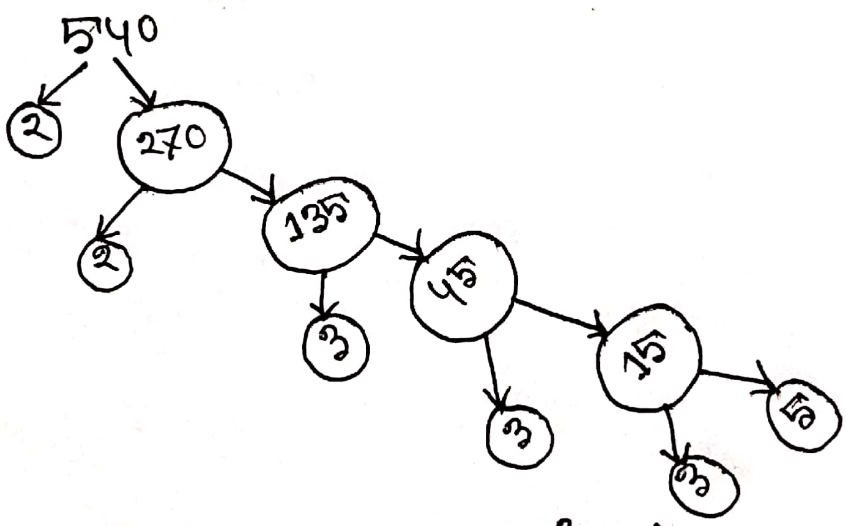
1. Write down the classification of number system.

- Number
  - Real Number
    - Rational Number
      - Integer
        - Negetive Number
        - Zero
        - Natural Number
          - Prime Number
          - Neither Prime nor Composite
          - Composite Number
      - Fraction
        - Proper Fraction
        - Improper Fraction
        - Mixed
    - Irrational Number
  - Complex Number
    - Purely Real
    - Purely Imaginary

2. Find the prime factorization of 540 using tree.



∴ Prime factor of 540 is : $2^2 . 3^3 . 5^1$

3. Find out the all factors of 540.

Ans: Total number of factor $540 = (2+1)\cdot(3+1)\cdot(1+1)$

$= 3 \cdot 4 \cdot 2$

$= 24$

Calculation of all factors:

$540 = 1 \times 540$

$= 2 \times 270$

$= 3 \times 180$

$= 4 \times 135$

$= 5 \times 108$

$= 6 \times 90$

$= 9 \times 60$

$= 10 \times 54$

$= 12 \times 45$

$= 15 \times 36$

$= 18 \times 30$

$= 20 \times 27$

$\therefore$ All the factors of 540: 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540

4. What is the GCD & LCM of 240 & 540

Ans: Given numbers are 240 & 540.
Therefore, the prime factorization of 240 and 540 are -

$240 = 2 \cdot 120 = 2^4 \cdot 5 \cdot 3$

$540 = 2 \cdot 270 = 2^2 \cdot 3^3 \cdot 5$

$\therefore$ G.C.D $= 2^2 \cdot 3 \cdot 5 = 60$

$\therefore$ L.C.M $= 2^4 \cdot 3^3 \cdot 5 = 2160$

5. Find the H.C.F and L.C.M of 42, 63, 140

Ans: Given numbers are 42, 63, 140.
There, the prime factorization of 42, 63 & 140 are -

$42 = 6 \cdot 7 = 2 \cdot 3 \cdot 7$

$63 = 7 \cdot 9 = 3^2 \cdot 7$

$140 = 2 \cdot 70 = 2^2 \cdot 5 \cdot 7$

$\therefore$ H.C.F. $= 7$

$\therefore$ L.C.M $= 2^2 \cdot 3^2 \cdot 5 \cdot 7 = 1260$

6. Find the H.C.F and L.C.M of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$

Calculation of Numerators

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

$10 = 2^1 . 5^1$

$\therefore$ L.C.M $= 2^4 . 5 = 80$

$\therefore$ H.C.M $= 2^1 = 2$

Calculation of Denominators

$3 = 3^1$

$9 = 3^2$

$81 = 3^4$

$27 = 3^3$

L.C.M $= 3^4 = 81$

H.C.F $= 3^1 = 3$

$\therefore$ L.C.M. of Fraction $= \frac{80}{3}$

$\therefore$ H.C.F. of fraction $= \frac{2}{81}$

7. Find the modulus and Argument of $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$ and also its polar, exponential form.

Ans: $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$

$$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$$

$$= \frac{1^2 + 2.1.\sqrt{3}i + (\sqrt{3}i)^2}{(1)^2 - (\sqrt{3}i)^2}$$

$$= \frac{1 + 2\sqrt{3}i + 3i^2}{1 - 3i^2}$$

$$= \frac{1+2\sqrt{3}i-3}{1+3}$$

$$= \frac{-2+2\sqrt{3}i}{4}$$

$$= -\frac{2}{4} + \frac{2\sqrt{3}}{4}i$$

$$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$= x + iy$$

where $x = -\frac{1}{2}$ & $y = \frac{\sqrt{3}}{2}$

Now, $r = \sqrt{x^2+y^2}$

$$= \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \sqrt{1/4 + 3/4}$$

$$= \sqrt{1}$$

$$= 1$$

And, $\theta = \pi - \tan^{-1}\left(\frac{y}{x}\right)$

$$= \pi - \tan^{-1}\left(\frac{\sqrt{3}/2}{-1/2}\right)$$

$$= \pi - \tan^{-1}\left(-\sqrt{3}\right)$$

$$= \pi - \tan^{-1}(\sqrt{3})$$

$$= \pi - \pi/3$$

$$= 2\pi/3$$

∴ So, the polar form is, $z = r(\cos\theta + i\sin\theta)$

$= \cancel{1 \cdot (\cos\theta}$

$= 1(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})$

$= (\cos 2\pi/3 + i\sin 2\pi/3)$

∴ Exponential form, $z = e^{i 2\pi/3}$

8. Evaluate $\sqrt{-16} \times \sqrt{-4}$ & $\frac{\sqrt{-16}}{\sqrt{-4}}$

Ans: We have $i^2 = -1$

Now, $\sqrt{-16} \times \sqrt{-4}$

$= \sqrt{16i^2} \times \sqrt{4i^2}$

$= \sqrt{4^2 \cdot i^2} \times \sqrt{2^2 \cdot i^2}$

$= 4i \times 2i$

$= 8i^2$

$= -8$

Again, $\frac{\sqrt{-16}}{\sqrt{-4}}$

$= \frac{\sqrt{16i^2}}{\sqrt{4i^2}}$

$= \frac{\sqrt{4^2 \cdot i^2}}{\sqrt{2^2 \cdot i^2}}$

$= \frac{4i}{2i}$

$= 2$ Ans!

9. Evaluate Modulous & Argument of $8z-z^2$ by replacing $z=2+i$

Ans: Here, $8z-z^2$

$$= 8(2+i) - (2+i)^2$$

$$= 16+8i - (4+4i+i^2)$$

$$= 16+8i - 4 - 4i - i^2$$

$$= 16+8i - 4 - 4i + 1$$

$$= 13+4i$$

$\therefore z = x+iy$

$\Rightarrow \quad = 13+4i$

$$\therefore r = \sqrt{x^2+y^2}$$

$$= \sqrt{(13)^2+(4)^2}$$

$$= \sqrt{169+16}$$

$$= \sqrt{185}$$

$$= 13.60147051$$

$$\therefore \theta = \tan^{-1}\left|y/x\right|$$

$$= \tan^{-1}\left|\frac{4}{13}\right|$$

$$= \tan^{-1}(0.3076923)$$

$$= 17.10272897$$

$\therefore$ So the modulus $r = 13.60147051$

and Argument, $\theta = 17.10272897$ (Ans:)

10. Express $1 + i\sqrt{3}$ in the form of $r(\cos\theta + i\sin\theta)$

Ans:

Here, $z = \cancel{1+i\sqrt{x}}\ x + iy$

$= 1 + i\sqrt{3}$

$\therefore x = 1$

& $y = \sqrt{3}$

$\therefore$ Modulus, $r = \sqrt{x^2 + y^2}$

$= \sqrt{(1)^2 + (\sqrt{3})^2}$

$= \sqrt{1+3}$

$= 2$

$\therefore$ Argument, $\theta = \tan^{-1} |y/x|$

$= \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$= \tan^{-1}(\sqrt{3})$

$= \pi/3$

$\therefore$ The polar form of $1 + i\sqrt{3}$ is $= 2(\cos\pi/3 + i\sin\pi/3)$

Ans