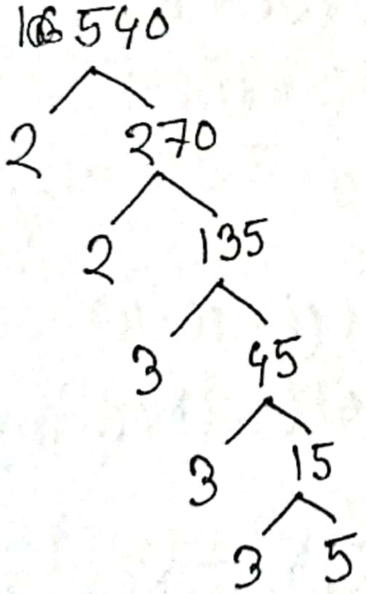
1. write down the classification of number system.

The classification of numbers are, real number, imaginary number, irrational number, integers, and natural numbers.

Real numbers are, numbers that land somewhere on a number line. Irrational numbers are numbers that cannot be written as a fraction, and include never-ending decimal numbers, like π.

2. Find the prime factorization of 540 using tree.

Tree Diagram:



There for the prime factonization of 540 is $= 2^2 \cdot 3^3 \cdot 5$

3.
There for the prime factorization of 540 is $= 2^2 \cdot 3^3 \cdot 5$.

So the total number of factors of 540 is

$$(2+1)(3+1)(1+1) = 24.$$

Calculation of all factors

$540 = 1 \times 540$
$= 2 \times 270$
$= 3 \times 180$
$= 4 \times 135$
$= 5 \times 108$
$= 6 \times 90$
$= 9 \times 60$
$= 10 \times 54$
$= 12 \times 45$
$= 15 \times 36$
$= 18 \times 30$
$= 20 \times 27$

The factors of 540 are
1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36,
45, 54, 60, 90, 108, 135, 180, 270, 540.

4. What is the GCD and LCM of 240 and 540.

Ans: Given numbers are 240, 540.

$240 = 2.120 = 2.2.60 = 2.2.2.30 = 2.2.2.2.15 = 2.2.2.2.3.5$

$= 2^4.3.5.$

$540 = 2 \times 270 = 2.2.135 = 2^2.3.45 = 2^2.3.3.15 = 2^2.3.3.3.5$

$= 2^2.3^3.5$

$LCM(240, 540) = 2^4.3^3.5 = 2160.$

$GCD(240, 540) = 2^2.3.5 = 60.$

5. Find the H.C.F and L.C.M. of 42, 63, 140.

$42 = 2.21 = 2.3.7.$ $\quad 140 = 2.70 = 2.2.35 = 2.2.5.7 = 2^2.5.7$

$63 = 7.9 = 3.3.7$

$L.C.M(42, 63, 140) = 2^2.3^2.5.7 = 420.$

$G.C.M(42, 63, 140) = 7$

6. Find the H.C.F & L.C.M of $2/3$, $8/9$, $16/81$ and $\frac{10}{27}$,

Ans:

Calculation for Numerators

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

$10 = 2.5$

Calculation for Denominators

$3 = 3^1$

$9 = 3^2$

$81 = 9.9 = 3.3.3.3 = 3^4$

$27 = 3.9 = 3.3.3 = 3^3$

$$\cancel{Lcm\ of\left(\frac{2}{3},\frac{8}{9},\frac{16}{81},\frac{10}{27}\right) = \frac{L.C.M\ of\ Numerators}{H}}$$

$L.C.M(2,8,16,10) = 2^4.5 = 80$

$HCF(2,8,16,10) = 2^1 = 2$

$L.cm(3,9,81,27) = 3^4 = 81$

$HCF(3,9,81) = 3^1 = 3$

$$L.C.M\ of\left(\frac{2}{3},\frac{8}{9},\frac{16}{81},\frac{10}{27}\right) = \frac{L.C.M\ of\ (2,8,16,10)}{H.C.F\ of\ (3,9,81,27)}$$

$$= \frac{80}{3}$$ (Ans)

$$H.C.F\ of\left(\frac{2}{3},\frac{8}{9},\frac{16}{81},\frac{10}{27}\right) = \frac{H.C.F\ of\ (2,8,16,10)}{L.C.M\ of\ (3,9,81,27)}$$

$$= \frac{2}{81}$$ (Ans)

7. Find the modulus and Argument of $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$ and also polar exponential form.

Given that

$$z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i} = \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)} = \frac{1+2\sqrt{3}i+3i^2}{1-3i^2}$$

$$\Rightarrow \frac{1-3+2\sqrt{3}}{1+3} \quad [i^2 = -1] \quad \Rightarrow \frac{-2+2\sqrt{3}}{4} \Rightarrow -\tfrac{1}{2} + \frac{\sqrt{3}i}{4}$$

$$\therefore z = x+iy = -\tfrac{1}{2} + \frac{\sqrt{3}i}{4}.$$

$\therefore$ modulus of z

$$|z| = r$$
$$= \sqrt{x^2+y^2}$$
$$= \sqrt{(-\tfrac{1}{2})^2 + (\tfrac{\sqrt{3}}{4})^2}$$
$$= \sqrt{\frac{1}{4} + \frac{3}{16}}$$
$$\Rightarrow \sqrt{\frac{4+3}{16}}$$
$$= \frac{\sqrt{7}}{4}$$

Argument of z

$$\arg(z) = \theta$$
$$= 180^\circ - \alpha$$
$$= 180^\circ - \tan^{-1}\left|\frac{y}{x}\right|$$
$$= 180^\circ - \tan^{-1}\left|\frac{\sqrt{3}/4}{-1/2}\right|$$
$$= 180^\circ - 40.\cancel{90}'$$
$$= \cancel{139.1} = 140$$

The polar form of z

$$z = r\cos\theta + ir\sin\theta$$
$$= r(\cos\theta + \sin\theta)$$
$$= \sqrt{7}/4\,(\cos 140 + \sin 140)$$

8. Evaluate $\sqrt{-16}\times\sqrt{-4}$ & $\frac{\sqrt{-16}}{\sqrt{-4}}$

: $\sqrt{-16}\times\sqrt{-4}$

$= i\sqrt{16}\times i\sqrt{4}$

$= 4i\times 2i$

$= 8i^2$

$= -8 \quad \because i^2=-1$

Ans

$\frac{\sqrt{-16}}{\sqrt{-4}}$

$\sqrt{-16} = i\sqrt{16}$

$= i\sqrt{4} = 4i$

$\sqrt{-4} = i\sqrt{4} = 2i$

$\therefore \frac{\sqrt{-16}}{\sqrt{-4}} = \frac{4i}{2i}$

$= 2i$ Ans

9. Evaluate Modulus and Argument of $8z - z^2$ by replacing

$z = 2+i$

Given that

$8z - z^2$

$\Rightarrow 8(2+i) - (2+i)^2$

$= 16+8i-4-4i-i^2$

$= 13+4i = z$

mod $|z| = r$

$= \sqrt{x^2+y^2}$

$= \sqrt{(13)^2+(4)^2}$

$= \sqrt{185}$

Arg $z = \theta$

$= \tan^{-1}\left(\frac{4}{13}\right)$

$= 17.10$ Ans

10. Express $1+i\sqrt{3}$ in the form of $r(\cos\theta + i\sin\theta)$.

Let $z = 1+i\sqrt{3}$

mod $|z| = r$

$= \sqrt{x^2+y^2}$

$= \sqrt{(1)^2+(\sqrt{3})^2}$

$= \sqrt{1+4}$

$= \sqrt{5}$

Arg $z = \theta = \tan^{-1}\left|\frac{y}{x}\right|$

$= \tan^{-1}\left|\frac{\sqrt{3}}{1}\right|$

$= 60°$

$= \pi/3$

$\therefore$ polar form $= r(\cos\theta + i\sin\theta)$

$= \sqrt{5}\,(\cos \pi/3 + i\sin \pi/3)$. Ans.

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