05 Feb 2022

1. Classification of Number System:

- Number
  - Real $(-2, 0, 2.4, \pi, \frac{1}{2})$
    - Rational $(0, 54, \frac{1}{4}, (-\frac{2}{5}))$
      - Integer $(-2, 0, 1)$
        - Negative $(-2, -5)$
        - Positive/Natural $(1, 2, 3)$
          - Prime
          - Neither prime nor composite
          - Composite
        - Zero
      - Fraction $(\frac{1}{2}, 0.53)$
        - Proper
        - Improper
        - Mixed
    - Irrational $(\sqrt{2}, \pi, \sqrt{.4343})$
  - Complex
    - Purely Real
    - imaginary $(\sqrt{-2}, i, 3i)$

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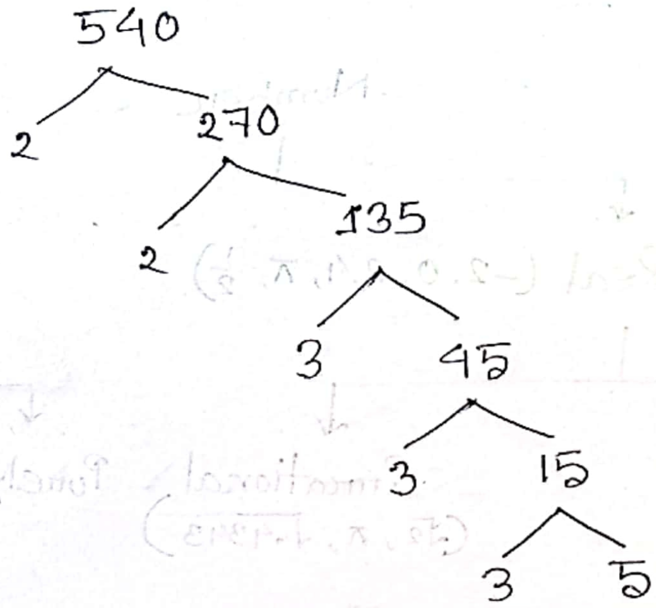
2. Tree diagram :



Prime factorization of $540 = 2^2 . 3^3 . 5^1$

3.

$$\begin{array}{r|l} 2 & 540 \\ \hline 2 & 270 \\ \hline 3 & 135 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

$\therefore 540 = 2^2 . 3^3 . 5^1$ [only prime]

$\therefore$ all factors $= (2+1).(3+1).(1+1)$

$= 3.4.2$

$= 24$

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Here,

$$
\begin{aligned}
540 &= 1 \times 540 \\
&= 2 \times 270 \\
&= 3 \times 180 \\
&= 4 \times 135 \\
&= 5 \times 108 \\
&= 6 \times 90 \\
&= 9 \times 60 \\
&= 10 \times 54 \\
&= 12 \times 45 \\
&= 15 \times 36 \\
&= 18 \times 30 \\
&= 20 \times 27
\end{aligned}
$$

$\therefore$ all factors of 540 = 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540

4.

$$
\begin{array}{r|l}
2 & 240 \\ \hline
2 & 120 \\ \hline
2 & 60 \\ \hline
2 & 30 \\ \hline
3 & 15 \\ \hline
 & 5
\end{array}
\qquad\qquad
\begin{array}{r|l}
2 & 540 \\ \hline
2 & 270 \\ \hline
3 & 135 \\ \hline
3 & 45 \\ \hline
3 & 15 \\ \hline
 & 5
\end{array}
$$

$\therefore$ prime factorization of $240 = 2^4 . 3^1 . 5^1$

and of $540 = 2^2 . 3^3 . 5^1$

$\therefore$ GCD $= 2^2 . 3^1 . 5^1 = 60$

$\therefore$ LCM $= 2^4 . 3^3 . 5^1 = 2160$

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5.

$$\begin{array}{r|l} 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 2 & 140 \\ \hline 2 & 70 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

$42 = 2 \times 3 \times 7$

$63 = 3 \times 3 \times 7$

$140 = 2 \times 2 \times 5 \times 7$

$\therefore$ HCF $= 7$

$\therefore$ LCM $= 2 \times 2 \times 3 \times 3 \times 7 = 252$

6. $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27}$

Factorization of Numerators:

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

$10 = 2^1 \times 5$

HCF of numerators $= 2$

LCM of numerators $= 80$

Factorization of Denominators:

$3 = 3^1$

$9 = 3^2$

$81 = 3^4$

$27 = 3^3$

HCF of denominators $= 3$

LCM of denominators $= 81$

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$\therefore$ HCF of ~~nomin numerators~~ $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$ $= \dfrac{\text{HCF}(2,8,16,10)}{\text{LCM}(3,9,81,27)}$

$$= \frac{2}{81}$$

$\therefore$ LCM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$ $= \dfrac{\text{LCM}(2,8,16,10)}{\text{HCF}(3,9,81,27)}$

$$= \frac{16}{3}$$

7. $z = \dfrac{1+\sqrt{3}i}{1-\sqrt{3}i}$

$$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$$

$$= \frac{(1+\sqrt{3}i)^2}{(1)^2-(\sqrt{3}i)^2}$$

$$= \frac{1^2+2.\sqrt{3}i+3i^2}{1+3}$$

$$= \frac{-2+2\sqrt{3}i}{4}$$

$$= -\frac{1}{2} + \frac{2\sqrt{3}}{4}i$$

$$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$= x+iy$ [where $x = -\frac{1}{2}$ and $y = \frac{\sqrt{3}}{2}$]

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now, $r = \sqrt{x^2+y^2}$

$= \sqrt{\frac{1}{4}+\frac{3}{4}}$

$= 1$

and, $\theta = \pi - \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right)$

$= \pi - \tan^{-1}(-\sqrt{3})$

$= \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

So, polar form is, $z = r(\cos\theta + i\sin\theta)$

$= 1.\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$= \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

and exponential form is, $z = e^{i.\frac{2\pi}{3}}$

8. $\sqrt{-16} \times \sqrt{-4}$

$= \sqrt{16i^2} \times \sqrt{4i^2}$ $[i^2 = -1]$

$= \sqrt{4^2 i^2} \times \sqrt{2^2 i^2}$

$= 4i \times 2i = 8i^2 = -8$

again, $\frac{\sqrt{-16}}{\sqrt{-4}}$

$= \frac{\sqrt{16i^2}}{\sqrt{4i^2}}$

$= \frac{4i}{2i}$

$= 2$

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9. $8z - z^2$

$= 8(2+i) - (2+i)^2 \quad [z = 2+i]$

$= 16 + 8i - 4 - 4i + 1$

$= 12 + 4i + 1$

$= 13 + 4i$

$= x + iy \quad [\text{where } x = 13,\ y = 4]$

modulus,

$r = \sqrt{13^2 + 4^2}$

$= \sqrt{185}$

argument,

$\theta = \tan^{-1} \frac{4}{13} = 17.102$

10. $1 + i\sqrt{3}$

Let, $z = 1 + i\sqrt{3}$

$z = x + iy$

$r = \sqrt{(\sqrt{3})^2 + 1}$

$= 2$

$\theta = \tan^{-1} \left| \frac{\sqrt{3}}{1} \right|$

$= \tan^{-1} \tan \frac{\pi}{3}$

$= \frac{\pi}{3}$

$\therefore r(\cos\theta + i\sin\theta)$ form is

$= 2\left(\cos \frac{\pi}{3} + i\sin \frac{\pi}{3}\right)$

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