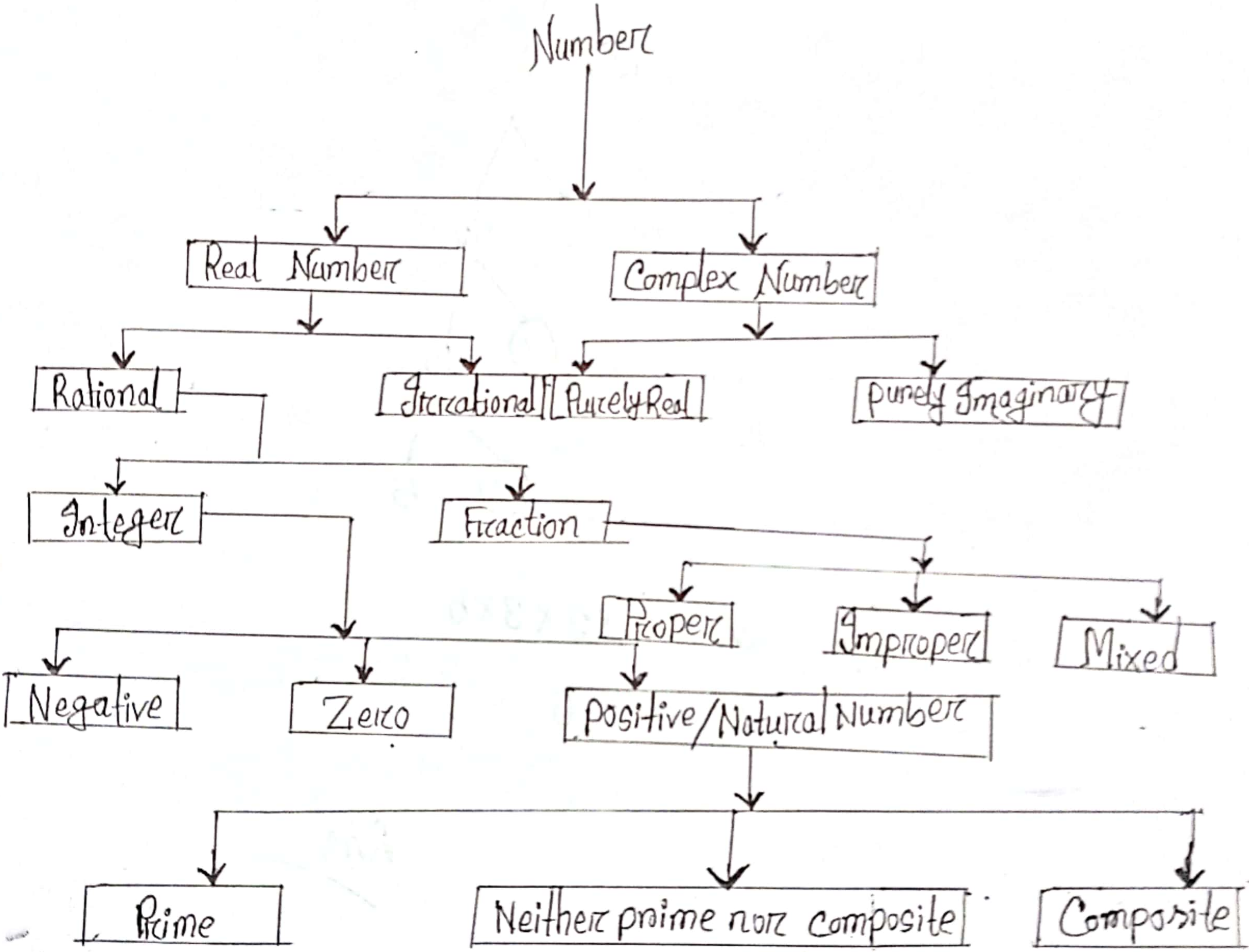
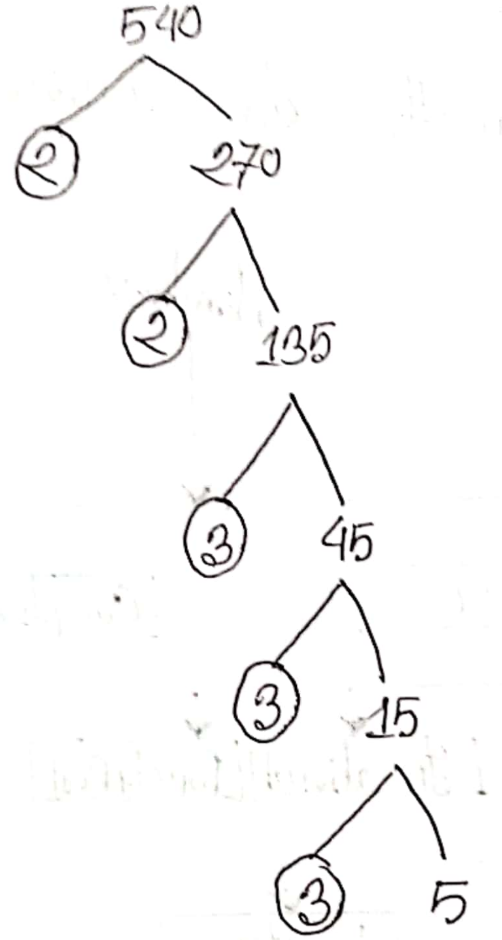
Number System

1. Write down the classification of number system.



2. Find the Prime factorization of 540 using tree.



$$540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$$
$$= 2^2 \ 3^3 \ 5$$

Ans

3. Find out the all factors of 540.

$$540 = 1 \times 540$$
$$= 2 \times 270$$
$$= 3 \times 180$$
$$= 4 \times 135$$
$$= 5 \times 108$$
$$= 6 \times 90$$
$$= 9 \times 60$$
$$= 10 \times 54$$
$$= 12 \times 45$$
$$= 15 \times 36$$
$$= 18 \times 30$$
$$= 20 \times 27$$

The factors of 540 are = ~~1, 2, 3, 4, 8, 1~~

∴ 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540.

Ans

3.

Or,

$$\begin{array}{r|l} 2 & 540 \\ \hline 2 & 270 \\ \hline 3 & 135 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

$$540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$$
$$= (2^2)(3^3)(5)$$

So, the total number of factors of 540 is

$$(2+1).(3+1).(1+1) = 3.4.$$
$$= 24$$

Ans

4. What is the GCD and LCM of 240 and 540.

$240 = 2.120 = 2.2.60 = 2^2.2.30 = 2^3.2.15 = 2^4.3.5$

$540 = 2.270 = 2.2.135 = 2^2.3.45 = 2^2.3.3.15$

$= 2^2.3.3.3.5$

$= 2^2.3^3.5$

$\therefore$ LCM (240 and 540) $= 2^4.3^3.5 = 2160$

$\therefore$ GCD = (240 and 540) $= 2^2.3.5 = 60$

5. Find the H.C.F and L.C.M of 42, 63 and 140.

$42 = 2 \times 21 = 2 \times 3 \times 7 = 2^1 \times 3^1 \times 7^1$

$63 = 3 \times 21 = 3 \times 3 \times 7 = 3^2 \times 7^1$

$140 = 2 \times 70 = 2 \times 2 \times 35 = 2^2 \times 5^1 \times 7^1$.

$\therefore$ L.C.M = (42, 63 and 140) $= 2^2.3^2.5.7 = 1260$

$\therefore$ H.C.F = (42, 63 and 140) $= \cancel{2.3.5.7 = 270} = 7$

$\therefore$ Therefore, HCF of 42, 63, 140 is 7 and LCM of 42, 63, 140 is 1260.

Ans

6. Find the H.C.F and L.C.M of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$.

calculation for Numerators

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

$10 = 2^1 \times 5^1$

calculation for Denominators

$3 = 3^1$

$9 = 3^2$

$81 = 3^4$

$27 = 3^3$

L.C.M $(2, 8, 16, 10) = 2^4 \times 5 = 80$

H.C.F $(2, 8, 16, 10) = 2$

L.C.M$(3, 9, 81, 27) = 3^4 = 81$

H.C.F$(3, 9, 81, 27) = 3$

H.C.F of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{\text{H.C.F}(2,8,16,10)}{\text{L.C.M}(3,9,81,27)}$

$= \frac{2}{81}$

L.C.M of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{\text{L.C.M}(2,8,16,10)}{\text{H.C.F}(3,9,81,27)}$

$= \frac{80}{3}$

Ans

7. Find the modulus and Argument of $Z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$ and also its polar, exponential form.

We have, $\frac{1+\sqrt{3}i}{1-\sqrt{3}i}$

$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$

$= \frac{1+2\sqrt{3}i-3}{1^2-(\sqrt{3}i)^2}$

$= \frac{-2+2\sqrt{3}i}{1+3}$

$= \frac{-2+2\sqrt{3}i}{4}$

$= \frac{-1+\sqrt{3}i}{2}$

$= -\frac{1}{2}+\frac{\sqrt{3}}{2}i$

Let $z = -\frac{1}{2}+\frac{\sqrt{3}}{2}i$

$|z| = \sqrt{(-\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}$

$r = \sqrt{\frac{1}{4}+\frac{3}{4}}$

$= \sqrt{\frac{4}{4}}$

$= \sqrt{1}$

$= 1$

$\therefore$ Modulus of z is $= 1$

And Argument of z will

$\theta = \pi - \tan^{-1}\left|\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right|$

$= \pi - \tan^{-1}(\sqrt{3})$

$= \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

polar Form $-\frac{1}{2}+\frac{\sqrt{3}}{2}i$

Exponential Form is $z = re^{i\theta}$

$= 1 \cdot e^{i\frac{2\pi}{3}}$

$= e^{\frac{2\pi}{3}i}$

8. Evaluate $\sqrt{-16} \times \sqrt{-4}$ and $\frac{\sqrt{-16}}{\sqrt{-4}}$

We have,

$$\sqrt{-16} \times \sqrt{-4}$$
$$= \sqrt{-16}\,i \times \sqrt{-4}\,i$$
$$= 4i \times 2i$$
$$= 8i^2$$
$$= -8$$

and

$$\frac{\sqrt{-16}}{\sqrt{-4}}$$
$$= \frac{4i}{2i}$$
$$= 2$$

Ans

9. Evaluate Modulus and Argument of $8z - z^2$ by replacing $z = 2+i$

We have, $z = 2+i$

$$\therefore 8z - z^2 = 8(2+i) - (2+i)^2$$
$$= 16 + 8i - (4 + 4i + i^2)$$
$$= 16 + 8i - 4 - 4i + 1$$
$$= 13 + 4i$$

so, $x = 13$ and $y = 4$

Modulus $r = \sqrt{(13)^2+(4)^2}$

$= \sqrt{169+16}$

$= \sqrt{185}$

$\theta = \tan^{-1}\left|\frac{4}{13}\right|$

$= \underline{17.102} = \tan^{-1}\frac{4}{3}$ Ans

10. Express $1+i\sqrt{3}$ in the form of $r(\cos\theta + i\sin\theta)$

Let, $z = 1+i\sqrt{3}$, $z = x+iy$ $|z| = \sqrt{x^2+y^2}$

$\theta = \tan^{-1}\left|\frac{y}{x}\right|$

$\therefore$ Modulus of $z = \sqrt{(1)^2+(\sqrt{3})^2}$

$= \sqrt{(1+3)}$

$= \sqrt{4}$

$= 2$

$\therefore r = 2$

Argument of $z = \tan^{-1}\left|\frac{y}{x}\right|$

$= \tan^{-1}\left|\frac{\sqrt{3}}{1}\right|$

$= \tan^{-1}\tan\frac{\pi}{3}$

$= \frac{\pi}{3}$

Therefore, $r(\cos\theta + i\sin\theta)$ form is $= \cancel{2\left(\cos\frac{\pi}{3} + i\,si\right.}$

$$= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$