_1 = {frequent items};
for (k = 1; L_k !=$\varnothing$; k++) **do begin**
  C_{k+1} = candidates generated from L_k;
  for each transaction t in database do
    increment the count of all candidates in C_{k+1} that are contained in t
  L_{k+1} = candidates in C_{k+1} with min_support
  end
return $\cup_k L_k$;

The Apriori Algorithm: Example

TID	List of Items
T100	I1, I2, I5
T100	I2, I4
T100	I2, I3
T100	I1, I2, I4
T100	I1, I3
T100	I2, I3
T100	I1, I3
T100	I1, I2 ,I3, I5
T100	I1, I2, I3

- Consider a database, D , consisting of 9 transactions.
- Suppose min. support count required is 2 (i.e. min_sup = 2/9 = 22 %)
- Let minimum confidence required is 70%.
- We have to first find out the frequent itemset using Apriori algorithm.
- Then, Association rules will be generated using min. support & min. confidence.

Step 1: Generating 1-itemset Frequent Pattern

Scan D for count of each candidate →

Itemset	Sup.Count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

C_1

Compare candidate support count with minimum support count →

Itemset	Sup.Count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

L_1

- The set of frequent 1-itemsets, L_1 , consists of the candidate 1-itemsets satisfying minimum support.
- In the first iteration of the algorithm, each item is a member of the set of candidate.

Step 2: Generating 2-itemset Frequent Pattern

Generate C_2 candidates from L_1

Itemset
{I1, I2}
{I1, I3}
{I1, I4}
{I1, I5}
{I2, I3}
{I2, I4}
{I2, I5}
{I3, I4}
{I3, I5}
{I4, I5}

C_2

Scan D for count of each candidate

Itemset	Sup. Count
{I1, I2}	4
{I1, I3}	4
{I1, I4}	1
{I1, I5}	2
{I2, I3}	4
{I2, I4}	2
{I2, I5}	2
{I3, I4}	0
{I3, I5}	1
{I4, I5}	0

C_2

Compare candidate support count with minimum support count

Itemset	Sup Count
{I1, I2}	4
{I1, I3}	4
{I1, I5}	2
{I2, I3}	4
{I2, I4}	2
{I2, I5}	2

L_2

Step 2: Generating 2-itemset Frequent Pattern

- To discover the set of frequent 2-itemsets, L_2 , the algorithm uses L_1 *Join* L_1 to generate a candidate set of 2-itemsets, C_2.
- Next, the transactions in D are scanned and the support count for each candidate itemset in C_2 is accumulated (as shown in the middle table).
- The set of frequent 2-itemsets, L_2 , is then determined, consisting of those candidate 2-itemsets in C_2 having minimum support.
- Note: We haven't used Apriori Property yet.

Step 3: Generating 3-itemset Frequent Pattern

Scan D for count of each candidate

Itemset
{I1, I2, I3}
{I1, I2, I5}

C_3

Scan D for count of each candidate

Itemset	Sup. Count
{I1, I2, I3}	2
{I1, I2, I5}	2

C_3

Compare candidate support count with min support count

Itemset	Sup Count
{I1, I2, I3}	2
{I1, I2, I5}	2

L_3

- The generation of the set of candidate 3-itemsets, C_3 , involves use of the Apriori Property.
- In order to find C_3, we compute L_2 *Join* L_2.
- C_3 = L2 *Join* L2 = {{I1, I2, I3}, {I1, I2, I5}, {I1, I3, I5}, {I2, I3, I4}, {I2, I3, I5}, {I2, I4, I5}}.
- Now, Join step is complete and Prune step will be used to reduce the size of C_3. Prune step helps to avoid heavy computation due to large C_k.

Step 3: Generating 3-itemset Frequent Pattern

- Based on the Apriori property that all subsets of a frequent itemset must also be frequent, we can determine that four latter candidates cannot possibly be frequent. How ?
- For example , lets take {I1, I2, I3}. The 2-item subsets of it are {I1, I2}, {I1, I3} & {I2, I3}. Since all 2-item subsets of {I1, I2, I3} are members of L_2, We will keep {I1, I2, I3} in C_3.
- Lets take another example of {I2, I3, I5} which shows how the pruning is performed. The 2-item subsets are {I2, I3}, {I2, I5} & {I3,I5}.
- BUT, {I3, I5} is not a member of L_2 and hence it is not frequent violating Apriori Property. Thus We will have to remove {I2, I3, I5} from C_3.
- Therefore, C_3 = {{I1, I2, I3}, {I1, I2, I5}} after checking for all members of result of Join operation for Pruning.
- Now, the transactions in D are scanned in order to determine L_3, consisting of those candidates 3-itemsets in C_3 having minimum support.

Step 4: Generating 4-itemset Frequent Pattern

- The algorithm uses L_3 *Join* L_3 to generate a candidate set of 4-itemsets, C_4. Although the join results in {{I1, I2, I3, I5}}, this itemset is pruned since its subset {{I2, I3, I5}} is not frequent.
- Thus, $C_4 = \varphi$, and algorithm terminates, having found all of the frequent items. This completes our Apriori Algorithm.
- What's Next ?

  These frequent itemsets will be used to generate strong association rules (where strong association rules satisfy both minimum support & minimum confidence).

Step 5: Generating Association Rules from Frequent Itemsets

- Procedure:
  - For each frequent itemset ***"l"***, generate all nonempty subsets of ***l.***
  - For every nonempty subset ***s*** of ***l***, output the rule **"s → (l-s)"** if **support_count(l) / support_count(s) >= min_conf** where min_conf is minimum confidence threshold.

- Back To Example:

  We had L = {{I1}, {I2}, {I3}, {I4}, {I5}, {I1,I2}, {I1,I3}, {I1,I5}, {I2,I3}, {I2,I4}, {I2,I5}, {I1,I2,I3}, {I1,I2,I5}}.
  - Lets take ***l*** = {I1,I2,I5}.
  - Its all nonempty subsets are {I1,I2}, {I1,I5}, {I2,I5}, {I1}, {I2}, {I5}.

Step 5: Generating Association Rules from Frequent Itemsets

- Let minimum confidence threshold is , say 70%.
- The resulting association rules are shown below, each listed with its confidence.
  - R1: I1 ^ I2 → I5
    - Confidence = sc{I1,I2,I5}/sc{I1,I2} = 2/4 = 50%
    - R1 is Rejected.
  - R2: I1 ^ I5 → I2
    - Confidence = sc{I1,I2,I5}/sc{I1,I5} = 2/2 = 100%
    - R2 is Selected.
  - R3: I2 ^ I5 → I1
    - Confidence = sc{I1,I2,I5}/sc{I2,I5} = 2/2 = 100%
    - R3 is Selected.

Step 5: Generating Association Rules from Frequent Itemsets

- R4: I1 → I2 ^ I5
  - Confidence = sc{I1,I2,I5}/sc{I1} = 2/6 = 33%
  - R4 is Rejected.
- R5: I2 → I1 ^ I5
  - Confidence = sc{I1,I2,I5}/{I2} = 2/7 = 29%
  - R5 is Rejected.
- R6: I5 → I1 ^ I2
  - Confidence = sc{I1,I2,I5}/ {I5} = 2/2 = 100%
  - R6 is Selected.

  In this way, We have found three strong association rules.

Methods to Improve Apriori's Efficiency

- Hash-based itemset counting: A *k*-itemset whose corresponding hashing bucket count is below the threshold cannot be frequent.
- Transaction reduction: A transaction that does not contain any frequent k-itemset is useless in subsequent scans.
- Partitioning: Any itemset that is potentially frequent in DB must be frequent in at least one of the partitions of DB.
- Sampling: mining on a subset of given data, lower support threshold + a method to determine the completeness.
- Dynamic itemset counting: add new candidate itemsets only when all of their subsets are estimated to be frequent.

Mining Frequent Patterns Without Candidate Generation

- Compress a large database into a compact, Frequent-Pattern tree (FP-tree) structure
  - highly condensed, but complete for frequent pattern mining
  - avoid costly database scans
- Develop an efficient, FP-tree-based frequent pattern mining method
  - A divide-and-conquer methodology: decompose mining tasks into smaller ones
  - Avoid candidate generation: sub-database test only!

FP-Growth Method : An Example

TID	List of Items
T100	I1, I2, I5
T100	I2, I4
T100	I2, I3
T100	I1, I2, I4
T100	I1, I3
T100	I2, I3
T100	I1, I3
T100	I1, I2 ,I3, I5
T100	I1, I2, I3

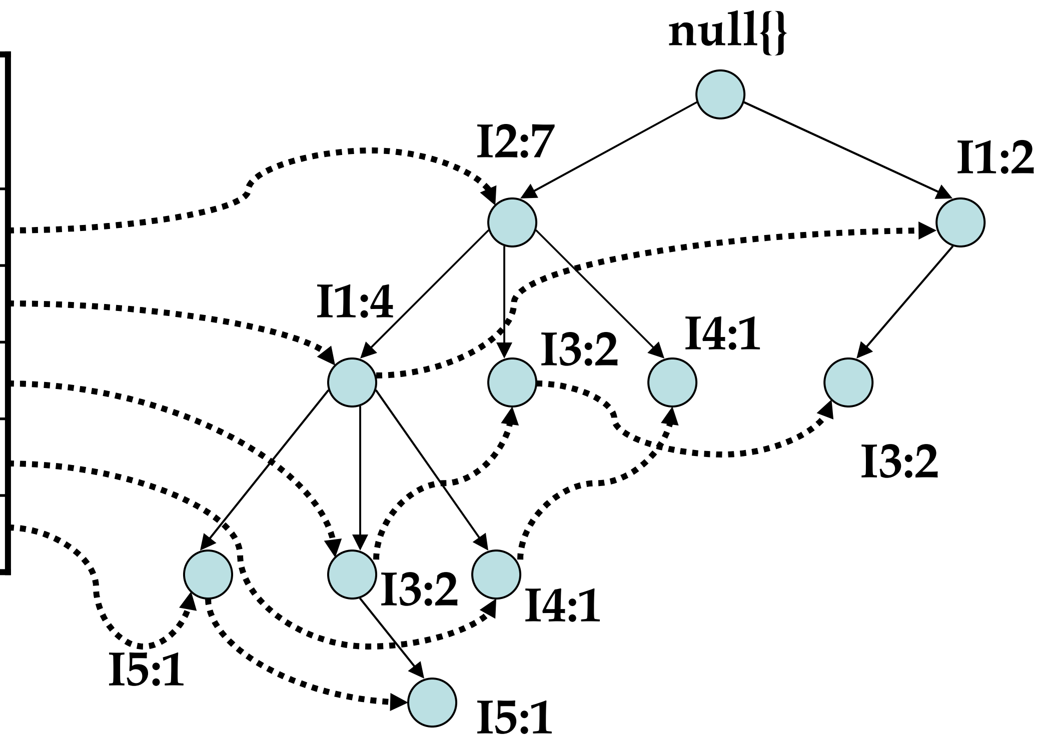
- Consider the same previous example of a database, D , consisting of 9 transactions.
- Suppose min. support count required is 2 (i.e. min_sup = 2/9 = 22 %)
- The first scan of database is same as Apriori, which derives the set of 1-itemsets & their support counts.
- The set of frequent items is sorted in the order of descending support count.
- The resulting set is denoted as L = {I2:7, I1:6, I3:6, I4:2, I5:2}

FP-Growth Method: Construction of FP-Tree

- First, create the root of the tree, labeled with “null”.
- Scan the database D a second time. (First time we scanned it to create 1-itemset and then L).
- The items in each transaction are processed in L order (i.e. sorted order).
- A branch is created for each transaction with items having their support count separated by colon.
- Whenever the same node is encountered in another transaction, we just increment the support count of the common node or Prefix.
- To facilitate tree traversal, an item header table is built so that each item points to its occurrences in the tree via a chain of node-links.
- Now, The problem of mining frequent patterns in database is transformed to that of mining the FP-Tree.

FP-Growth Method: Construction of FP-Tree

Item Id	Sup Count	Node-link
I2	7	
I1	6	
I3	6	
I4	2	
I5	2	



An FP-Tree that registers compressed, frequent pattern information

Mining the FP-Tree by Creating Conditional (sub) pattern bases

Steps:

1. Start from each frequent length-1 pattern (as an initial suffix pattern).
2. Construct its conditional pattern base which consists of the set of prefix paths in the FP-Tree co-occurring with suffix pattern.
3. Then, Construct its conditional FP-Tree & perform mining on such a tree.
4. The pattern growth is achieved by concatenation of the suffix pattern with the frequent patterns generated from a conditional FP-Tree.
5. The union of all frequent patterns (generated by step 4) gives the required frequent itemset.

FP-Tree Example Continued

Item	Conditional pattern base	Conditional FP-Tree	Frequent pattern generated
I5	{(I2 I1: 1),(I2 I1 I3: 1)}	<I2:2 , I1:2>	I2 I5:2, I1 I5:2, I2 I1 I5: 2
I4	{(I2 I1: 1),(I2: 1)}	<I2: 2>	I2 I4: 2
I3	{(I2 I1: 1),(I2: 2), (I1: 2)}	<I2: 4, I1: 2>,<I1:2>	I2 I3:4, I1, I3: 2 , I2 I1 I3: 2
I2	{(I2: 4)}	<I2: 4>	I2 I1: 4

Mining the FP-Tree by creating conditional (sub) pattern bases

Now, Following the above mentioned steps:

- Lets start from I5. The I5 is involved in 2 branches namely {I2 I1 I5: 1} and {I2 I1 I3 I5: 1}.
- Therefore considering I5 as suffix, its 2 corresponding prefix paths would be {I2 I1: 1} and {I2 I1 I3: 1}, which forms its conditional pattern base.

FP-Tree Example Continued

- Out of these, Only I1 & I2 is selected in the conditional FP-Tree because I3 is not satisfying the minimum support count.

  For I1 , support count in conditional pattern base = 1 + 1 = 2

  For I2 , support count in conditional pattern base = 1 + 1 = 2

  For I3, support count in conditional pattern base = 1

  Thus support count for I3 is less than required min_sup which is 2 here.
- Now , We have conditional FP-Tree with us.
- All frequent pattern corresponding to suffix I5 are generated by considering all possible combinations of I5 and conditional FP-Tree.
- The same procedure is applied to suffixes I4, I3 and I1.
- Note: I2 is not taken into consideration for suffix because it doesn't have any prefix at all.

Why Frequent Pattern Growth Fast ?

- Performance study shows
  - FP-growth is an order of magnitude faster than Apriori, and is also faster than tree-projection
- Reasoning
  - No candidate generation, no candidate test
  - Use compact data structure
  - Eliminate repeated database scan
  - Basic operation is counting and FP-tree building