" Complex Number "

Ans: (1) Classification of Number System.

- Number
  - Real Number
    - Retional Number
      - Intreger
        - Negative
        - Zero
        - Positive
          - Prime Number
          - Composite
      - Fraction
        - proper
        - mixed
        - Improper
    - Irretional
  - Complex Number
    - Pure Real
    - Imaginary

(2) Prime factorization of 540 with Tree Structure.

540
2 270
2 135
5 27
3 9
3 3

So, Prime factorization of 540 is $= 2^2 \cdot 3^3 \cdot 5$

(3) Find the all factor of 540 are–

$$540 = 1\times540$$
$$= 2\times270$$
$$= 3\times180$$
$$= 4\times135$$
$$= 5\times108$$
$$= 6\times90$$
$$= 9\times60$$
$$= 10\times54$$
$$= 12\times45$$
$$= 15\times36$$
$$= 18\times30$$
$$= 20\times27$$

Therefore, all factor of 540 are: 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 60, 90, 108, 135, 180, 270, 540

4) GCD and Lcm of 240 and 540:

2 | 240
2 | 120
2 | 60
2 | 30
3 | 15
5

∴ prime factorization of $240 = 2^4 \cdot 3^1 \cdot 5^1$

2 | 540
2 | 270
5 | 135
3 | 27
3 | 9
3

∴ Prime factorization $540 = 2^2 \cdot 3^3 \cdot 5^1$

Therefore, GCD and Lcm are:- $GCD = 2^2 \cdot 3^1 \cdot 5^1$
$= 4\times3\times5 = 60$

and, $Lcm = 2^4 \cdot 3^3 \cdot 5^1$
$= 2160$

Answer;

(5) HCF and LCM of 42, 63, 140

$42 = 2 \times 21 = 2 \times 3 \times 7$

$63 = 7 \cdot 9 = 7 \cdot 3 \cdot 3 = 7 \cdot 3^2$

$140 = 2 \cdot 70 = 2 \cdot 2 \cdot 35 = 2 \cdot 2 \cdot 5 \cdot 7 = 2^2 \cdot 5 \cdot 7$

Therefore, HCF and LCM of 42, 63, 140

HCF = 7

$LCM = 2^2 \cdot 3^2 \cdot 5 \cdot 7 = 1260$

(6) Finding the LCM and HCF of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$

Numerator Side,

$2 = 2^1$

$8 = 2 \cdot 4 = 2 \cdot 2 \cdot 2 = 2^3$

$16 = 2 \cdot 8 = 2 \cdot 2 \cdot 4 = 2 \cdot 2 \cdot 2 \cdot 2 = 2^4$

$10 = 2 \cdot 5$

$\therefore$ LCM $(2, 8, 16, 10) = 2^4 \cdot 5 = 80$

HCF $= 2^1 = 2$

Denominator side,

$3 = 3^1$

$9 = 3 \cdot 3 = 3^2$

$81 = 9 \cdot 9 = 3 \cdot 3 \cdot 3 \cdot 3 = 3^4$

$27 = 3 \cdot 9 = 3 \cdot 3 \cdot 3 = 3^3$

$\therefore$ LCM of $(3, 9, 81, 27) = 3^4 = 81$

HCF $= 3^1 = 3$

Therefore, LCM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{80 \text{ (LCM)}}{3}$

HCM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{2 \text{ (HCF)}}{81}$

Ans:-

(7) Finding modulus, Argument and polar;

$$z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$$

$$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$$

$$= \frac{(1+\sqrt{3})^2}{1^2+(\sqrt{3})^2}$$

$$= \frac{1^2+2\sqrt{3}i+(\sqrt{3}i)^2}{1+3}$$

$$= \frac{1+2\sqrt{3}i-3}{4} = \frac{-2+2\sqrt{3}i}{4} = -\frac{1}{2}+\frac{\sqrt{3}}{2}i \quad [x+i$$

So, $x = -\frac{1}{2}$ and $y = \frac{\sqrt{3}}{2}$

$\therefore$ Modulus, $|z| = \sqrt{x^2+y^2}$

$$= \sqrt{\left(-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \sqrt{\frac{1}{4}+\frac{3}{4}}$$

$$= 1$$

Argument,

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$= \pi - \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right)$$

$$= \cancel{\pi}\tan^{-1}(-\sqrt{3})$$

$$= \pi - \tan^{-1}(\sqrt{3})$$

$$= \pi - 60^\circ = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Polar form,

$$z = r(\cos\theta + i\sin\theta)$$

$$= 1 \cdot \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

$$= \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

Ans:-

(8) Evolute :-

$$\sqrt{-16} \times \sqrt{-4}$$
$$= i\sqrt{16} \times i\sqrt{4}$$
$$= i4 \times 2i = 8i$$
$$= 8(-1) = -8$$

and, $\frac{\sqrt{-16}}{\sqrt{-4}}$

$$= \frac{i\sqrt{16}}{i\sqrt{4}} = \frac{i4}{i2}$$
$$\Rightarrow \frac{4i}{2i} = 2$$ Ans:-

(9) Evolute modulus and Argument :-

$$8z - z^2 \quad [z = 2+i]$$
$$= 8(2+i) - (2+i)^2$$
$$= 16 + 8i - \cancel{4}\, 2^2 - 2 \cdot 2 \cdot i + (i)^2$$
$$= 16 + 8i - 4 + 4i + (1)^2$$
$$= 12 + 4i + 1$$
$$= 13 + 4i$$ so, $x = 13$ and $y = 4$

Therefore, Modulus $|z| = \sqrt{13^2 + 4^2}$
$$= \sqrt{169 + 16}$$
$$= \sqrt{185}$$

Argument,
$$\theta = \tan^{-1} \left| \frac{4}{3} \right|$$
$$= \tan^{-1} \frac{4}{3}$$

Ans:-

10) Express :- $r(\cos\theta + i\sin\theta)$ from, $1 + i\sqrt{3}$

So, $x = 1$ and, $y = \sqrt{3}$

we know, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$= \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$= \tan^{-1}(\sqrt{3})$

$= 60° = \frac{\pi}{3}$

So, $z = r(\cos\theta + i\sin\theta)$

$= \cancel{1(\cos 60° + i\sin 60°)}$

$= 1\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

Therefore, from of $1 + i3 = \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ Ans: