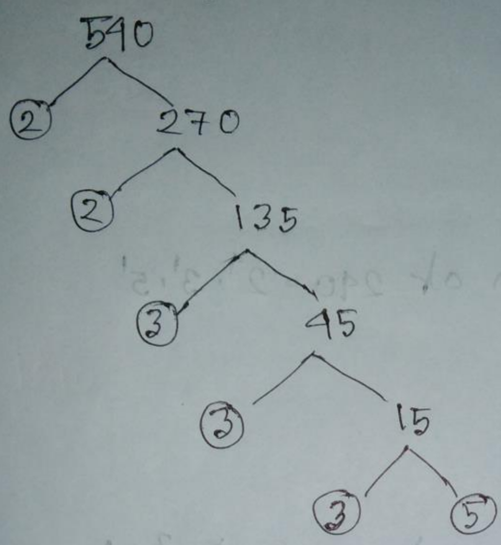
② Tree Diagram :-



③

$540 = 1 \times 540$
$= 2 \times 270$
$= 3 \times 180$
$= 4 \times 135$
$= 5 \times 108$
$= 6 \times 90$
$= 9 \times 60$
$= 10 \times 54$
$= 12 \times 45$
$= 15 \times 36$
$= 18 \times 30$
$= 20 \times 27$
$= 30 \times 18$
$= 36 \times 15$

The prime factors are :-
(1,2,3,4,5,6,9,10,12,15,18,20,30,
36,45,54,60,90,108,135,
180,270,540)

Ans:-

4 GCD and LCM of 240 and 540:

$$\begin{array}{r|l} 2 & 240 \\ \hline 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

$\therefore$ Prime factorization of $240 = 2^4 \cdot 3^1 \cdot 5^1$

$$\begin{array}{r|l} 2 & 540 \\ \hline 2 & 270 \\ \hline 5 & 135 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$\therefore$ Prime factorization of $540 = 2^2 \cdot 3^3 \cdot 5^1$

Therefore, GCD and LCM are :-

$GCD = 2^2 \cdot 3^1 \cdot 5^1$ and $LCM = 2^4 \cdot 3^3 \cdot 5^1$

$= 4 \times 3 \times 5$ $= 2160$

$= 60$

5 HCF and LCM of 42, 63 and 140

$42 = 2 \times 21 = 2 \times 3 \times 7$

$63 = 7 \cdot 9 = 7 \cdot 3 \cdot 3 = 7 \cdot 3^2$

$140 = 2 \cdot 70 = 2 \cdot 2 \cdot 35 = 2 \cdot 2 \cdot 5 \cdot 7 = 2^2 \cdot 5 \cdot 7$

Therefore, HCF and LCM of 42, 63, 140

$HCF = 7$

$LCM = 2^2 \cdot 3^2 \cdot 5 \cdot 7 = 1260$

⑥ Finding the LCM and HCF of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$

Numerator side,

$2 = 2^1$

$8 = 2 \cdot 4 = 2 \cdot 2 \cdot 2 = 2^3$

$16 = 2 \cdot 8 = 2 \cdot 2 \cdot 4 = 2 \cdot 2 \cdot 2 \cdot 2 = 2^4$

$10 = 2 \cdot 5$

$\therefore$ LCM$(2, 8, 16, 10) = 2^4 \cdot 5 = 80$

HCF $= 2^1 = 2$

Denominator side,

$3 = 3^1$

$9 = 3 \cdot 3 = 3^2$

$81 = 9 \cdot 9 = 3 \cdot 3 \cdot 3 \cdot 3 = 3^4$

$27 = 3 \cdot 9 = 3 \cdot 3 \cdot 3 = 3^3$

$\therefore$ LCM$(3, 9, 81, 27) = 3^4 = 81$

HCF $= 3^1 = 3$

Therefore, LCM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{80}{3}$

HCF of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{2}{81}$.

⑦ Finding Modulus, Argument and Polar:

$$z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$$

$$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)\ (1+\sqrt{3}i)}$$

$$= \frac{(1+\sqrt{3}i)^2}{1^2 - (\sqrt{3}i)^2}$$

$$= \frac{1+2\sqrt{3}i+(\sqrt{3}i)^2}{1+3}$$

$$= \frac{1+2\sqrt{3}i-3}{4}$$

$$= \frac{-2+2\sqrt{3}i}{4}$$

$$= -\frac{1}{2} + \frac{\sqrt{3}i}{2} \quad (x+iy)$$

So, $x = -\frac{1}{2}$ and $y = \frac{\sqrt{3}}{2}$

Modulus, $|z| = \sqrt{x^2+y^2}$

$$= \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \sqrt{\frac{1}{4} + \frac{3}{4}}$$

$$= 1$$

Argument,

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$= \pi - \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right)$

$= \pi - \tan^{-1}(-\sqrt{3})$

$= \pi - \tan^{-1}(\sqrt{3})$

$= \pi - 60^\circ$

$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Polar form,

$z = r(\cos\theta + i\sin\theta)$

$= 1\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$= \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$ Ans.

⑧ Evalute :-

$\sqrt{-16} \times \sqrt{-4}$

$= i\sqrt{16} \times i\sqrt{4}$

$= i4 \times 2i$

$= 8i^2$

$= 8(-1)$

$= -8$

and,

$$\frac{\sqrt{-16}}{\sqrt{-4}}$$

$$= \frac{i\sqrt{16}}{i\sqrt{4}}$$

$$= \frac{i4}{i2} = 2$$ Ans:-

(9) Evolute modulus and Argument:-

$8z - z^2$

$= 8(2+i) - (2+i)^2 \quad [z = 2+i]$

$= 16 + 8i - 2^2 - 2\cdot 2\cdot i + (i)^2$

$= 16 + 8i - 4 - 4i + i^2$

$= 12 + 4i + 1$

$= 13 + 4i$ So, $x = 13$ and $y = 4$

Therefore,

Modulus, $|z| = \sqrt{x^2 + y^2}$

$= \sqrt{13^2 + 4^2}$

$= \sqrt{169 + 16}$

$= \sqrt{185}$

Argument,

$\theta = \tan^{-1}\left|\frac{y}{x}\right|$

$= \tan^{-1} \frac{4}{13}$

(10) Express :- $r(\cos\theta + i\sin\theta)$ From, $1 + i\sqrt{3}$

So, $x = 1$ and $y = \sqrt{3}$

We know,

$$\theta = \tan^{-1} \left|\frac{y}{x}\right|$$

$$= \tan^{-1} \left|\frac{\sqrt{3}}{1}\right|$$

$$= \tan^{-1} \sqrt{3}$$

$$= \tan^{-1} 60^\circ$$

$$= \frac{\pi}{3}$$

So, $z = r(\cos\theta + i\sin\theta)$

$$= 1\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

Therefore,

from of $1 + i\sqrt{3} = \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

Ans-