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PHY 111/113: Mechanics, Heat and Thermodynamics, Wave and Oscillation and Optics.

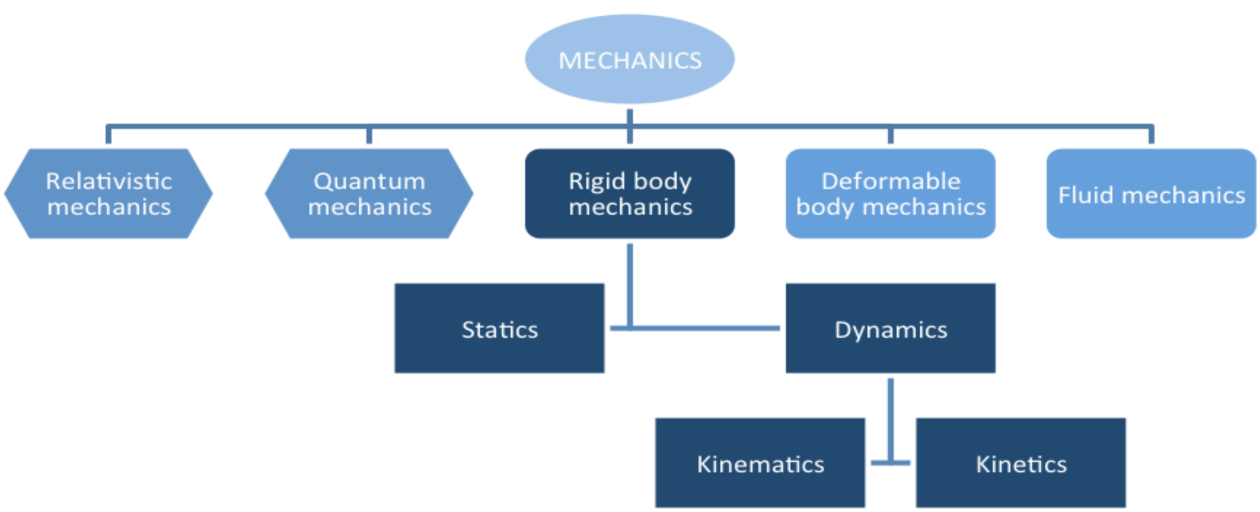
Chapter -1: Mechanics

1. What is mechanics?

The study of ***Physics*** begins with mechanics. Mechanics is the science which describes and predicts the conditions of rest or motion of bodies under the action of forces. In a word, it is the branch of Physics dealing with the study of static body and its motion.

(Mechanics is the foundation of most engineering sciences and is a vital requirement to their study)

Mechanics can be divided into 2 areas Statics and Dynamics. Dynamics divides into two sections - **kinematics,** which dealing with describing motions, and **kinetics,** which dealing with the causes of motion. **Kinematics** is the branch of classical mechanics which describes the motion of points, bodies (objects) and systems of bodies (groups of objects) without consideration of the causes of motion. **Kinetics** is a term for the branch of classical mechanics that is concerned with the relationship between the motion of bodies and its causes, namely forces and torques.



Reference point and Frame of Reference:

To measure motion, we must first measure position. We measure position relative to some fixed point O, called the origin.

A reference point is something that we use to determine the position of an object but the reference point has to be something that doesn't move at all. This point is also known as "origin" and in fig 1 the reference point is denoted by "O".

On the other hand, frame of reference refers to a system of coordinates and measures of the body position and other properties in it. Frame of reference is relative to all motions.

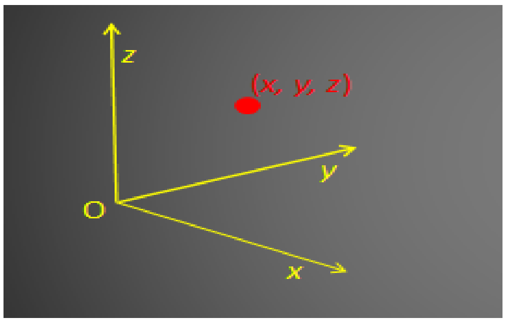


Fig 1: Frame of reference.

We give the ball's location as (x, y, z): we reach it from O by moving x meters along the x-axis, followed by y parallel to the y-axis and finally z parallel to the z-axis.

Motion:

The motion of a particle is defined as the change of its position with respect to time and its reference point. Motion is typically described in terms of displacement, direction, velocity, acceleration, and time. Motion is observed by attaching a frame of reference to a body and measuring its change in position relative to that frame.

Displacement and one dimentional motion:

Consider a body moving in 1 dimension: *e.g.*, a train traveling down a straight railroad track. Suppose that we have a team of observers who continually report the location of this body to us as time progresses. To be more exact, our observers report the distance x of the body from some arbitrarily chosen reference point located on the track on which it is constrained to move. This point is known as the ***origin*** of our coordinate system. Here, x implies that the body is located x meters to the*right* of the origin and considered as the *displacement* of the body from the origin.

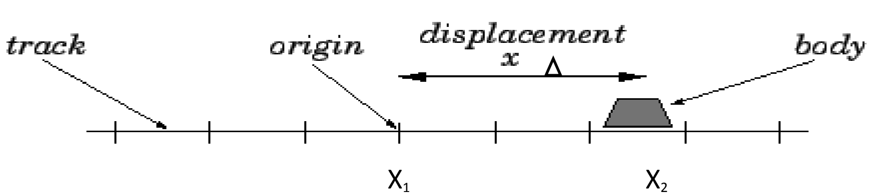


Fig-2: Motion in one dimention.

In fig-2, the displacement along the railroad has both magnitude and direction. That means displacement is a vector.If the displacement $x = x_2 - x_1$, its magnetude can be writtend as

$$|\Delta x| = |x_2 - x_1|.$$

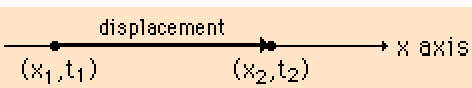
Whereas, direction is indicated by attaching an arrowhead.

Motion in two dimentions or motion in a plane:

In this case the motion of a particle can b described in dimentions by a numbers of variables, such as positions vector, time, velocity and accelaration etc. The polar co-ordinator systems are mainly used to deduce the relationship between the variables.

Velocity:

Average velocity can be defined as the displacement divided by the time. For the special case of straight line motion in the X direction, the average velocity takes the form



$$V_{average} = v = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

The units for velocity can be implied form the defination to be meters(m)/Second(s) or in general any distance unit over any time unit.

Whereas, the instaneous velocity at any point on the path by taking the limit as the time interval gets smaller and smaller.Such a limiting process is called a derivative and the instantanous velocity can be defined as

$$V_{instantaneous} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

The terms velocity and speed are often confused with one another. A velocity can be either positive or negative, depending on the direction of motion. The conventional definition of *speed* is that it is the magnitude of velocity (*i.e.*, it is v with the sign stripped off). It follows that a body can never possess a negative speed.

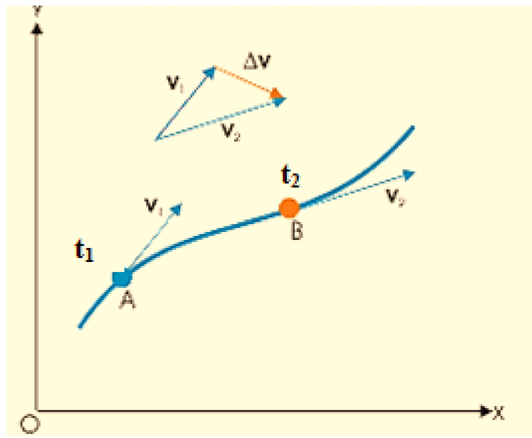
Acceleration:

The velocity of a particle may change in magnitude or in direction as it moves. The change in velocity with time is called acceleration. Acceleration is inherently a vector quantity, and an object will have non-zero acceleration of its speed and/or direction is changing. The units for acceleration can be implied from the definition to be meters/second square, usually m/s^2.

We can define the average acceleration as the change in velocity per unit time, or mathematically it can be written as

$$\text{Average acceleration} = \frac{\text{change in velocity}}{\text{time interval}}$$

$$\vec{a}_{avg} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}$$



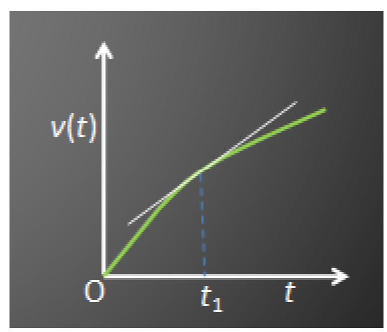
The direction of the average acceleration $\bar{\mathbf{a}}_{av}$ is the same as the direction of .

Instantaneous acceleration at any time may be obtained by taking the limit of the average acceleration as the time interval approaches zero. This is the derivative of the velocity with respect to time:

$$\vec{a}_{instantaneous} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

The acceleration at time t_1 is the slope of the

Velocity graph V (t) at that time.



Constant acceleration means the rate of change of velocity is constant. Mathematically,

Constant acceleration, a = = constant.

The equations of motion:

The equations of motion are valid only when acceleration is constant and motion is constrained to a straight line. Our goal in this section is to derive new equations that can be used to describe the motion of an object in terms of its three kinematic variables: velocity, displacement, and time. There are three ways to pair them up: **velocity-time**, **displacement-time**, and **velocity-displacement.** In this order, they are also often called the first, second, and third equations of motion, but there is no compelling reason to learn these names. Since we are dealing with motion in a straight line, the symbol *x* will be used for displacement. Direction will be indicated by the sign (positive quantities point in $+x$ direction, while negative quantities point in the $-x$ direction).

Property	Symbol	Unit
Displacement/distance	s/x	meter (m)
Initial velocity	u/v_0	Meter/second (ms^{-1})
Final velocity	v	Meter/second (ms^{-1})
Acceleration	a	Meter/second^2 (ms^{-2})
Time	t	Second (s)

Velocity-time equation:

Consider a body of mass "m" having initial velocity "v_0".Let after time "t" its final velocity becomes "v" due to uniform acceleration "a".

According to the definition of acceleration,

Acceleration = change in velocity/Time taken

Acceleration = Final velocity-Initial velocity / time taken

a = =

……….. (1)

This is our first equation of motion.

Second equation of motion:

Let the distance travelled by the body be "s".
We know that
Distance = Average velocity X Time

Also, Average velocity =

Distance, s = …….eq. (2)

Again we know that:
$v = v_0 + at$
substituting this value of "v" in eq.(2), we get

s = x t
s = X t
s =
s = $v_0t + 1/2at^2$ ……………………. (3)

This is the 2nd equation of motion.

3^{rd} equation of motion:

We start with squaring equation (1). Thus we have

$$v^2 = (v_0+at)^2$$

$$\Rightarrow \quad v^2 = v_0^2 + a^2t^2 + 2v_0at$$

$$\Rightarrow \quad v^2 = v_0^2 + 2v_0at + a^2t^2$$

$$\Rightarrow \quad v^2 = v_0^2 + 2a\,(v_0t + (1/2)at^2$$

Now, using equation 2, we have

$$\Rightarrow \quad v^2 = v_0^2 + 2as \qquad \textbf{(4)}$$

As you can see, the above equation gives a relation between the final velocity v of the body and the distance s traveled by the body.

Thus, we have the the three Newton's equations of Motion as

1) $v = \mathbf{v_0 + at}$

2) $\mathbf{s = v_0t + (1/2)at^2}$

3) $\mathbf{v^2 = v_0^2 + 2as}$

Newton's law of motion:

Isaac Newton (a 17th century scientist) put forth a variety of laws that explain why objects move (or don't move) as they do. These three laws have become known as Newton's three laws of motion.

First law: Newton's first law states that an object will remain at rest or in uniform motion in a straight line unless acted upon by an external force.

The tendency of a body to remain at rest or in uniform linear motion is called inertia, and Newton's first law is often called the law of inertia.

Basically what Newton's First Law is saying is that objects behave predictably. If a ball is sitting on your table, it isn't going to start rolling or fall off the table unless a force acts upon it to cause it to do so. Moving objects don't change their direction unless a force causes them to move from their path.

Newton's Second Law of Motion:

Newton's Second Law of Motion states that when a force acts on an object, it will cause the object to accelerate. The larger the mass of the object, the greater the force will need to be to cause it to accelerate. This Law may be written as force = mass x acceleration or:

$F = m a$

If you apply more force to an object, it accelerates at a higher rate.

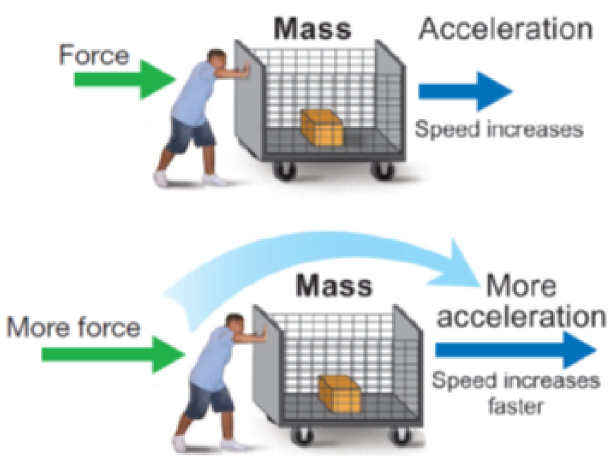


Fig: Example of Newton's second law

If the same force is applied to an object with greater mass, the object accelerates at a slower rate because mass adds inertia.

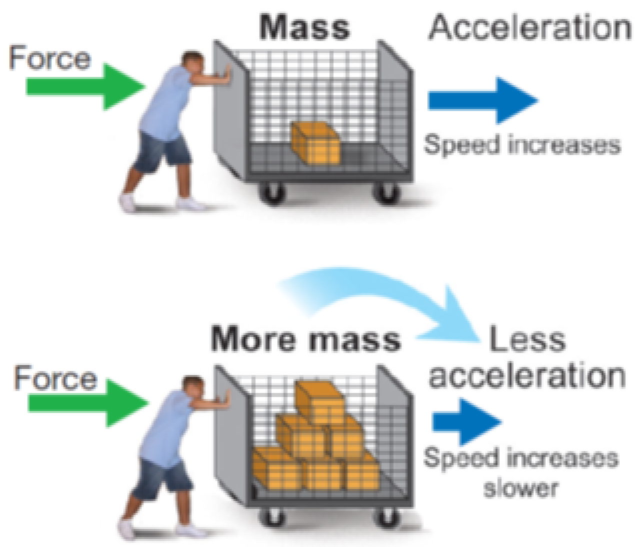


Fig: Example of Newton's second law

Newton's Third Law of Motion

Netwon's Third Law of Motion states that for every action, there is an equal and opposite reaction.

What this means is that pushing on an object causes that object to push back against you, the exact same amount, but in the opposite direction.

Force:

Force is a kind of impact, external or internal which tends to change or in real sense it changes the inertia of any object.
Or, force is what we call a push or a pull, or any action that has the ability to change an object's situation.

Forces can be used to increase the speed of an object, decrease the speed of an object, or change the direction in which an object is moving.

The unit of force is Newton (N). All forces in nature are classified into four basic forces:

1. The gravitational force, which originates with the presence of matter.

2. The electromagnetic force, which includes basic electric and magnetic interactions and is responsible for the binding of the atoms and structure of solids

3. The week nuclear force, which causes certain radioactive decay processes and certain reactions among the fundamental particles, and

4. The strong forces, which operates among the fundamental particles and is responsible for binding the nucleus together.

Projectile motion:

Projectile motion is a form of motion where an object moves in parabolic path; the curved path that the object follows is called its trajectory. Projectiles move in TWO dimensions
Since a projectile moves in 2- dimensions, it therefore has 2 components just like a resultant vector.
Horizontal and
Vertical component.

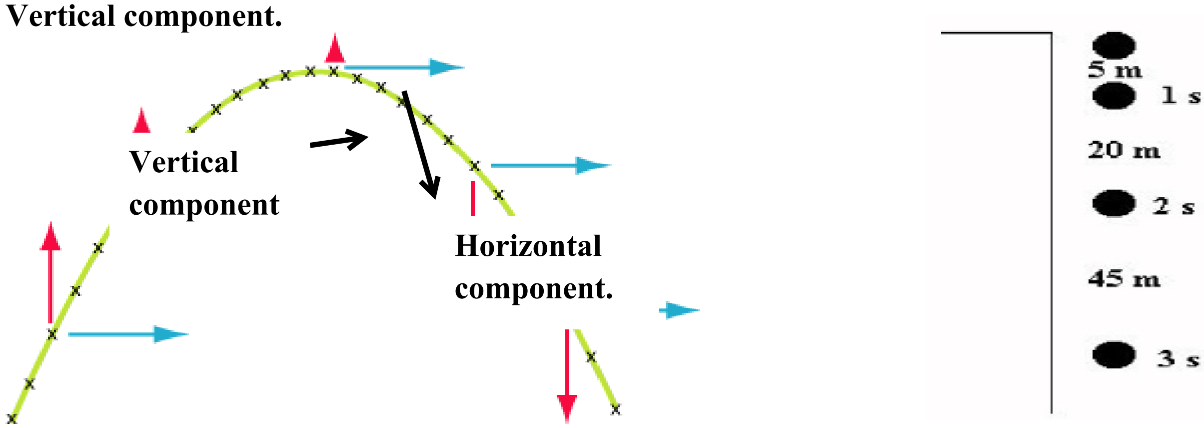


Fig: Trajectory of a projectile.

Horizontal “Velocity” Component neverchanges, covers equal displacements in equal time periods. This means the initial horizontal velocity equals the final horizontal
Velocity In other words, the horizontalvelocity is constant. But why?
Gravity doesnot work horizontally to increase or decrease the velocity.
On the other hand, vertical velocity component Changes (due to gravity), does notcover equal displacements in equal time periods. Both the magnitude and direction change. Asthe projectile moves up the magnitude decreases and its direction is upward. As it moves down the magnitude increases and the direction is downward. Together, these components produce what is called a **trajectory** or path. This path is **parabolic** in nature.

Equation of projectile motion:

Let us consider an object is projected with an initial velocity v_0 directed at an angle form the horizontal. We will separate the motion into horizontal motion (motion along x-axis) and vertical motion (motion along y-axis)

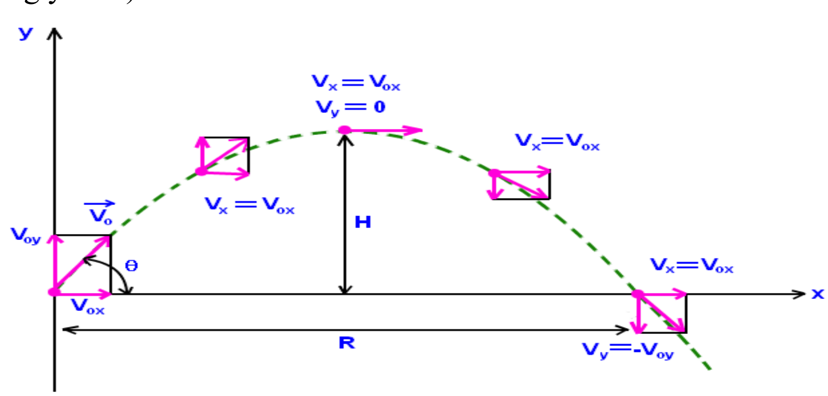
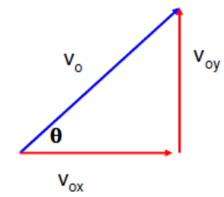


Fig: Projectile motion of an object.

Components of initial velocity along x and y axes are:



$v_{x0} = v_0 \cos$

$V_{y0} = v_0 \sin$ .... (1)

Acceleration along x-axis

$\mathbf{a_x = 0}$

Because no force is acting along the horizontal direction.

And the Acceleration along y-axis

$\mathbf{a_y = -g = -9.8 m/s^2}$(g is negative as it is acting in the downward direction)

Component of velocity along the x-axis at any instant **t**.

$\mathbf{v_x = v_{xo} + a_x t}$

$\mathbf{= v_o \cos + 0}$

$\mathbf{= v_o \cos}$**……………..(2)**

This means that the horizontal component of velocity does not change throughout the projectile motion.

Whereas, for y axis we can write,

$\mathbf{v_y = v_{yo} + a_y t}$

$\mathbf{v_y = v_o \sin - gt}$ **……………….. (3)**

The displacement along x-axis at any instant, **t**

$\mathbf{x = v_{xo} t + (1/2)\, a_x t^2}$

$\mathbf{x = v_0 \cos . t}$ **………………….(4)**

and for y axis

$\mathbf{y = v_{yo} t + (1/2)\, a_y t^2}$

$\mathbf{y = v_o \sin . t - (1/2) g t^2}$ **………………… (5)**

At any instant **t,**

$\mathbf{x = v_o \cos . t}$

t=

Also, $\mathbf{y = v_o \sin . t - (1/2) g t^2}$

Substituting the value of t in eq. (5),

y= -

y= tan -

This equation is of the form $\mathbf{y = ax + bx^2}$ where 'a' and 'b are constants.This is the equation of a parabola.Thus,the path of a projectile is a parabola.

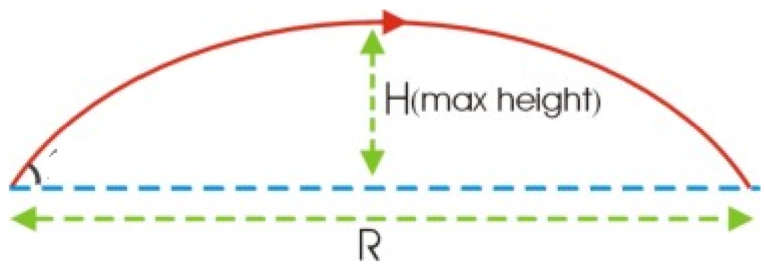
At any point, the magnitude of the velocity is

v=

and its direction is given by,

tan =

Where is the angle that the resultant velocity(v) makes with the horizontal at any instant.



Time of flight T:

Angular Projectile motion is symmetrical about the highest point.The object will reach the highest point in timeT/2.At the highest point,the vertical component of velocity $\mathbf{v_y}$ becomes equal to zero.
By using eq. (3)
$v_y = v_o$ sin - gt
at **t=T/2, v_y= 0**
0= v_o sin–

***T*=**

Maximum height H

Equation for vertical distance (y component)
y= $v_o$sin.t - (1/2)gt^2
At, **t =T/2, y=H**

H= - g $(T/2)^2$

Substituting **T,**

H= $v_o$sin. $v_o$sin/g - (1/2) g$(v_o\text{sin}/g)^2$
= (v_o^2sin^2)/g - (v_o^2sin^2)/2g

H= (v_o^2sin^2)/2g

<u>Range R</u>

Range is the total horizontal distance covered during the time of flight.
From equation for horizontal motion,
x=v_{xo}t

When **t=T, x=R**

R= $v_{xo}T$
= $v_o$cos.2$v_o$sin/g
= v_o^2 2sincos/g
= v_o^2sin2/g [using 2sincos= sin2]
R= (v_o^2sin2)/g
When,sin2the value of R will be maximum.

sin2= 1
2 =
=
Putting this value in eq.1, we can write,
R_{max} =

R_{max} =

Problems:

1. A soccer player kicks a ball at an angle of 37^0 from the horizontal with an initial speed of 20 m/s. Assume that the ball moves in a vertical plane

a) Find the time t_1 at which the ball reaches the highest point of its trajectory.

b) How high does the ball go?

c) What is the horizontal range of the ball and how long is it in the air?

d) What is the velocity of the ball as it strikes ground?

2. In a contest to drop a package on a target, one contest's plane is flying at a constant horizontal velocity of 155 Km/h at an elevation of 225 m toward a point directly above the target. At what angle of sight should the package be released to strike the target?

3. A bomber is flying at a constant horizontal velocity of 820 miles/hr at an elevator of 52000 ft toward a point directly above its target. At what angle of sight should a bomb be released to strike the target? (g= 32ft/sec^2)

4. A rescue plane flies at 123 miles/hr and constant height h=1640 ft. toward a point directly over a victim, where a rescue capsule is to land. What should be the angle of the pilot's line of sight to the victim when the capsule release is made? (Take g= 32 ft/s^2)