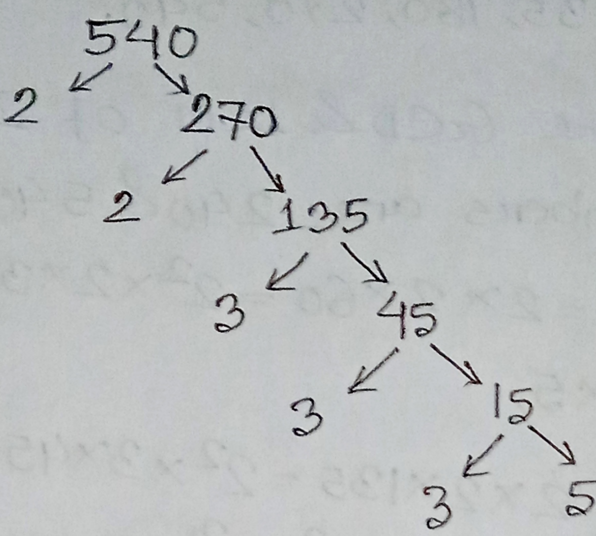
1. Write down the classification of number system.

⇒ The classification of number system

- Number
  - Real
    - Rational
      - Integer
        - Negative
        - Zero
        - Positive / Natural number
          - Prime
          - Neither prime nor ~~Complex~~ composite
          - composite.
      - Fraction
        - Proper
        - Improper
        - Mixed
    - Irrational
  - Complex
    - Purely real
    - Purely imaginary

2. Find the Prime factorization of 540 using tree.

⇒ Tree Diagram:



Prime factor of $540 = 2^2 . 3^3 . 5^1$.

3. Find out the all factors of 540.

Total number of factors; $540 = (2+1)(3+1)(1+1)$

$= 3.4.2$

$= 24$

Calculation of all factors:

$540 = 1 \times 540$

$= 2 \times 270$

$= 3 \times 180$

$= 4 \times 135$

$= 5 \times 108$

$= 6 \times 90$

$= 9 \times 60$

$= 10 \times 54$

$= 12 \times 45$

$= 15 \times 36$

$= 18 \times 30$

$= 20 \times 27$

All the factors of 540 are:
1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54,
60, 90, 108, 135, 180, 270, 540.

4. What is the GCD & LCM of 240 & 540.

⇒ Given numbers are; 240 & 540

$240 = 2 \times 120 = 2 \times 2 \times 60 = 2^2 \times 2 \times 30 = 2^3 \times 2 \times 15$
$= 2^4 \times 3 \times 5.$

$540 = 2 \times 270 = 2 \times 2 \times 135 = 2^2 \times 3 \times 45 = 2^2 \times 3 \times 3 \times 15$
$= 2^2 \times 3^2 \times 3 \times 5 = 2^2 \times 3^3 \times 5$

$GCD(240, 540) = 2^2 \times 3 \times 5 = 60$
$LCM(240, 540) = 2^4 \times 3^3 \times 5 = 2160.$

5. Find the HCF & LCM of 42, 63 & 140.

⇒ Given numbers are; 42, 63 & 140

$42 = 2 \times 21 = 2 \times 3 \times 7.$

$63 = 3 \times 21 = 3 \times 3 \times 7 = 3^2 \times 7$

$140 = 2 \times 70 = 2 \times 2 \times 35 = 2^2 \times 5 \times 7$

$HCF(42, 63, 140) = 7$

$LCM(42, 63, 140) = 2^2 \times 3^2 \times 5 \times 7 = 1260.$

6. Find the HCF & LCM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$.

$\Rightarrow$ Given numbers are; $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$

Calculation of Numerators

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

$10 = 2 \times 5$

$LCM(2, 8, 16, 10) = 2^4 \times 5 = 80$

$HCF(2, 8, 16, 10) = 2^1 = 2$

Calculation of Denominator.

$3 = 3^1$

$9 = 3^2$

$81 = 3 \times 27 = 3 \times 3 \times 9$

$= 3^2 \times 3 \times 3 = 3^4$

$27 = 3^3$

$LCM(3, 9, 81, 27) = 3^4 = 81$

$HCF(3, 9, 81, 27) = 3^1 = 3$

$$LCM \text{ of } \left(\frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27}\right) = \frac{LCM \text{ of } (2, 8, 16, 10)}{HCF \text{ of } (3, 9, 81, 27)} = \frac{80}{3}$$

$$HCF \text{ of } \frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27} = \frac{HCF \text{ of } (2, 8, 16, 10)}{LCM \text{ of } (3, 9, 81, 27)} = \frac{2}{81}$$

7. Find the modulus and Argument of $\frac{1+\sqrt{3}i}{1-\sqrt{3}i}$ and also its, Polar, exponential form.

→ We have, $\frac{1+\sqrt{3}i}{1-\sqrt{3}i}$

$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$

$= \frac{(1)^2+2\cdot1\cdot\sqrt{3}i+(\sqrt{3}i)^2}{(1)^2-(\sqrt{3}i)^2}$

$= \frac{1+2\sqrt{3}i+3i^2}{1-3i^2}$

$= \frac{1+2\sqrt{3}i-3}{1+3}$

$= \frac{-2+2\sqrt{3}i}{4}$

$= \frac{-2}{4}+\frac{2\sqrt{3}i}{4}$

$= -\frac{1}{2}+\frac{\sqrt{3}i}{2}$

Let,

$z = -\frac{1}{2}+\frac{\sqrt{3}}{2}i$

$|z| = r$

$r = \sqrt{\left(-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$

$= \sqrt{\frac{1}{4}+\frac{3}{4}}$

$= \sqrt{\frac{4}{4}}$

$= 1$

Modulus of z is = 1

Argument, $\theta = \pi - \tan^{-1}\left|\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right|$

$= \pi - \tan\sqrt{3}$

$= \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$.

∴ Polar form $= -\frac{1}{2}+\frac{\sqrt{3}}{2}i$

Argument of z is $\frac{2\pi}{3}$.

Exponential form of $z = re^{i\theta}$

$= 1e^{i\frac{2\pi}{3}} = e^{\frac{2\pi}{3}i}$.

8. Evalute $\sqrt{-16} \times \sqrt{-4}$ & $\frac{\sqrt{-16}}{\sqrt{-4}}$.

$\Rightarrow$ We have, $\sqrt{-16} \times \sqrt{-4}$ & $\frac{\sqrt{-16}}{\sqrt{-4}}$

$= \sqrt{16}i \times \sqrt{4}i$

$= 4i \times 2i$

$= 8i^2$

$= -8$

$= \frac{\sqrt{16}i}{\sqrt{4}i}$

$= \frac{4i}{2i}$

$= 2$

9. Evalute Modulus & Argument of $8z - z^2$ by replacing $z = \cancel{2i}\ 2+i$.

$\Rightarrow$ We have, $8z - z^2$

$= 8(2+i) - (2+i)^2$

$= \cancel{8}\ 16 + 8i - \{(2)^2 + 2.2i + i^2\}$

$= 16 + 8i - \{4 + 4i + i^2\}$

$= 16 + 8i - \{4 + 4i - 1\}$

$= 16 + 8i - \{4i + 3\}$

$= 16 + 8i - 4i - 3$

$= \cancel{4i + 13}\ 13 + 4i$

Modulus ~~of (4i+13)~~ $= \sqrt{(4)^2 + (13)^2}$

$= \sqrt{16 + 169}$

$= \sqrt{185}$

Argument, $\theta = \tan^{-1} \frac{4}{13}$

$= 17.102'$

10. Express $1+\sqrt{3}i$ in the form of $r(\cos\theta + i\sin\theta)$

⇒ Modulus, $r = \sqrt{(1)^2 + (\sqrt{3})^2}$

$= \sqrt{1+3}$

$= \sqrt{4}$

$= 2$

Argument, $\theta = \tan^{-1} \left| \frac{\sqrt{3}}{1} \right|$

$= \tan^{-1} \tan \frac{\pi}{3}$

$= \frac{\pi}{3}$

Therefore, $r(\cos\theta + i\sin\theta) = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$