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Exercise Number system:

① Write down the classification of number system?

Ans:

- Number
  - Real number
    - Rational
    - Irrational
  - Complex number
    - Purely Real
    - Purely Imaginary

2. Find the prime factorization of 540 using tree.

Ans:

Division method:

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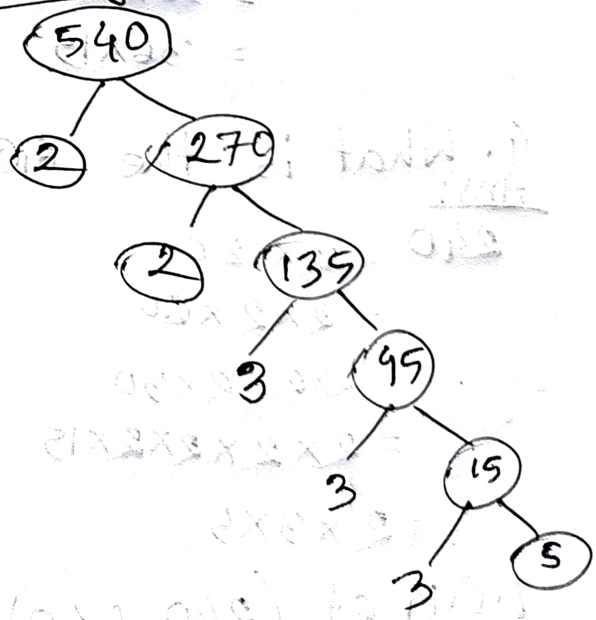
2|540
 2|270
  2|135
   3| 45
    3| 15
     | 5
```

Tree diagram:



Multiplication Method:

$540 = 2 \times 270$

$= 2 \times 2 \times 135$

$= 2^2 \times 3 \times 45$

$= 2^2 \times 3^2 \times 15$

$= 2^2 \times 3^3 \times 5$

3. Find out the all factors of 540.

Ans:

$$\begin{aligned}540 &= 1\times 540\\ &= 2\times 270\\ &= 3\times 180\\ &= 4\times 135\\ &= 5\times 108\\ &= 6\times 90\\ &= 9\times 60\\ &= 10\times 54\\ &= 12\times 45\\ &= 15\times 36\\ &= 18\times 30\\ &= 20\times 27\\ &= 30\times 18\\ &= 36\times 15\end{aligned}$$

The prime factors are:

(1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540)

Ans:

4. What is the GCD and LCM of 240 and 540.

Ans:

$$\begin{aligned}240 &= 2\times 120\\ &= 2\times 2\times 60\\ &= 2\times 2\times 2\times 30\\ &= 2\times 2\times 2\times 2\times 15\\ &= 2^4\times 3\times 5\end{aligned}$$

$$\begin{aligned}540 &= 2\times 270\\ &= 2\times 2\times 135\\ &= 2\times 2\times 3\times 45\\ &= 2\times 2\times 3\times 3\times 15\\ &= 2^2\times 3^3\times 5\end{aligned}$$

L.C.M of $(240, 540) = 2^4 \cdot 3^3 \cdot 5 = 2150$

GCD of $(240, 540) = 2^2 \cdot 3 \cdot 5 = 60$

Ans.

5. Find the H.C.F and L.C.M of 42, 63, 140 :

Ans.

$42 = 2 \times 21$
$= 2 \times 3 \times 7$

$63 = 7 \times 9$
$= 7 \times 3$

$140 = 2 \times 70$
$= 2 \times 2 \times 35$
$= 2 \times 5 \times 7$

$\therefore$ H.C.F of $(42, 63, 140) = 7$

L.C.M of $(42, 63, 140) = 2 \cdot 3 \cdot 5 \cdot 7$
$= 1260$

Ans:

6. Find the H.C.F and L.C.M of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$.

Ans:

Calculation of number:

$2 = 2^1$
$8 = 2^3$
$16 = 2^4$
$10 = 2 \cdot 5$

$\therefore$ H.C.F $= 2$

L.C.M $= 2^4 \cdot 5 = 80$

Calculation of Denominators

$3 = 3^1$
$9 = 3$
$81 = 3^4$
$27 = 3^3$

$\therefore$ H.C.F $= 3$

L.C.M $= 3^4 = 81$

H.C.F of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27}$ $= \dfrac{\text{H.C.F of } (2, 8, 16, 10)}{\text{L.C.M of } (3, 9, 81, 27)} = \dfrac{2}{81}$

L.C.M of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27}$ $= \dfrac{\text{L.C.M of } (2, 8, 16, 10)}{\text{H.C.F. of } (3, 9, 81, 27)} = \dfrac{80}{3}$

7. Find the modulus and Argument of $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$ and also its polar, exponential from.

Ans:

$$Z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$$

Notation $= |Z|$

Rule $|Z| = \sqrt{x^2+y^2}$

Supose,

$$Z_1 = 1+\sqrt{3}i, \quad Z_2 = 1-\sqrt{3}i$$

$$|Z_1| = \sqrt{(1)^2+(\sqrt{3})^2} = \sqrt{4} = 2$$

$$|Z_2| = \sqrt{(1)^2+(-\sqrt{3})^2} = \sqrt{4} = 2$$

$\therefore$ Modulas of $|Z| = \frac{2}{2} = 1$

Z_1 argument is $\theta_1 = \tan^{-1}\frac{y}{x}$

$$= \tan^{-1}\sqrt{3}$$

$$= 60^\circ$$

8. Evaluate $\sqrt{-16} \times \sqrt{-4}$ and $\frac{\sqrt{-16}}{\sqrt{-4}}$

Ans:

$$\sqrt{-16} \times \sqrt{-4}$$
$$= \sqrt{(4i)^2} \times \sqrt{(2i)^2}$$
$$= 4i \times 2i$$
$$= 8i^2$$
$$= -8$$

$$\frac{\sqrt{-16}}{\sqrt{-4}}$$
$$= \frac{\sqrt{(4i)^2}}{\sqrt{(2i)^2}}$$
$$= \frac{4i}{2i}$$
$$= 2$$

9. Evaluate Modulus and Argument of $8z-z^2$ by replacing $z = 2+i$

Now,

$$8z - z^2$$
$$= 8(2+i) - (2+i)^2$$
$$= 16 + 8i - (4 + 4i + i^2)$$
$$= 16 + 8i - 4 - 4i + 1$$
$$= 13 + 4i$$

Modulus $= \sqrt{(13)^2 + (4)^2} = \sqrt{185}$

Argument $= \tan^{-1}\left(\frac{4}{13}\right) =$

10. Express $1+i\sqrt{3}$ in the from of $r(\cos\theta + i\sin\theta)$

Now,

$$1+i\sqrt{3}$$

$$r = \sqrt{(1)^2 + (\sqrt{3})^2}$$

$$= \sqrt{4} = 2$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$$

$$= \frac{\pi}{3}$$

$$\therefore 1+i\sqrt{3} = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$