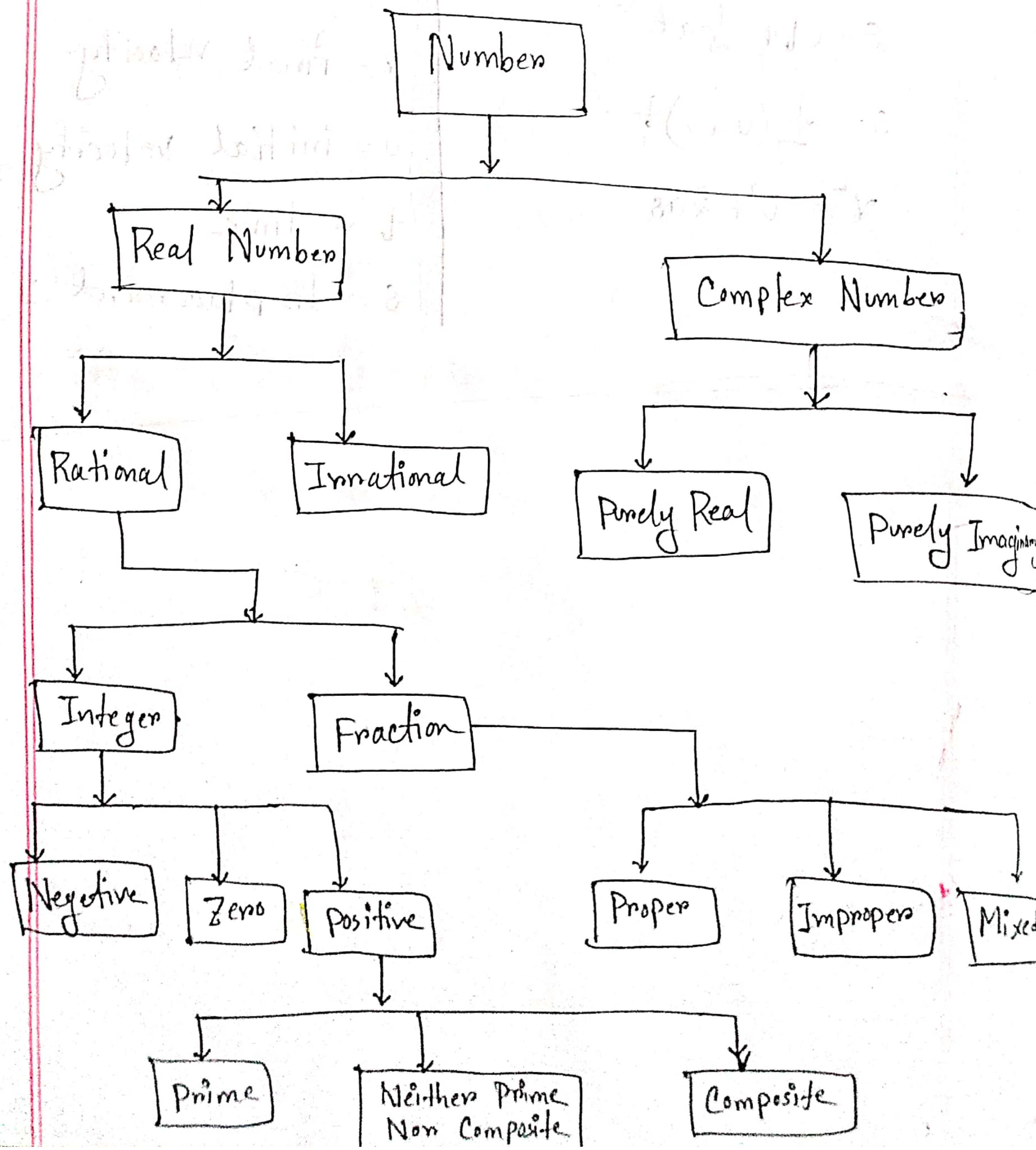
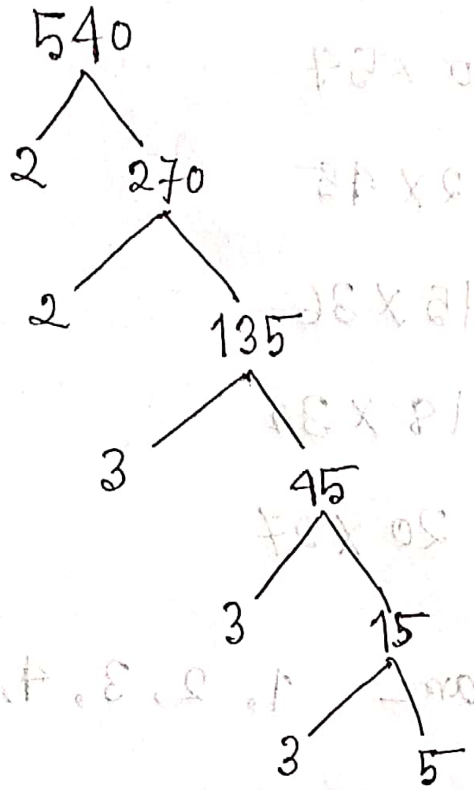
Math

Ans to the Q. NO. 1

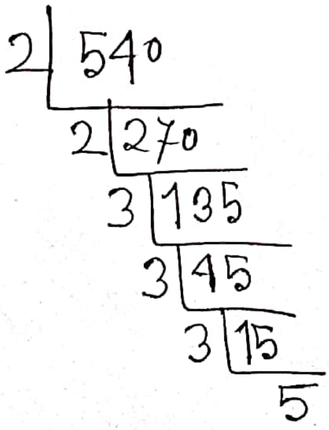


Ans to the Q.NO. 2



∴ The Prime factorization of 540 is $= 2^2 \cdot 3^3 \cdot 5$

Ans to the Q. NO. 3



Calculation for all factors

$540 = 1 \times 540$

$= 2 \times 270$

$= 3 \times 180$

$= 4 \times 135$

$= 5 \times 108$

The total number of factors of 540 is $(2+1)\cdot(3+1)\cdot(1+1)$

$= 3 \cdot 4 \cdot 2$

$= 24$.

$= 6 \times 90$

$= 9 \times 60$

$= 10 \times 54$

$= 12 \times 45$

$= 15 \times 36$

$= 18 \times 30$

$= 20 \times 27$

The factors of 540 are 1, 2, 3, 4, 5, 6, ~~7~~9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540.

Ans to the Q. NO. 4

$$\begin{array}{r|l} 2 & 240 \\ \hline 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 540 \\ \hline 2 & 270 \\ \hline 3 & 135 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

$\therefore 240 = 2^4 \cdot 3 \cdot 5$ $\qquad$ $\therefore 540 = 2^2 \cdot 3^3 \cdot 5$

$\therefore$ L.C.M of $(240, 540) = 2^4 \cdot 3^3 \cdot 5 = 2160$

$\therefore$ GCD of $(240, 540) = 2^2 \cdot 3 \cdot 5 = 60$

Ans.

Ans to the Q. NO. 5

$42 = 2 \times 21 = 2 \times 3 \times 7$

$63 = 3 \times 21 = 3 \times 3 \times 7 = 3^2 \cdot 7$

$140 = 2 \times 70 = 2 \times 2 \times 35 = 2^2 \cdot 5 \cdot 7$

$\therefore$ LCM of $(42, 63, 140) = 2^2 \cdot 3^2 \cdot 7 \cdot 5 = 1260$

$\therefore$ HCF of $(42, 63, 140) = 7$.

Ans.

Ans to the Q. NO: 6

Calculation of Numerators. Denominators.

$2 = 2^1$	$3 = 3^1$
$8 = 2^3$	$9 = 3^2$
$16 = 2^4$	$81 = 3^4$
$10 = 2 \cdot 5$	$27 = 3^3$.

L.C.M of Numerator $= 2^4 \cdot 5$

$= 80$

HCF of Numerator $= 2$.

L.C.M of Denominator $= 3^4 = 81$.

HCF of Denominator $= 3$.

H.CF of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{2}{81}$

L.CM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{80}{3}$

Ans.

Ans to the Q. NO. 7

$$z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i} = \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)} = \frac{(1+\sqrt{3}i)^2}{(1)^2-(\sqrt{3}i)^2}$$

$$= \frac{1^2+2\cdot 1\cdot\sqrt{3}i+(\sqrt{3}i)^2}{1-3i^2} = \frac{1+2\sqrt{3}i-3}{4}$$

$$= \frac{-2+2\sqrt{3}i}{4} = -\frac{2}{4}+\frac{2\sqrt{3}i}{4}$$

$$= -\frac{1}{2}+\frac{\sqrt{3}i}{2}$$

Where,

$x = -\frac{1}{2}$ and $y = \frac{\sqrt{3}}{2}$

$$\therefore r = \sqrt{x^2+y^2}$$

$$= \sqrt{\left(-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \sqrt{\frac{1}{4}+\frac{3}{4}}$$

$= \sqrt{1}$

$= 1.$

$\therefore \theta = \pi - \tan^{-1}\left(\frac{y}{x}\right)$

$= \pi - \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right)$

$= \pi - \tan^{-1}(\sqrt{3})$

$= \pi - \tan^{-1} \tan \frac{\pi}{3}$

$= \left(\pi - \frac{\pi}{3}\right)$

$= \frac{2\pi}{3}$

So, the polar form is $z = r \left(\cos\theta + i\sin\theta\right)$

$= 1\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$= \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}.$

Exponential form $z = e^{i\frac{2\pi}{3}}$ (Ans).

Ans to the Q.NO. 8

$\therefore \sqrt{16} \times \sqrt{-4}$

$= \sqrt{16i} \times \sqrt{4i}$

$= 4i \times 2i$

$= 8i^2$

$= -8.$

$\frac{\sqrt{-16}}{\sqrt{-9}}$

$= \frac{4i}{2i}$

$= 2$

Ans to the Q.NO. 9

Given that, $z = 2+i$

$\therefore 8z - z^2 = 8(2+i) - (2+i)^2$

$= 16+8i-(4+4i+i^2)$

$= 16+8i-4-4i-i^2$

$= 16+8i-4-4i+1$

$= 13+4i$

Modulus, $r = \sqrt{x^2 + y^2}$

$= \sqrt{(13)^2 + (4)^2}$

$= \sqrt{169 + 16}$

$= \sqrt{185}$ (Ans.)

Argument,

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$= \tan^{-1}\left(\frac{4}{13}\right)$

$= 17.102$ (Ans)

Ans to the Q. NO. 10

Here,

$$z = x + iy$$

$$= 1 + i\sqrt{3}$$

$\therefore x = 1$

$y = \sqrt{3}$

Modulus, $r = \sqrt{x^2 + y^2}$

$$= \sqrt{1^2 + (\sqrt{3})^2}$$

$$= \sqrt{1+3}$$

$$= 2.$$

Argument, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$$= \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$$

$$= \tan^{-1} \tan\frac{\pi}{3}$$

$$= \frac{\pi}{3}$$

Therefore,

$r(\cos\theta + i\sin\theta)$ form is $= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

Ans: