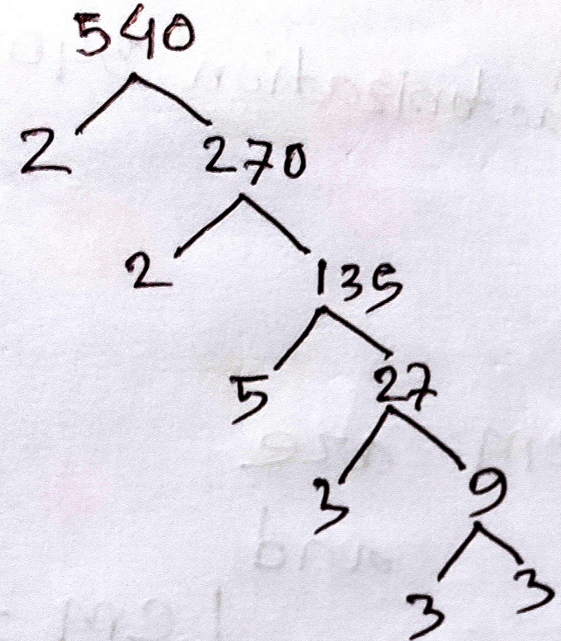
"Complex Number"

Ans:(1) Classification of Number System.

- Number
  - Real number
    - Retional number
      - Intreger
        - Negative
        - Zero
        - Positive
          - Prime
          - Composite
      - Fraction
        - Proper
        - mixed
        - Improper
    - Irretional number
  - Complex Number
    - Pure real
    - Imaginary

(2) Prime factorization of 540 with tree structure.



So,
Prime Factorization of 540 is = $2^2 \cdot 3^3 \cdot 5$

③ All Factors of 540 are,

$540 = 1 \times 540$

$= 2 \times 270$

$= 3 \times 180$

$= 4 \times 135$

$= 5 \times 108$

$= 6 \times 90$

$= 9 \times 60$

$= 10 \times 54$

$= 12 \times 45$

$= 15 \times 36$

$= 18 \times 30$

$= 20 \times 27$

Therefore, all factors of 540 are: 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 60, 90, 108, 135, 180, 270, 540.

④ GCD and LCM of 240 and 540 :-

$$
\begin{array}{r|l}
2 & 240 \\ \hline
2 & 120 \\ \hline
2 & 60 \\ \hline
2 & 30 \\ \hline
3 & 15 \\ \hline
 & 5
\end{array}
$$

$\therefore$ Prime factorization of $240 = 2^4 \cdot 3^1 \cdot 5^1$

$$
\begin{array}{r|l}
2 & 540 \\ \hline
2 & 270 \\ \hline
5 & 135 \\ \hline
3 & 27 \\ \hline
3 & 9 \\ \hline
 & 3
\end{array}
$$

$\therefore$ Prime factorization $540 = 2^2 \cdot 3^3 \cdot 5^1$

Therefore, GCD and LCM are

$GCD = 2^2 \cdot 3^1 \cdot 5^1$

$= 4 \cdot 3 \cdot 5$

$= 60$

and, $LCM = 2^4 \cdot 3^3 \cdot 5^1$

$= 2160$

(Ans.)

(5) HCF and LCM of 42, 63, 140

$42 = 2 \times 21 = 2 \times 3 \times 7$

$63 = 7 \times 9 = 7 \cdot 3 \cdot 3 = 7 \cdot 3^2$

$140 = 2 \times 70 = 2 \cdot 2 \cdot 35 = 2 \cdot 2 \cdot 5 \cdot 7 = 2^2 \cdot 5 \cdot 7$

therefore, HCF and LCM of 42, 63, 140

HCF = 7

$LCM = 2^2 \cdot 3^2 \cdot 5 \cdot 7$

$= 1260$

(6) Finding the LCM and HCF of $\frac{2}{3}$, $\frac{8}{9}$, $\frac{16}{81}$ and $\frac{10}{27}$

Numerator side,

$2 = 2^1$

$8 = 2 \cdot 4 = 2 \cdot 2 \cdot 2 = 2^3$

$16 = 2 \cdot 8 = 2 \cdot 2 \cdot 4 = 2 \cdot 2 \cdot 2 \cdot 2 = 2^4$

$10 = 2 \cdot 5$

$\therefore$ LCM ; $2, 8, 16, 10 = 2^4 \cdot 5$

$= 80$

$HCF = 2^1 = 2$

enominator side,

$3 = 3^1$

$9 = 3 \cdot 3 = 3^2$

$81 = 9 \cdot 9 = 3 \cdot 3 \cdot 3 \cdot 3 = 3^4$

$27 = 3 \cdot 9 = 3 \cdot 3 \cdot 3 = 3^3$

$\therefore$ LCM of $(3, 9, 81, 27) = 3^4$

$= 81$

$HCF = 3^1 = 3$

therefore, LCM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{80}{3}$ (LCM)

HCM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{2}{81}$ (HCF)

(Ans.)

⑦ Finding modulus, Argument, and Polar;

$$z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$$

$$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$$

$$= \frac{(1+\sqrt{3})^2}{1+(\sqrt{3})^2}$$

$$= \frac{1+2\sqrt{3}i+(\sqrt{3}i)^2}{1+3}$$

$$= \frac{1+2\sqrt{3}i-3}{4} = \frac{-2+2\sqrt{3}i}{4} = -\frac{1}{2}+\frac{\sqrt{3}}{2}i$$

SO, $x = -\frac{1}{2}$ and $y = \frac{\sqrt{3}}{2}$

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$x = -\frac{1}{2}$ and $y = \frac{\sqrt{3}}{2}$

∴ Modulus, $|z| = \sqrt{x^2+y^2}$

$$= \sqrt{\left(-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \sqrt{\frac{1}{4}+\frac{3}{4}}$$

$$= 1$$

Argument,

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$= \pi - \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right)$$

$$= \pi - \tan^{-1}(\sqrt{3})$$

$$= \pi - \tan^{-1}(\sqrt{3})$$

$$= \pi - 60^\circ = \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

Polar form,

$$z = r(\cos\theta + \sin\theta)$$

$$= 1\cdot\left(\cos\frac{2\pi}{3} + \sin\frac{2\pi}{3}\right)$$

$$= \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

(Ans.)

(8) Evalute :

$\sqrt{-16} \times \sqrt{-4}$

$= i\sqrt{16} \times i\sqrt{4}$

$= i4 \times 2i = 8i$

$= 8(-1) = -8$

and, $\dfrac{\sqrt{-16}}{\sqrt{-4}}$

$= \dfrac{i\sqrt{16}}{i\sqrt{4}} = \dfrac{i4}{i2}$

$\Rightarrow \dfrac{4i}{2i} = 2$ (Ans)

(9) Evalute Modulus and Argument :

$8z - z^2 \quad [z = 2+i]$

$= 8(2+i) - (2+1)^2$

$= 16 + 8i - 2^2 - 2\cdot2\cdot1 + (i)^2$

$= 16 + 8i - 4 + 4i + (i)^2$

$= 12 + 4i + i$

$= 13 + 4i$ so, $x = 13$ and $y = 4$

therefore,

Modulus $|z| = \sqrt{13^2 + 4^2}$

$= \sqrt{169 + 16}$

$= \sqrt{185}$

Argument,

$\theta = \tan^{-1} \left| \frac{4}{3} \right|$

$= \tan^{-1} \frac{4}{3}$ (Ans)

(10) Express :-

$r(\cos\theta + i\sin\theta)$ from, $1 + i\sqrt{3}$

so, $x = 1$ and, $y = \sqrt{3}$

we know, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$= \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$= \tan^{-1}(\sqrt{3})$

$= 60^\circ = \frac{\pi}{3}$

So, $z = r(\cos\theta + i\sin\theta)$

$= 1\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

therefore, from of $1 + i3 = \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

(Ans.)