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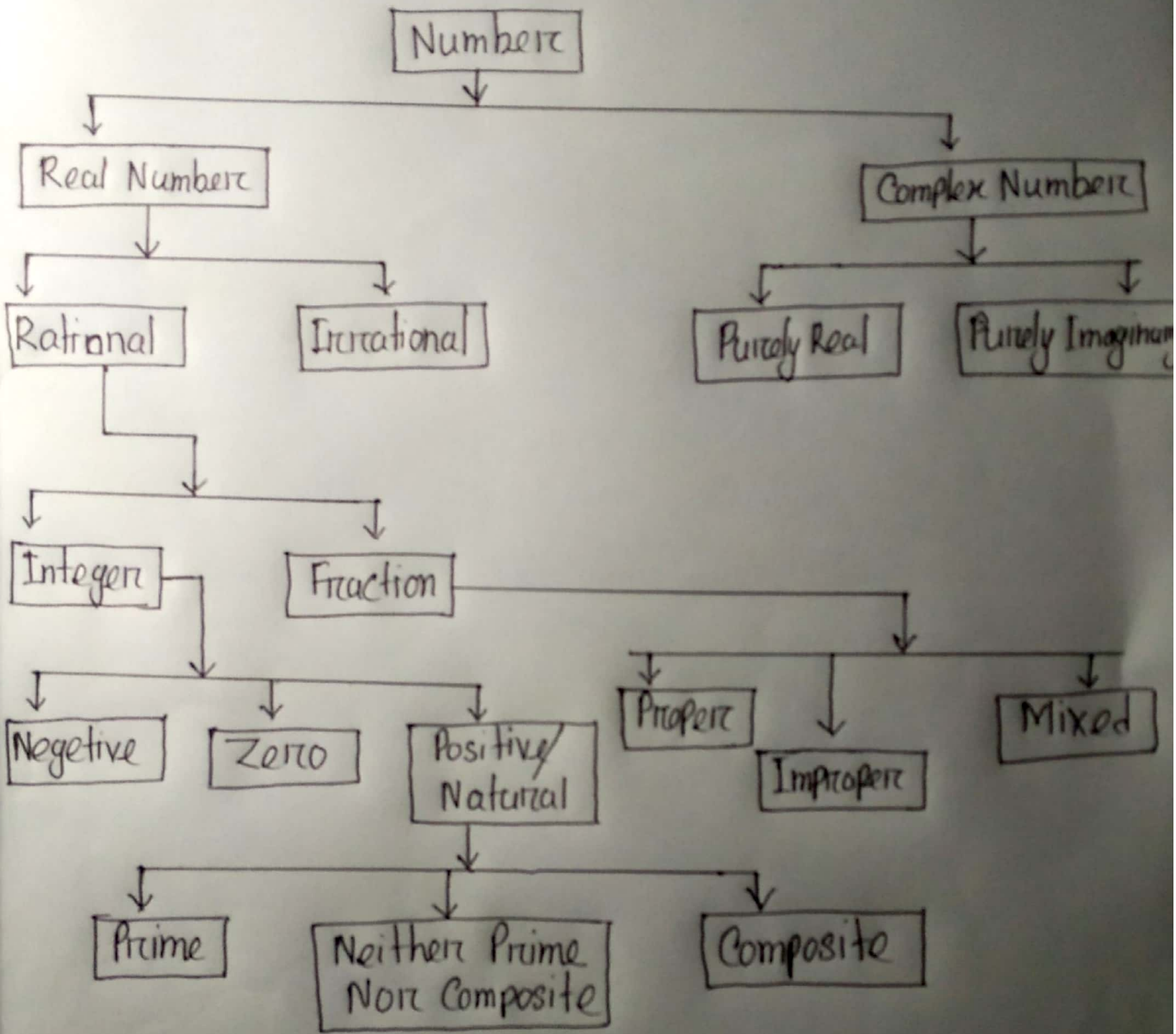
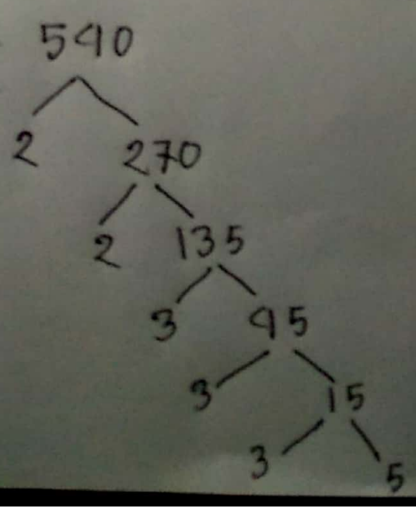
Ans the Q. NO:-1



Ans the Q NO:- 2



∴ The Prime Factorization of 540 is $= 2^2 \cdot 3^3 \cdot 5$

Ans the Q. NO:- 3

$$\begin{array}{r|l} 2 & 540 \\ \hline 2 & 270 \\ \hline 3 & 135 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

Therefore, the Prime factorization of 540 is $= 2^2 . 3^3 . 5$

So, the total number of factors of 540 is

$= (2+1)(3+1)(1+1)$

$= 3.4.2$

$= 24$

Calculation for all factors

$540 = 1 \times 540$
$= 2 \times 270$
$= 3 \times 180$
$= 4 \times 135$
$= 5 \times 108$
$= 6 \times 90$
$= 9 \times 60$
$= 10 \times 54$
$= 12 \times 45$
$= 15 \times 36$
$= 18 \times 30$
$= 20 \times 27$

The factors of 540 are

1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540.

Ans the Q. NO:- 4

$$\begin{array}{r|l} 2 & 240 \\ \hline 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 540 \\ \hline 2 & 270 \\ \hline 3 & 135 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

$\therefore 240 = 2^4 \times 3 \times 5$

$\therefore 540 = 2^2 \times 3^3 \times 5$

LCM of $(240, 540) = 2^4 . 3^3 . 5 = 2160$

and ~~HCF~~ GCD of $(240, 540) = 2^2 . 3 . 5 = 60$ (Ans)

Ans the Q. NO:- 5

$42 = 2 \times 21 = 2 \times 3 \times 7$

$63 = 3 \times 21 = 3 \times 3 \times 7 = 3^2 \times 7$

$140 = 2 \times 70 = 2 \times 2 \times 35 = 2^2 \times 5 \times 7$

$\therefore$ LCM of $(42, 63, 140) = 2^2 . 3^2 . 5 . 7 = 1260$

and HCF of $(42, 63, 140) = 7$ (Ans)

Ans the Q. NO: 6

Find the H.C.F and L.C.M of $2/3$, $8/9$, $16/81$ and $10/27$

Calculation of Numerator

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

$10 = 2 \times 5$

Calculation of Denominator

$3 = 3^1$

$9 = 3^2$

$81 = 3^4$

$27 = 3^3$

LCM of Numerator $= 2^4 \times 5$
$= 80$

HCF of Numerator $= 2$

LCM of Denominator $= 3^4 = 81$

HCF of Denominator $= 3$

~~LCM of~~

HCF of $2/3$, $8/9$, $16/81$ and $10/27$ $= \frac{2}{81}$

LCM of $2/3$, $8/9$, $16/81$, $10/27$ $= \frac{80}{3}$ (Ans).

Ans the Q. NO: 7

$$Z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i} = \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)} = \frac{(1+\sqrt{3}i)^2}{(1)^2-(\sqrt{3}i)^2}$$

$$= \frac{1^2+2.1.\sqrt{3}i+(\sqrt{3}i)^2}{(1)^2-(\sqrt{3}i)^2} = \frac{1+2\sqrt{3}i+3i^2}{1-3i^2} = \frac{1+2\sqrt{3}i-3}{1+3}$$

$$= \frac{-2+2\sqrt{3}i}{4} = -2/4 + \frac{2\sqrt{3}i}{4} = -\frac{1}{2} + \frac{\sqrt{3}i}{2}$$

Where, $x = -\frac{1}{2}$ and $y = \frac{\sqrt{3}}{2}$

$r = \sqrt{x^2+y^2}$

$= \sqrt{(-\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}$

$= \sqrt{\frac{1}{4}+\frac{3}{4}}$

$= \sqrt{1} = 1$

$\therefore \theta = \pi - \tan^{-1}(y/x)$

$= \pi - \tan^{-1}\left(\frac{\sqrt{3}/2}{-1/2}\right)$

$= \pi - \tan^{-1}(\sqrt{3})$

$= \pi - \tan^{-1}\tan \pi/3$

$= \pi - \pi/3$

$= 2\pi/3$

So, the polar form is, $z = r(\cos\theta + i\sin\theta)$

$= 1\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$= \cos 2\pi/3 + i\sin 2\pi/3$

and Exponential form $z = e^{i\frac{2\pi}{3}}$. (Ans).

Ans the Q. NO: 8

Evaluate $\sqrt{-16} \times \sqrt{-4}$ & $\frac{\sqrt{-16}}{\sqrt{-4}}$

$\therefore \sqrt{-16} \times \sqrt{-4}$

$= \sqrt{16i} \times \sqrt{4i}$

$= 4i \times 2i$

$= 8i^2$

$= -8$

and $\frac{\sqrt{-16}}{\sqrt{-4}}$

$= \frac{4i}{2i}$

$= 2$

Ans the Q. NO: 9

Here given that,

$z = 2+i$

$\therefore 8z - z^2 = 8(2+i) - (2+i)^2$

$= 16 + 8i - (4 + 4i + i^2)$

$= 16 + 8i - 4 - 4i - i^2$

$= 16 + 8i - 4 - 4i + 1$

$= 13 + 4i$

Modulus, $r = \sqrt{x^2+y^2} = \sqrt{(13)^2 + (4)^2}$

$= \sqrt{169 + 16}$

$= \sqrt{185}$ (Ans)

Argument,

$\therefore \theta = \tan^{-1}(y/x) = \tan^{-1}(4/13)$

$= 17.102$ (Ans)

Ans the Q. NO :- 10

Here, $z = x + iy$

$= 1 + i\sqrt{3}$

$\therefore x = 1$

$\therefore y = \sqrt{3}$

Modulus, $r = \sqrt{x^2 + y^2}$

$= \sqrt{1^2 + (\sqrt{3})^2}$

$= \sqrt{1+3}$

$= 2$

Argument, $\theta = \tan^{-1}(y/x)$

$= \tan^{-1}(\sqrt{3}/1)$

$= \tan^{-1} \tan \pi/3$

$= \pi/3$

Therefore, $r(\cos\theta + i\sin\theta)$ Form is $= 2(\cos \pi/3 + i \sin \pi/3)$

(Ans).