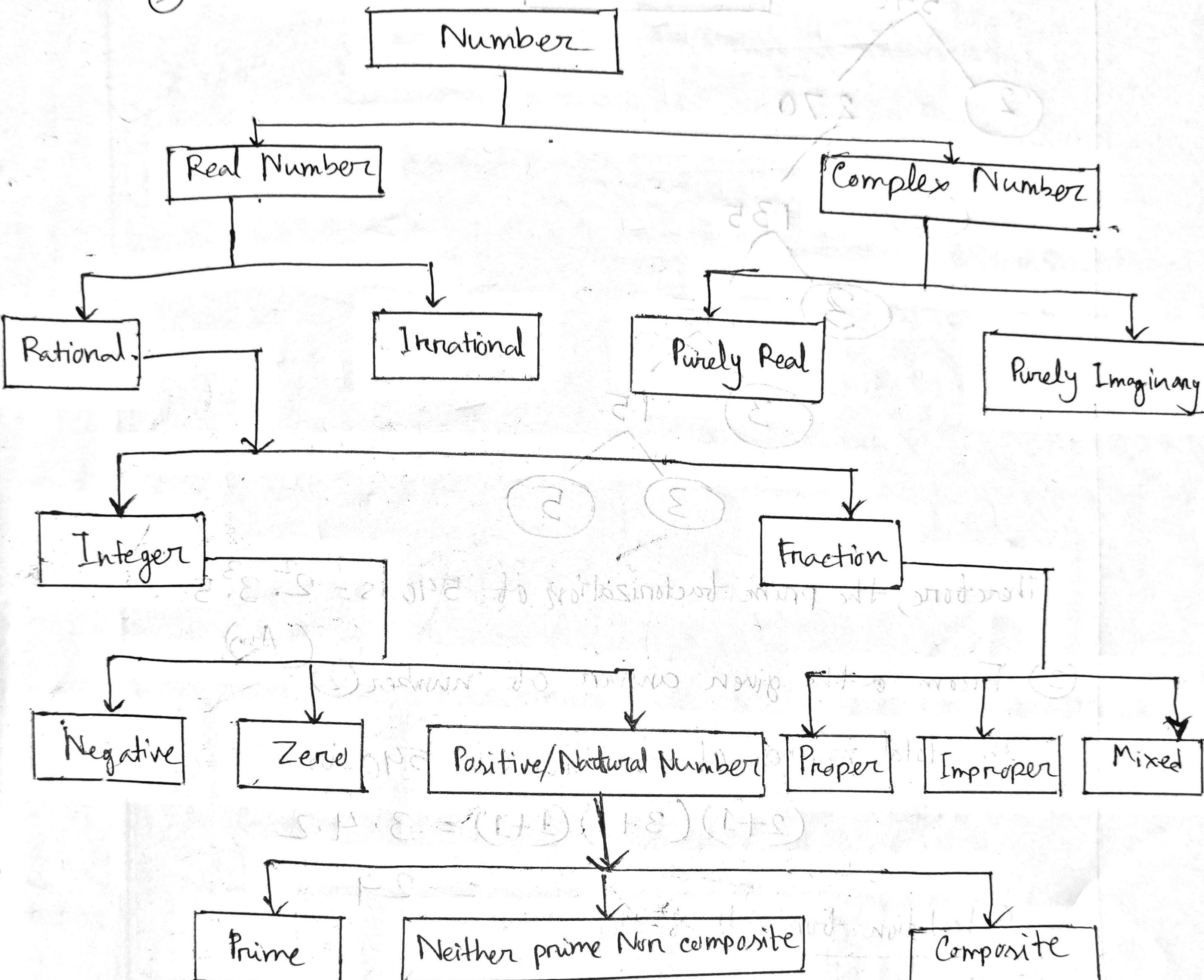
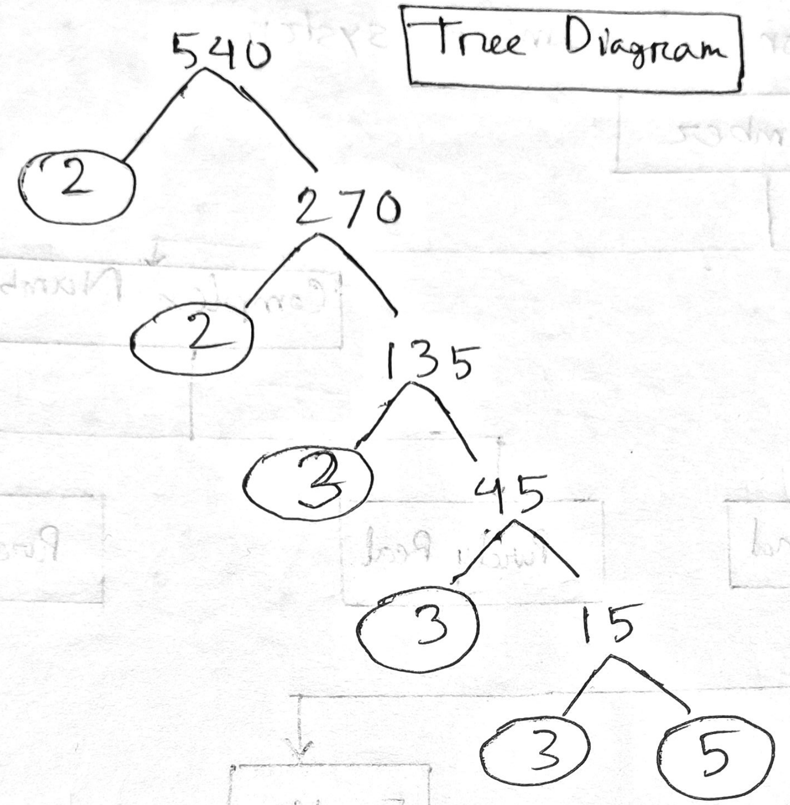
Complex Number

(1) the classification of number system



② Find the prime factorization of 540 using tree.

Tree Diagram



Therefore, the prime factorization of 540 is $= 2^2 \cdot 3^3 \cdot 5$ (Ans)

③ From ~~a~~ the given answer of number ②

The total number of factors of 540 is

$$(2+1)(3+1)(1+1) = 3 \cdot 4 \cdot 2$$
$$= 24$$

Calculation for all factors

$540 = 1 \times 540$
$= 2 \times 270$
$= 3 \times 180$
$= 4 \times 135$
$= 5 \times 108$
$= 6 \times 90$
$= 9 \times 60$
$= 10 \times 54$
$= 12 \times 45$
$= 15 \times 36$
$= 18 \times 30$
$= 20 \times 27$

the factors of 540 are,

1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540

(Ans)

(4) Given numbers are 240 and 540

$240 = 2\cdot120 = 2\cdot2\cdot60 = 2\cdot2\cdot2\cdot30 = 2\cdot2\cdot2\cdot2\cdot3\cdot5$

$= 2^4\cdot3\cdot5$

$540 = 2\cdot270 = 2\cdot2\cdot135 = 2\cdot2\cdot3\cdot45 = 2\cdot2\cdot3\cdot3\cdot15 = 2\cdot2\cdot3\cdot3\cdot3\cdot5$

$= 2^2\cdot3^3\cdot5$

$GCD(240, 540) = 2^2\cdot3\cdot5 = 60$

& $LCM(240, 540) = 2^4\cdot3^3\cdot5$

$= 2160$ (Ans)

(5) Given numbers are 42, 63 & 140

$42 = 2\cdot21 = 2\cdot3\cdot7$

$63 = 3\cdot21 = 3\cdot3\cdot7 = 3^2\cdot7$

$140 = 2\cdot70 = 2\cdot2\cdot35 = 2\cdot2\cdot5\cdot7 = 2^2\cdot5\cdot7$

$H.C.F(42, 63, 140) = 7$

$L.C.M(42, 63, 140) = 2^2\cdot3^2\cdot5\cdot7$

$= 1260$ (Ans)

(6) Given numbers are $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$

Calculation for Numerators

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

$10 = 2 \cdot 5$

$L.C.M (2, 8, 16, 10) = 2^4 \cdot 5$

$= 80$

$H.C.F (2, 8, 16, 10) = 2$

Calculation for Denominators

$3 = 3^1$

$9 = 3^2$

$81 = 3^4$

$27 = 3^3$

$L.C.M (3, 9, 81, 27) = 3^4$

$= \cancel{3^4}\ 81$

$H.C.F (3, 9, 81, 27) = 3$

$$\text{L.C.M of } \left(\frac{2}{3}, \frac{8}{9}, \frac{16}{81} \text{ and } \frac{10}{27}\right) = \frac{80}{3}$$

$$\text{H.C.F of } \left(\frac{2}{3}, \frac{8}{9}, \frac{16}{81} \text{ and } \frac{10}{27}\right) = \frac{2}{81}$$

(Am)

(7) Finding modulus, argument and polar

$$z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$$

$$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$$

$$= \frac{(1+\sqrt{3}i)^2}{1^2-(\sqrt{3}i)^2}$$

$$= \frac{1 + 2\sqrt{3}i + (\sqrt{3}i)^2}{1 + (\sqrt{3})^2}$$

$$= \frac{1 + 2\sqrt{3}i - 3}{1 + 3}$$

$$= \frac{2\sqrt{3}i - 2}{4}$$

$$= -\frac{1}{2} + \frac{\cancel{2}\sqrt{3}}{2} i \quad [x + iy]$$

So, $x = -\frac{1}{2}$ and $y = \frac{\sqrt{3}}{2}$

Modulus, $|z| = \sqrt{x^2 + y^2}$

$$= \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \sqrt{\frac{1}{4} + \frac{3}{4}}$$

$$= \sqrt{\frac{1+3}{4}}$$

$$= \sqrt{\frac{4}{4}} = 1$$

Argument,

$$\theta = \pi - \tan^{-1}\left(\frac{y}{x}\right)$$

$$= \pi - \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right)$$

$$= \pi - \tan^{-1}(-\sqrt{3})$$

$$= \pi - \tan^{-1}(\sqrt{3})$$

$$= \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

The polar form is,

$$z = r(\cos\theta + i\sin\theta)$$

$$z = 1\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

$$= \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

(Ans)

(8) $\sqrt{-16} \times \sqrt{-4}$ and $\frac{\sqrt{-16}}{\sqrt{-4}}$

$= i\sqrt{16} \times i\sqrt{4}$

$= 4i \times 2i$

$= 8i$

$= -8$

$= \frac{i\sqrt{16}}{i\sqrt{4}}$

$= \frac{4i}{2i}$

$= 2$

(Ans)

(9) Here's given, $z = 2+i$

So,

$8z - z^2$

$= 8(2+i) - (2+i)^2$ $[z = 2+i]$

$= 16 + 8i - 4 - 4i - i^2$

$= 12 + 4i - i^2$

$= 12 + 4i + 1$

$= 13 + 4i$ $[x + iy]$

So, $x = 13$ and $y = 4$

Modulus, $|z| = \sqrt{(13)^2 + (4)^2}$

$= \sqrt{169+16}$

$= \sqrt{185}$

Argument,

$\theta = \tan^{-1}\left(\frac{4}{13}\right)$

$= \tan^{-1}\frac{4}{13}$

(Ans)

(10) Here, by comparing $1+i\sqrt{3}$ with $z = x + iy$ we get,

$x = 1$ and $y = \sqrt{3}$

So, ~~the~~ modulus, $|z| = \sqrt{x^2+y^2}$

$= \sqrt{1^2+(\sqrt{3})^2}$

$= \sqrt{1+3}$

$= \sqrt{4}$

$= 2$

Argument, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$= \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$= \tan^{-1}(\sqrt{3})$

$= \frac{\pi}{3}$

Therefore, the polar form of $1+i\sqrt{3}$ is,

$r(\cos\theta + i\sin\theta)$

$= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

(Ans)