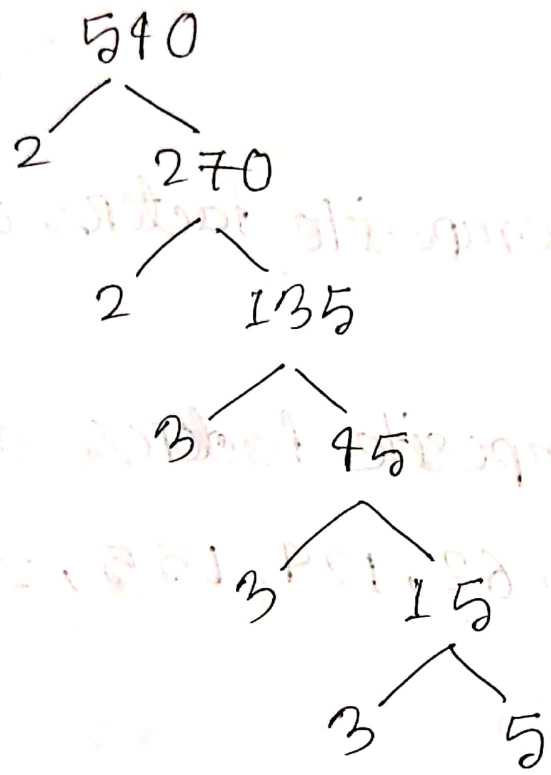
(1) Write down the classification of Number system.

Soln:-

- Number
  - Real Number
    - Rational
      - Integer
        - Negative
        - Zero
        - Positive or Natural Number
          - Prime
          - Neither prime nor composite
          - composite
      - Fraction
        - Proper
        - Improper
        - Mixed
    - Irrational
  - Complex Number
    - Purely Real
    - Purely Imaginary

(2) Find the prime factorization of 540 using tree.

Soln:-



$\therefore$ The prime factorization of 540 is,

$540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$

$= 2^2 \cdot 3^3 \cdot 5^1$

(3) Find ~~out~~ the all factors of 540.

Sol^n:-

From '2' we find the prime factorization of 540 is, $540 = 2^2 \cdot 3^3 \cdot 5^1$

$\therefore$ The total number of factors of 540 are

$$(2+1)\cdot(3+1)\cdot(1+1)$$
$$= 3 \times 4 \times 2$$
$$= 24$$

Here,

$$540 = 1 \times 540$$
$$= 2 \times 270$$
$$= 3 \times 180$$
$$= 4 \times 135$$
$$= 5 \times 108$$
$$= 6 \times 90$$
$$= 9 \times 60$$
$$= 10 \times 54$$
$$= 12 \times 45$$
$$= 15 \times 36$$
$$= 18 \times 30$$
$$= 20 \times 27$$

$\therefore$ The factors of 540 are,

1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540.

④ What is the GCD & LCM of 240 & 540.

Sol^n:-

$$\begin{array}{r|l} 2 & 240 \\ \hline 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 540 \\ \hline 2 & 270 \\ \hline 3 & 135 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

$\therefore 240 = 2\times2\times2\times2\times3\times5 = 2^4 \cdot 3^1 \cdot 5^1$

$540 = 2\times2\times3\times3\times3\times5 = 2^2 \cdot 3^3 \cdot 5^1$

$\therefore$ The GCD of 240 & 540 is $= 2^2 \cdot 3^1 \cdot 5^1$

$= 4\times3\times5$

$= 60$

and, The LCM of 240 & 540 is $= 2^4 \cdot 3^3 \cdot 5^1$

$= 16 \cdot 27 \cdot 5$

$= 2160$

⑤ Find the HCF & LCM of 42, 63 & 140.

Sol^n :-

$$\begin{array}{r|l} 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 2 & 140 \\ \hline 2 & 70 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

$42 = 2^1 \cdot 3^1 \cdot 7^1$

$63 = 3^2 \cdot 7^1$

$140 = 2^2 \cdot 5^1 \cdot 7^1$

$\therefore$ The HCF of 42, 63 & 140 is $= 7^1 = 7$

and, the LCM of 42, 63 & 140 is $= 2^2 \cdot 3^2 \cdot 5^1 \cdot 7^1$

$= 4 \times 9 \times 5 \times 7$

$= 1260.$

⑥ Find the HCF & LCM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ & $\frac{10}{27}$.

Solⁿ:-

Factorization of Numerators,

$2 = 2^1$

$8 = 2\times2\times2 = 2^3$

$16 = 2\times2\times2\times2 = 2^4$

$10 = 2\times5 = 2^1 \cdot 5^1$

Factorization of Denominators

$3 = 3^1$

$9 = 3\times3 = 3^2$

$81 = 3\times3\times3\times3 = 3^4$

$27 = 3\times3\times3 = 3^3$

$HCF(2, 8, 16 \& 10) = 2$

$LCM(2, 8, 16 \& 10) = 2^4 \cdot 5^1$

$= 16\times5$

$= 80$

$HCF(3, 9, 81 \& 27) = 3$

$LCM(3, 9, 81 \& 27) = 3^4$

$= 81$

$\therefore$ The HCF of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ & $\frac{10}{27}$ is $= \frac{HCF(2, 8, 16 \& 10)}{LCM(3, 9, 81 \& 27)}$

$= \frac{2}{81}$

&, the LCM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ & $\frac{10}{27}$ is $= \frac{LCM(2, 8, 16 \& 10)}{HCF(3, 9, 81 \& 27)}$

$= \frac{80}{3}$

(7) Find the modulus and Argument of $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$ and also its polar, exponential form.

Solⁿ:-

$$z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i} = \frac{(1+\sqrt{3}i)^2}{(1+\sqrt{3}i)(1-\sqrt{3}i)} = \frac{1+2\sqrt{3}i-3}{1+3}$$

$$= \frac{-2+2\sqrt{3}i}{4} = \frac{2(-1+\sqrt{3}i)}{4} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

Here,

If we compare $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$, with $a+ib$

then, $a = -\frac{1}{2}$ and $b = \frac{\sqrt{3}}{2}$

Now, modulus, $r = \sqrt{a^2+b^2} = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}}$

$$= \sqrt{\frac{4}{4}} = \sqrt{1} = 1$$

and, argument, $\theta = \tan^{-1}\frac{b}{a} = \tan^{-1}\frac{\sqrt{3}/2}{-\frac{1}{2}} = \tan^{-1}(-\sqrt{3})$

$$= \pi - \tan(\sqrt{3}) = \pi - \frac{\pi}{3} = \frac{3\pi - \pi}{3}$$

$$= \frac{2\pi}{3}$$

Again,

the polar form is, $z = r(\cos\theta + i\sin\theta)$

$$= 1\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

& Exponential form, $z = e^{i\theta}$

$= e^{i\frac{2\pi}{3}}$

⑧ Evaluate $\sqrt{-16} \times \sqrt{-4}$ & $\frac{\sqrt{-16}}{\sqrt{-4}}$.

Solⁿ:-

Now, $\sqrt{-16} \times \sqrt{-4}$

$= i\sqrt{16} \times i\sqrt{4}$

$= i^2\sqrt{4^2} \times \sqrt{2^2}$

$= i^2 \times 4 \times 2$

$= -8$

Again,

$\frac{\sqrt{-16}}{\sqrt{-4}}$

$= \frac{i\sqrt{4^2}}{i\sqrt{2^2}}$

$= \frac{4}{2}$

$= 2$

(9) Evaluate Modulus & Argument of $8z - z^2$ by ~~using~~ replacing $z = 2 + i$

Sol^n:

Now $8z - z^2$

$= 8(2+i) - (2+i)^2$ $[\because z = 2+i]$

$= 16 + 8i - (4 + 4i + i^2)$

$= 16 + 8i - 4 - 4i + 1$

$= 13 + 4i$.

Here, $a = 13$ & $b = 4$

$\therefore$ Modulus, $r = \sqrt{a^2 + b^2}$

$= \sqrt{13^2 + 4^2}$

$= \sqrt{169 + 16}$

$= \sqrt{185}$

Argument, $\theta = \tan^{-1}\frac{b}{a} = \tan^{-1}\frac{4}{13}$

(10) Express $1+i\sqrt{3}$ in the form of $r(\cos\theta + i\sin\theta)$.

Soln.:-

Let, $z = 1 + i\sqrt{3}$

Here, $a = 1,\ b = \sqrt{3}$

$\therefore r = \sqrt{a^2+b^2} = \sqrt{1+3} = \sqrt{4} = 2$

$\theta = \tan^{-1}\frac{\sqrt{3}}{1} = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

$\therefore r(\cos\theta + i\sin\theta) = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$