Complex Number System

1. Classification of Number System:

- Number
  - Real Number
    - Rational
      - Integer
        - Negative
        - Zero
        - Positive/ Netural Number
          - Prime
          - Neither Prime Nor Composite
          - Composite
      - Fraction
        - Proper
        - Improper
        - Mixed
    - Irrational
  - Complex Number
    - Rarely Real
    - Purely Imaginary

2. Find the Prime factorisation of 540 using tree.

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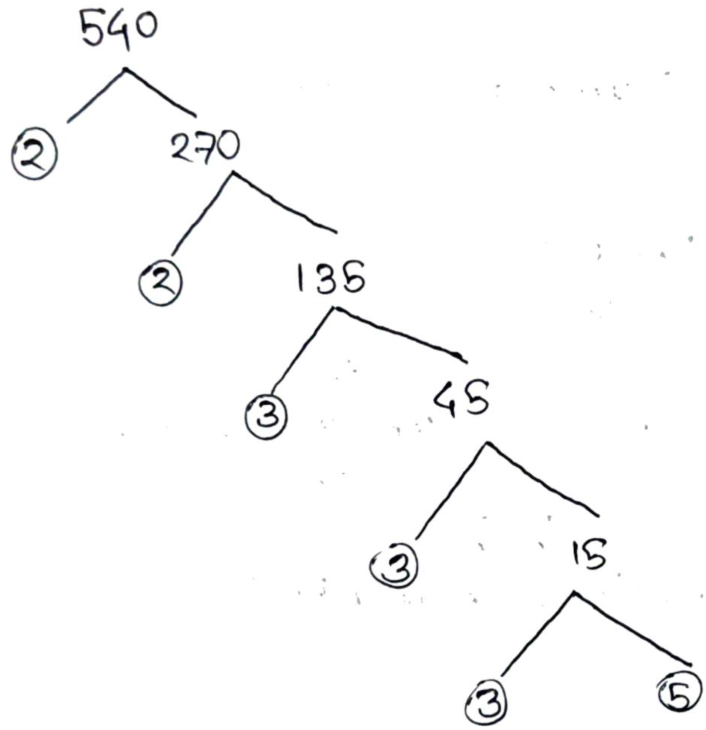

Therefore, the Prime factorization of 540 is

$= 2^2 \cdot 3^3 \cdot 5$

3. Find out the all factors of 540.

→

Got it from number '2',

The prime factorisation of 540 is $= 2^2 \cdot 3^3 \cdot 5$

So, the total number of factors of 540 is

$= (2+1) \cdot (3+1) \cdot (1+1) = 3 \cdot 4 \cdot 2 = 24$

Calculation for all factors,

$540 = 1 \times 540$

$= 2 \times \cancel{2\times}270$

$= 4 \times 135$

$= 3 \times 180$

$= 6 \times 90$

$= 12 \times 45$

$= 5 \times 108$

$= 10 \times 54$

$= 20 \times 27$

$= 9 \times 60$

$= 18 \times 30$

$= 36 \times 15$

The factors of 540 are,

1, 2, 4, 3, 6, 12, 5, 10, 20, 9, 18, 36, 15, 30, 60, 28, 54
108, 45, 90, 180, 135, 270, 540.

4. What is the GCD & LCM of 240 & 540.

→

Given numbers are 240 & 540

$240 = 2 \cdot 120 = 2 \cdot 2 \cdot 60 = 2^2 \cdot 2 \cdot 30 = 2^3 \cdot 2 \cdot 15 = 2^4 \cdot 3 \cdot 5$

$540 = 2 \cdot 270 = 2 \cdot 2 \cdot 135 = 2^2 \cdot 3 \cdot 45 = 2^2 \cdot 3^2 \cdot 15 = 2^2 \cdot 3^3 \cdot 5$

LCM of $(240, 540) = 2^4 \cdot 3^3 \cdot 5 = 2160$

GCM of $(240, 540) = 2 \cdot 3 \cdot 5 = 30$

5. Find the HCF and LCM of 42, 63 and 140.

→

Given numbers are 42, 63, 140

Therefore, the Prime factorisation of 42, 63, 140 are

$42 = 2 \cdot 21 = 2 \cdot 3 \cdot 7$

$63 = 3 \cdot 21 = 3^2 \cdot 7$

$140 = 2 \cdot 70 = 2^2 \cdot 35 = 2^2 \cdot 5 \cdot 7$

LCM of $(42, 63, 140) = 2^2 \cdot 3^2 \cdot 5 \cdot 7 = 1260$

HCF of $(42, 63, 140) = 7$

6. Find the HCF and LCM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27}$

→

Given numbers are $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27}$

Calculation for Numerators	Calculation for Denominators
$2 = 2^1$	$3 = 3^1$
$8 = 2^3$	$9 = 3^2$
$16 = 2^4$	$81 = 3^4$
$10 = 2 \cdot 5$	$27 = 3^3$

LCM of $(2, 8, 16, 10) = 2^4 \cdot 5$ LCM of $(3, 9, 81, 27) = 3^4$

HCF of $(2, 8, 16, 10) = 2^1 = 2$ HCF of $(3, 9, 81, 27) = 3^1 = 3$

LCM of $\left(\frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27}\right) = \frac{2^4 \cdot 5}{3} = \frac{80}{3}$

HCF of $\left(\frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27}\right) = \frac{3^4}{2} = \frac{81}{2}$

7. Find the modulus and Argumet of = $\frac{1+\sqrt{3}i}{1-\sqrt{3}i}$ and also Polar, ~~expot~~ exponential form.

→

We have, $\frac{1+\sqrt{3}i}{1-\sqrt{3}i}$

$$= \frac{(1+\sqrt{3}i)(1-\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$$

$$= \frac{(1+\sqrt{3}i)^2}{1^2-(\sqrt{3}i)^2}$$

$$= \frac{1+2\sqrt{3}i-3}{1+3}$$

$$= \frac{-2+2\sqrt{3}i}{4}$$

$$= \frac{-1+\sqrt{3}i}{2}$$

$$= -\frac{1}{2}+\frac{\sqrt{3}}{2}i$$

Let $z = -\frac{1}{2}+\frac{\sqrt{3}}{2}i$

$r = |z| = \sqrt{(-\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}$

$= \sqrt{\frac{1}{4}+\frac{3}{4}}$

$= \sqrt{4/4}$

$= 1$

∴ Modulus of $z = 1$

And argument of z is,

$$\theta = \pi - \tan^{-1}\left|\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right|$$

$$= \pi - \tan^{-1}(\sqrt{3})$$

$$\cancel{= \pi - \tan^{-1}(\sqrt{\ }}$$

$$= \pi - \frac{\pi}{3}$$

$$= \frac{3\pi - \pi}{3}$$

$$= \frac{2\pi}{3}$$

Polar form of $z = r(\cos\theta + i\sin\theta)$

$$= 1\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

Exponential form is $z = re^{i\theta}$

$$= 1 \cdot e^{i \cdot \frac{2\pi}{3}}$$

$$= e^{\frac{2\pi}{3}i}$$

8. Evaluate $\sqrt{-16} \times \sqrt{-4}$ & $\frac{\sqrt{-16}}{\sqrt{-4}}$

→ Given numbers are $\sqrt{-16} \times \sqrt{-4}$ & $\frac{\sqrt{-16}}{\sqrt{-4}}$

$\sqrt{-16} \times \sqrt{-4}$

$= \sqrt{-16 \times (-1)} \times \sqrt{4 \times (-1)}$

$= \sqrt{16 \times i^2} \times \sqrt{4 \times i^2}$

$= \sqrt{16}\, i \times \sqrt{4}\, i$

$= 4i \times 2i$

$= 8i^2$

$= -8$

then,

$\frac{\sqrt{-16}}{\sqrt{-4}}$

$= \frac{4i}{2i}$

$= 2$

9. Evaluate Modulus and Argument of $8z - z^2$ by replacing $z = 2+i$

→ Given is $z = 2+i$

$\therefore 8z - z^2 = \cancel{8(+}\,8\,(2+i) - (2+i)^2$

$= 16 + 8i - (4 + 4i + i^2)$

$= 16 + 8i - 4 - 4i + 1$

$= 13 + 4i$

Modulus $r = \sqrt{13^2 + (4)^2}$

$= \sqrt{169+16}$

$= \sqrt{185}$

Argument,

$\theta = \tan^{-1}\left|\frac{4}{13}\right|$

$= 17.102^{\circ}$

10. Express $1+i\sqrt{3}$ in the form of $r(\cos\theta + i\sin\theta)$

→ Let,

$z = 1 + i\sqrt{3}$

Modulus of z,

$r = \sqrt{1^2 + (\sqrt{3})^2}$

$= \sqrt{1+3}$

$= 2$

Argument of z,

$\theta = \tan^{-1}\left|\frac{y}{x}\right|$

$= \tan^{-1}\left|\frac{\sqrt{3}}{1}\right|$

$= \tan^{-1}(\sqrt{3})$

$= \frac{\pi}{3}$

Therefore, $r(\cos\theta + i\sin\theta)$ form is,

$= 2(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$