CE 414: Prestressed Concrete
Lecture 8
Flexural Analysis (Contd. II)

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Contents

- Ultimate flexural strength
- Reinforcement index
- Stress block and kern
- I shape beam ultimate resistance moment

A simple method for determining ultimate flexural strength following the ACI Code is presented herewith, based on the results of the aforementioned tests as well as others. This method is limited to the following conditions.

1. The failure is primarily a flexural failure, without shear, bond, or anchorage failure which might decrease the strength of the section.
2. The beams are bonded. Unbonded beams possess different ultimate strength and are discussed later.
3. The beams are statically determinate. Although the discussions apply equally well to individual sections of continuous beams, the ultimate strength of continuous beams as a whole is explained by the plastic hinge theory to be discussed in Chapter 10.
4. The load considered is the ultimate load obtained as the result of a short static test. Impact, fatigue, or long-time loadings are not considered.

materials presently used in prestressed work, the reinforcement index, ω_p, which approximates the limiting value to assure that the prestressed steel (A_{ps}) will be slightly into its yield range, is given by the ACI Code as follows:

$$\omega_p = \rho_p f_{ps} / f'_c \leqslant 0.30 \qquad (5\text{-}13)$$

where

$$\rho_p = A_{ps} / bd$$

There are situations where prestressing steel (A_{ps}) and ordinary reinforcing bars (A_s) are used together in a prestressed beam. In this case the total of all the tension steel is considered along with the possibility of compression steel (A'_s). The limiting reinforcement ratio is given as

$$(\omega + \omega_p - \omega') \leq 0.30 \tag{5-14}$$

where

$$\omega = \rho f_y / f'_c \qquad \text{and} \qquad \rho = A_s / bd$$

$$\omega' = \rho' f_y / f'_c \qquad \text{and} \qquad \rho' = A'_s / bd$$

ACI Code Bonded Beams. For underreinforced bonded beams following the ACI Code, the steel is stressed to a stress level which approaches its ultimate strength at the point of failure for the beam in flexure. For the purpose of practical design, it will be sufficiently accurate to assume that the steel is stressed to the stress level, f_{ps}, given by the equation for bonded beams from the ACI Code which closely approximates test results.[6] Provided the effective prestress, f_{se}, is not less than $0.5f_{pu}$, the following approximate value for the steel stress at ultimate moment capacity for the beam is applicable for bonded beams:

$$f_{ps} = f_{pu}\left(1 - 0.5\rho_p \frac{f_{pu}}{f'_c}\right) \tag{5-15}$$

The computation of the ultimate resisting moment is a relatively simple matter and can be carried out as follows. Referring to Fig. 5-16, the ultimate compressive force in the concrete C' equals the ultimate tensile force in the steel T', thus,

$$C' = T' = A_s f_{ps}$$

Let a' be the lever arm between the forces C' and T'; then the ultimate resisting moment is given by

$$M' = T'a' = A_s f_{ps} = M_n \text{ (ACI Code nominal strength)}$$

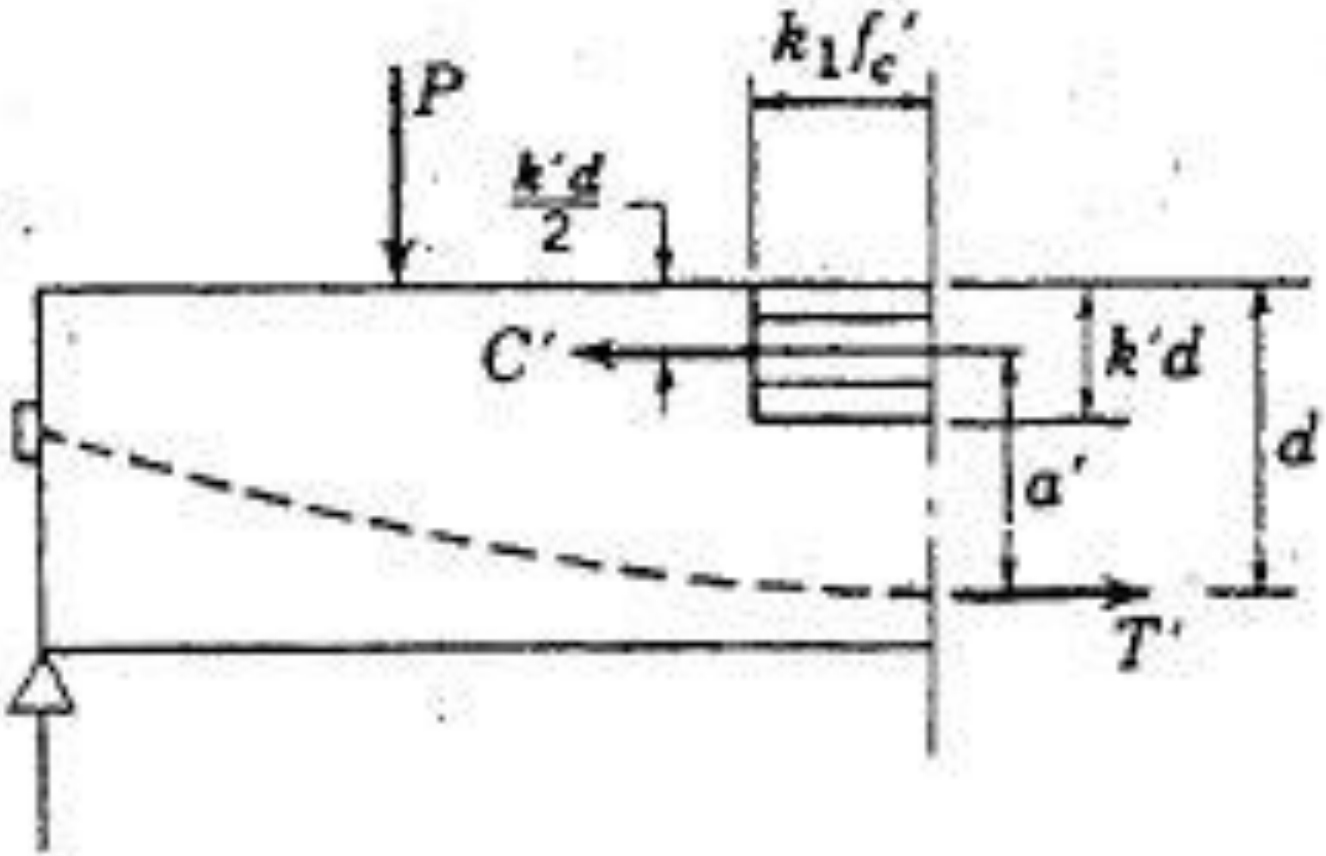

Fig. 5-16. Ultimate moment.

ACI Code

$k'd = a$

$k_1 = 0.85$

Hence, the ultimate resisting moment is

$$M' = A_{ps} f_{ps} d\left(1 - \frac{k'}{2}\right) = M_n \text{ (ACI notation)} \qquad (5\text{-}18)$$

$$k' = \frac{A_{ps} f_{ps}}{0.85 f'_c b d} \qquad (5\text{-}19)$$

$$M_n = A_{ps} f_{ps}\left(d - \frac{a}{2}\right)$$

The alternative equation (5-22) written directly in terms of the T' and C' force couple becomes the following ACI Code design ultimate moment equation:

$$M_u = \phi\left[A_{ps} f_{ps}\left(d - \frac{a}{2}\right)\right] \tag{5-24}$$

EXAMPLE 5-8

An I-shaped beam is prestressed with $A_{ps} = 2.75$ in.^{2} as prestressing steel with an effective stress, f_{se}, of 160 ksi. The c.g.s. of the strands which supply the prestress is 4.5 in. above the bottom of the beam as shown in Fig. 5-17 along with the shape of the concrete cross section. Material properties are: $f_{pu} = 270$ ksi; $f'_c = 7000$ psi. Find the ultimate resisting moment for the section for design following the ACI Code. ($A_{ps} = 1{,}774$ mm^2, $f_{se} = 1{,}103$ N/mm^2, $f_{pu} = 1{,}862$ N/mm^2, and $f'_c = 48$ N/mm^2.)

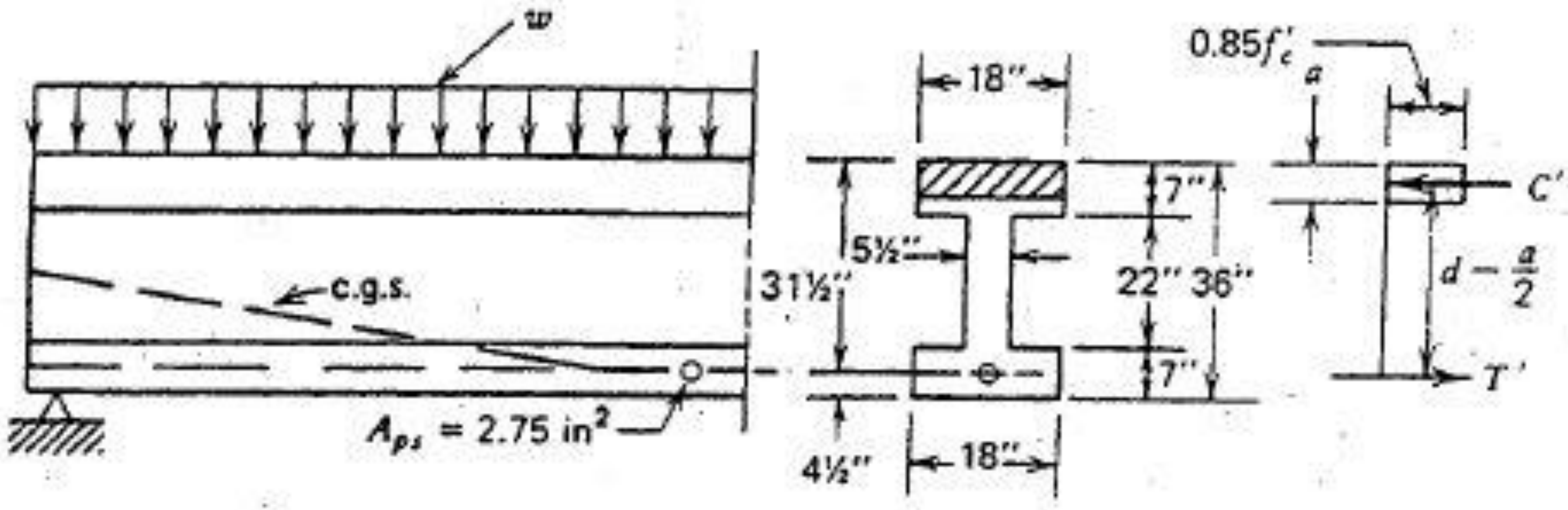

Fig. 5-17. Example 5-8.

Solution:

$$\rho_p = \frac{2.75}{(18)(31.5)} = 0.00485$$

Estimate steel stress at ultimate by the ACI equation (5-15) which is valid to use since $f_{se} = 160$ ksi ($1103\ \text{N/mm}^2$) $> 0.5 f_{pu} = 135$ ksi ($931\ \text{N/mm}^2$).

$$f_{ps} = 270{,}000\left[1 - (0.5)(0.00485)\left(\frac{270{,}000}{7{,}000}\right)\right] \qquad (5\text{-}15)$$

$$f_{ps} = 245{,}000 \text{ psi} = 245 \text{ ksi } (1689\ \text{N/mm}^2)$$

Check the reinforcement index

$$\omega_p = \frac{(0.00485)(245{,}000)}{7000} = 0.17 < 0.30 \qquad (5\text{-}13)$$

Referring to Fig. 5-17 sketch of section

$$T' = A_{ps} f_{ps} = 2.75 \times 245 = 674 \text{ k } (2{,}998 \text{ kN})$$

$$C' = 0.85 f_c' \times 18 \times a = 674 \text{ k } (2{,}998 \text{ kN})$$

$$a = \frac{674}{(0.85)(7)(18)} = 6.29 \text{ in.} < 7 \text{ in. } O.K. \text{ rectangular section behavior}$$

$$M_n = T'\left(d - \frac{a}{2}\right) = 674\left(31.5 - \frac{6.29}{2}\right) = 19{,}100 \text{ in.-k. } (2{,}158 \text{ kN-m}) \qquad (5\text{-}22)$$

$$M_u = 0.9 M_n = 17{,}200 \text{ in.-k. } (1944 \text{ kN-m}) \qquad (5\text{-}24)$$