Example 11.7a

Find an spectrum of M where

(a). $M = \begin{pmatrix} 1 & 2 & 0 \\ 2 & -7 & 1 \\ 2 & 1 & 7 \end{pmatrix}$; (c). $M = \begin{pmatrix} -1 & 0 & 7 \\ -8 & 0 & -1 \\ -5 & 4 & -6 \end{pmatrix}$.

Solution(a):

Let 'M' have an eigenvalue λ.

$\Longrightarrow$ characteristic matrix $= M - \lambda I_n = \begin{pmatrix} 1-\lambda & 2 & 0 \\ 2 & -7-\lambda & 1 \\ 2 & 1 & 7-\lambda \end{pmatrix}$.

$\Longrightarrow$ characteristic polynomial $= |M - \lambda I_n| = -74 + 54\lambda + \lambda^2 - \lambda^3$.

$\Longrightarrow$ characteristic equation : $\lambda^3 - \lambda^2 - 54\lambda + 74 = 0$.

$\Longrightarrow$ eigenvalues : $\lambda \approx 1.384, 7.123, -7.51$ [programmed calculator]

$\Longrightarrow$ an spectrum :{ 1.384, 7.123, –7.51 }. ◙

Solution(b):

Let 'M' have eigenvalue λ.

$\Longrightarrow$ characteristic matrix $= M - \lambda I_n = \begin{pmatrix} -1-\lambda & 0 & 7 \\ -8 & -\lambda & -1 \\ -5 & 4 & -6-\lambda \end{pmatrix}$.

$\Longrightarrow$ characteristic polynomial $= |M - \lambda I_n| = -228 - 45\lambda - 7\lambda^2 - \lambda^3$.

$\Longrightarrow$ characteristic equation : $\lambda^3 + 7\lambda^2 + 45\lambda + 228 = 0$.

$\Longrightarrow$ eigenvalues : $\lambda \approx -5.9118, -0.544 \pm 6.18634i$ [programmed calculator]

$\Longrightarrow$ an spectrum :{$-5.9118, -0.5443 + 6.18634i, -0.544 - 6.18634i$ }. ◙

SQ

2. A singular matrix must have 0 as the only eigenvalue: True or False ?
3. A nonsingular matrix must not have 0 as an eigenvalue: True or False ?
4. $|M| = -5 \Rightarrow \lambda \neq 0$: True or False ?
7. *M* has all nonzero characteristic roots. Is *M* nonsingular ?
8. M is diagonal and *diag*(M) = {-4,3,0,-1}. What is the spectrum of M ?
10. M is scalar and *diag*(M) = {1}. What is the spectrum of M ?
14. M has a latent root 5. What will be a characteristic root of M^4: 5^4 or 4^5 ?
15. If M is real & 2×2, then λ is also real: True or False ?

Exercise – 11

1. Find all eigenvalues of the following matrices:

(a). $\begin{pmatrix}4 & 3\\ 2 & 5\end{pmatrix}$ (b). $\begin{pmatrix}1 & 2\\ 3 & 2\end{pmatrix}$ (c). $\begin{pmatrix}3 & -1\\ 1 & 1\end{pmatrix}$ (d). $\begin{pmatrix}0 & 4\\ -5 & 0\end{pmatrix}$ (e). $\begin{bmatrix}-4 & 3\\ 2 & -5\end{bmatrix}$ (f). $\begin{pmatrix}1 & 4\\ 9 & 1\end{pmatrix}$.

[You are asked to answer in *real numbers* only.]

ans: (a). λ = 2, λ = 7, (b). λ = 4, λ = -1, (c). λ = 2,2, (f). λ = -5, λ = 7, [

2. A matrix M is mentioned below. Find :

(i). an spectrum of M ; (ii). an spectrum of M^T, M^2, M^5 ;

(iii). which matrices have inverses [answer with the help of eigenvalue] ;

(iv). an spectrum of (if possible) M^{-1}, M^{-5} ;

(a). $\begin{pmatrix}9 & 0 & 0\\ 0 & -5 & 0\\ 0 & 0 & 1\end{pmatrix}$ (b). $\begin{pmatrix}4 & 0 & 0\\ 5 & 9 & 0\\ -1 & 2 & 1\end{pmatrix}$ (c). $\begin{pmatrix}0 & -1 & 6\\ 0 & 2 & 5\\ 0 & 0 & -1\end{pmatrix}$ (d). $\begin{pmatrix}0 & -1 & 7\\ 1 & 0 & 2\\ -7 & -2 & 0\end{pmatrix}$;

(e). $\begin{pmatrix}0 & 4 & -5\\ -4 & 0 & -1\\ 5 & 1 & 0\end{pmatrix}$ (f). $\begin{pmatrix}5 & 0 & 4\\ -1 & -1 & 2\\ 0 & 2 & 3\end{pmatrix}$ (g). $\begin{pmatrix}3 & -1 & 2\\ 0 & 1 & -2\\ 2 & 0 & -1\end{pmatrix}$ (h). $\begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 4\\ 3 & 4 & 5\end{pmatrix}$;

(i). $\begin{bmatrix}7 & -2 & 8\\ 3 & 5 & 0\\ 14 & 8 & 8\end{bmatrix}$ (j). $\begin{pmatrix}1 & 0 & 1\\ -1 & -1 & 0\\ 0 & 1 & 1\end{pmatrix}$ (k). $\begin{pmatrix}-2 & 0 & 3\\ 1 & 5 & 2\\ 3 & 5 & -1\end{pmatrix}$ (l). $\begin{pmatrix}0 & i & 1-i\\ -i & 0 & 4\\ 1+i & 4 & 0\end{pmatrix}$.

Answers:[only an spectrum of M is mentioned. Yes/Not refers to the case of invertibility.]

(e). { 0 , $\pm 6.48i$ } , Not ; (f). {-2.023, 4.5115 $\pm$ 0.949 i } , Yes ; (g). {4.0514, -1.5341,0.4827 } , Yes ; (h). { 0, -0.62, 9.62 }, Not ;

3. A matrix M is mentioned below.

Find all that mentioned in the sample of the PPP:

(a). $M = \begin{pmatrix}3 & 1 & 1\\ 2 & 4 & 2\\ 1 & 1 & 3\end{pmatrix}$ ans: λ = 2,2; λ = 6

(b). $M = \begin{pmatrix}1 & 0 & -2\\ 0 & 0 & 0\\ -2 & 0 & 4\end{pmatrix}$ ans: λ = 0,0 λ = 5

(c). $M = \begin{bmatrix}4 & 6 & 6\\ 1 & 3 & 2\\ -1 & -4 & -3\end{bmatrix}$ ans: λ = 4, λ = 1 λ = -1,

(e). $M = \begin{pmatrix}1 & 2 & -1\\ 0 & -2 & 0\\ 0 & -5 & 2\end{pmatrix}$ ans: λ = 2 λ = 1, λ = -2 ,

(f). $M = \begin{pmatrix}5 & 0 & 0\\ -1 & 3 & 0\\ 0 & 2 & -1\end{pmatrix}$ ans: λ = 5 , λ = 3, λ = -1

(g). $M = \begin{pmatrix}1 & 2 & 3\\ 0 & -4 & 2\\ 0 & 0 & 7\end{pmatrix}$ ans: λ = 7, λ = 1, λ = -4 ,

(h). $M = \begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$ ans: λ = 0, λ = 1, λ = -1