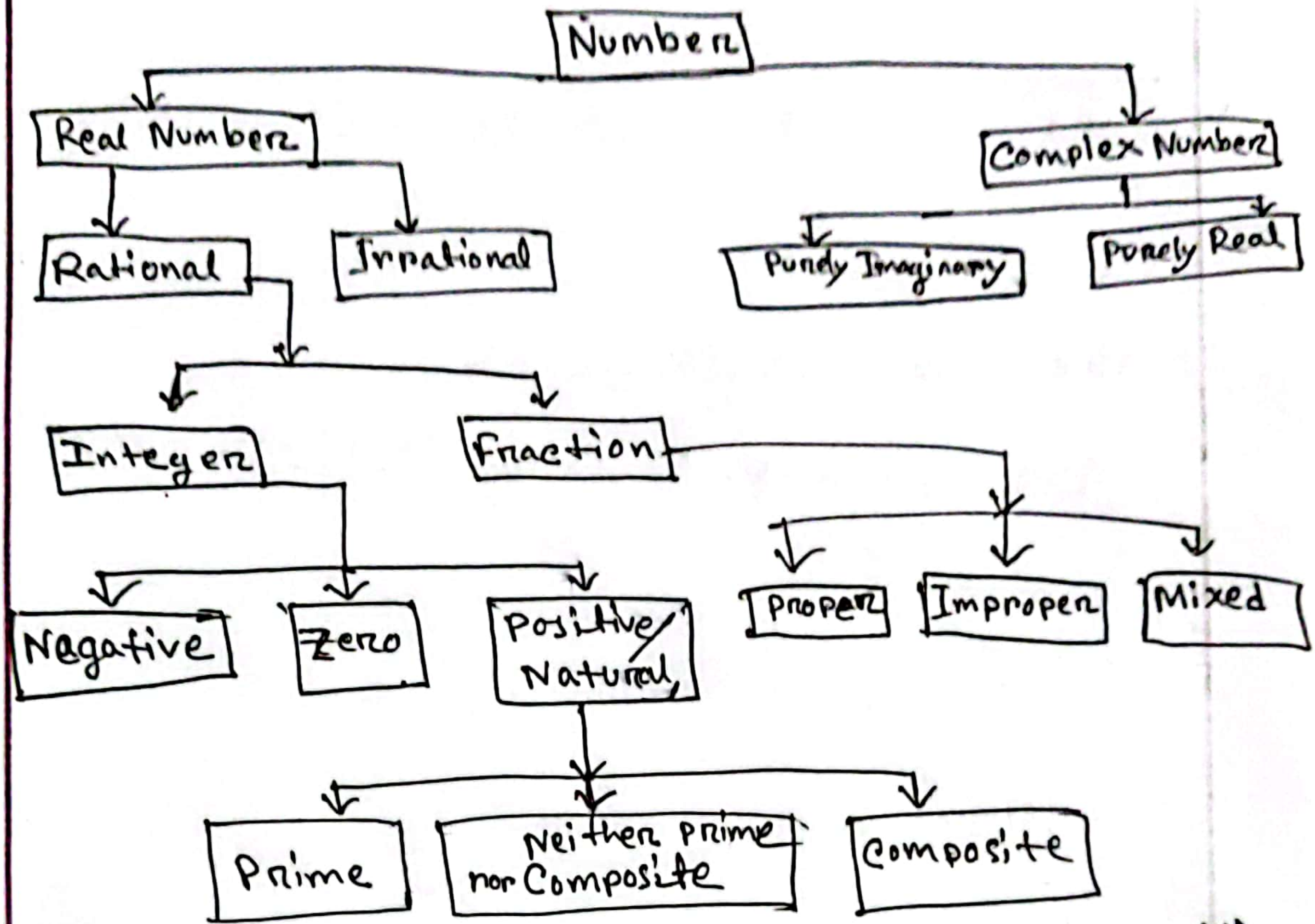
Mathematics - III

H.W

Complex Number

1| Write down the Classification of Number System?

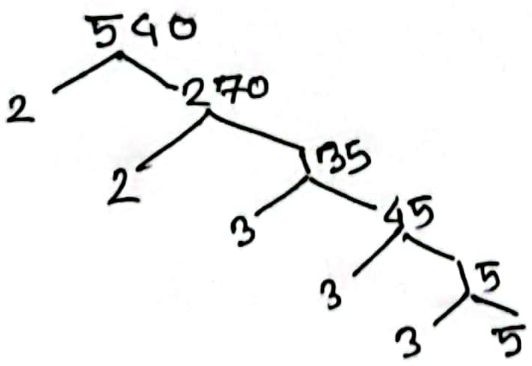
⇒ Classification of Number System-



Ans

2| Find the prime factorization of 540 using tree.

⇒



∴ The prime factorization of 540 is $= 2^2 \cdot 3^3 \cdot 5^1$.

Ans

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3. Find out the ~~pr~~ all factors of 540.

$\Rightarrow$

$$\begin{array}{r|l} 2 & 540 \\ \hline 2 & 270 \\ \hline 3 & 135 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

Calculation for all factor 540 = 1 × 540
= 2 × 270
= 3 × 180
= 4 × 135
= 5 × 108
= 6 × 90
= 9 × 60
= 10 × 54
= 12 × 45
= 15 × 36
= 18 × 30
= 20 × 27

∴ The prime factorization of 540 is = $2^2 \cdot 3^3 \cdot 5$

So, the total number of factors of 540 is = $(2+1)(3+1)(1+1)$
= $3 \cdot 4 \cdot 2$
= 24

The factor of 540 are -
1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540.

Ans

4. What is the GCD and LCM of 240 and 540.

$\Rightarrow$ $240 = 2 \times 120 = 2 \times 2 \times 60 = 2 \times 2 \times 2 \times 30 = 2 \times 2 \times 2 \times 2 \times 15 = 2^4 \times 3 \times 5$

$540 = 2 \times 270 = 2 \times 2 \times 135 = 2^2 \times 3 \times 45 = 2^2 \times 3 \times 3 \times 15 = 2^2 \times 3^3 \times 5$

$\therefore LCM(240, 540) = 2^4 \cdot 3^3 \cdot 5 = 2160$

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$\therefore$ $GCD = 2^2 \cdot 3 \cdot 5 = 60$.

Ans

5) Find the H.C.F and L.C.M of 42, 63, 140.

$\Rightarrow$

$42 = 2 \times 21 = 2 \times 3 \times 7$

$63 = 3 \times 21 = 3 \times 3 \times 7 = 3^2 \times 7$

$140 = 2 \times 70 = 2 \times 2 \times 35 = 2^2 \times 5 \times 7$

$\therefore$ $LCM(42, 63, 140) = 2^2 \times 3^2 \times 5 \times 7 = 1260$

and $H.C.F(42, 63, 140) = 7$

Ans

6) Find the H.C.F and L.C.M of $\frac{2}{3}$, $\frac{8}{9}$, $\frac{16}{81}$ and $\frac{10}{27}$.

$\Rightarrow$

Calculation ~~for~~ of Numerators

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

$10 = 2^1 \times 5^1$

$LCM = (2, 8, 16, 10) = 2^4 \times 5 = 80$

$HCF = (2, 8, 16, 10) = 2$

Calculation of Denominators

$3 = 3^1$

$9 = 3^2$

$81 = 3^4$

$27 = 3^3$

$LCM = (3, 9, 81, 27) = 3^4 = 81$

$HCF = (3, 9, 81, 27) = 3$

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HCF of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{HCF(2,8,16,10)}{LCM(3,9,81,27)} = \frac{2}{81}$

LCM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{LCM(2,8,16,10)}{HCF(3,9,81,27)} = \frac{80}{3}$.

Ans

7. Find the modulus and Argument of $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$ and also its polar, exponential form.

⇒

$$z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$$

$$= \frac{(1+\sqrt{3}i)\ (1+\sqrt{3}i)}{(1-\sqrt{3}i)\ (1+\sqrt{3}i)}$$

$$= \frac{1^2+2.1.\sqrt{3}i+(\sqrt{3}i)^2}{(1)^2-(\sqrt{3}i)^2}$$

$$= \frac{1+2\sqrt{3}i+3i^2}{1-3i^2}$$

$$= \frac{1+2\sqrt{3}i-3}{1+3}$$

$$= \frac{-2+2\sqrt{3}i}{4}$$

$$= \frac{-2}{4}+\frac{2\sqrt{3}i}{4}$$

$$= -\frac{1}{2}+\frac{\sqrt{3}i}{2}$$

where $x = -\frac{1}{2}$ and $y = \frac{\sqrt{3}i}{2}$

Now, $r=\sqrt{x^2+y^2}$

$$=\sqrt{\left(-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$=\sqrt{\frac{1}{4}+\frac{3}{4}}$$

$$=\sqrt{\frac{1+3}{4}}$$

$$=\sqrt{1}$$

$$=1$$

and $\theta=\pi-\tan^{-1}\left(\frac{y}{x}\right)$

$$=\pi-\tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right)$$

$$=\pi-\tan^{-1}(-\sqrt{3})$$

$$=\pi-\tan^{-1}(\sqrt{3})$$

$$=\pi-\pi/3$$

$$=\frac{2\pi}{3}$$

so, the polar form is $z=r(\cos\theta+i\sin\theta)$

$$=1.\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$$

$$=\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}$$

and. Exponential form $z=e^{i\frac{2\pi}{3}}$.

Ans

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8) Evaluate $\sqrt{-16} \times \sqrt{-4}$ and $\frac{\sqrt{-16}}{\sqrt{-4}}$

$\Rightarrow$ we know, $i^2 = -1$

Now, $\sqrt{-16} \times \sqrt{-4}$

$= \sqrt{16 i^2} \times \sqrt{4 i^2}$

$= 4i \times 2i$

$= 8i^2$

$= -8$

and $\frac{\sqrt{-16}}{\sqrt{-4}}$

$= \frac{\sqrt{16 i^2}}{\sqrt{4 i^2}}$

$= \frac{4i}{2i}$

$= 2$

Ans.)

9) Evaluate Modulus and Argument of $8z - z^2$ by replacing $z = 2+i$.

$\Rightarrow$ Here, $8z - z^2$

$= 8(2+i) - (2+i)^2$

$= 16 + 8i - (4 + 4i + i^2)$

$= 16 + 8i - 4 - 4i - i^2$

$= 16 + 8i - 4 - 4i + 1$

$\therefore z = 13 + 4i$

Modulus $r = \sqrt{x^2 + y^2}$

$= \sqrt{(13)^2 + 4^2}$

$= \sqrt{169 + 16}$

$= \sqrt{185}$ $= 13.60147051$

$\therefore \theta = \tan^{-1} \frac{y}{x}$

$= \tan^{-1}\left(\frac{4}{13}\right)$

$= \tan^{-1}(0.307692$

$= 17.10272897$

$\therefore$ So, the modulus $r = 13.60147051$

and Argument $\theta = 17.10272897$

Ans

10| Express $1+i\sqrt{3}$ in the form of $r(\cos\theta + i\sin\theta)$

$\Rightarrow$

Here, $z = x+iy$

$= 1 + i\sqrt{3}$

$\therefore x = 1$

and $y = \sqrt{3}$

Modulus, $r = \sqrt{x^2+y^2}$

$= \sqrt{1^2+(\sqrt{3})^2}$

$= \sqrt{1+3}$

$= 2$

Argument $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$= \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$= \tan^{-1}(\sqrt{3})$

$= \frac{\pi}{3}$

Therefore, The polar form of $1+i\sqrt{3}$ is $= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

(Ans)

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