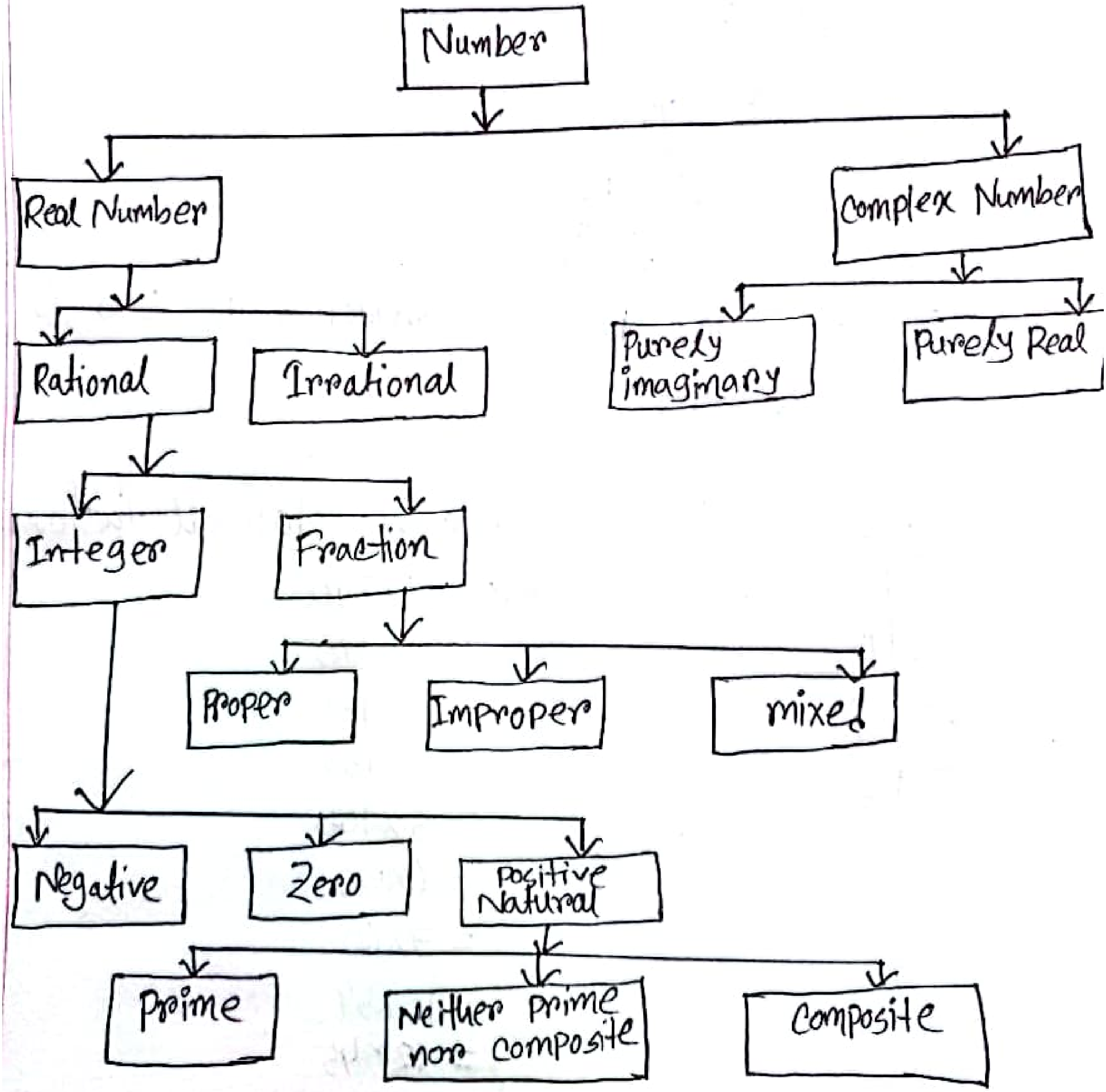
Complex Number

1. classification of number system :



2. Find the prime factorization of 540 with.

$$540 \to 2 \times 270$$
$$270 \to 2 \times 135$$
$$135 \to 3 \times 45$$
$$45 \to 3 \times 15$$
$$15 \to 3 \times 5$$

Therefore the prime factorization of 540 is

$= 2^2 \cdot 3^3 \cdot 5$

3.

2	540
2	270
3	135
3	45
3	15
	5

calculation for all factors-

$540 = 1 \times 540$
$= 2 \times 270$
$= 3 \times 180$
$= 4 \times 135$
$= 5 \times 108$
$= 6 \times 90$
$= 9 \times 60$
$= 10 \times 54$
$= 12 \times 45$
$= 15 \times 36 = 18 \times 30 = 20 \times 27$

Therefore, the Prime factorization of 540 is

$\Rightarrow = 2^2 \cdot 3^3 \cdot 5$

So, the total number of factors of 540

is $= (2+1)(3+1)(1+1)$

$= 3 \cdot 4 \cdot 2$

$= 24$

The factors of 540 are : 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540

4) $240 = 2 \times 120 = 2 \times 2 \times 60 = 2 \times 2 \times 2 \times 2 \times 15$

$= 2^4 \times 3 \times 5$

$540 = 2 \times 270 = 2 \times 2 \times 135 = 2 \times 2 \times 3 \times 45 = 2 \times 2 \times 3 \times 3 \times 15$

$= 2^2 \times 3^3 \times 5$

$\therefore$ LCM $(240, 540) = 2^4 \cdot 3^3 \cdot 5 = 2160$

HCF or GCD $(240, 540) = 2^2 \cdot 3 \cdot 5 = \cancel{30}\ 60$

5] $42 = 2 \times 21 = 2 \times 3 \times 7$

$63 = 3 \times 21 = 3 \times 3 \times 7 = 3^2 \times 7$

$140 = 2 \times 70 = 2 \times 2 \times 35 = 2^2 \times 5 \times 7$

$\therefore$ LCM $(42, 63, 140) = 2^2 \times 3^2 \times 5 \times 7 = 1260$

HCF $(42, 63, 140) = \cancel{7 = 14} = 7$

6] calculation for Numerators

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

$10 = 2^1 \times 5^1$

LCM $(2, 8, 16, 10) = 2^4 \times 5$
$= 80$

HCF $(2, 8, 16, 10) = 2$

Calculation for denominators

$3 = 3^1$

$9 = 3^2$

$81 = 3^4$

$27 = 3^3$

LCM $(3, 9, 81, 27) = 3^4$
$= 81$

HCF $(3, 9, 81, 27) = 3$

HCF of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$ $= \dfrac{\text{HCF}(2, 8, 16, 10)}{\text{LCM}(3, 9, 81, 27)}$

$= \dfrac{2}{81}$

LCM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{\text{LCM }(2,8,16,10)}{\text{HCF }(3,9,81,27)}$

$= \frac{80}{3}$

7] We have, $\frac{1+\sqrt{3}i}{1-\sqrt{3}i}$

$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$

$= \frac{1+2\sqrt{3}i-3}{1^2-(\sqrt{3}i)^2}$

$= \frac{-2+2\sqrt{3}i}{1+3}$

$= \frac{2(-1+\sqrt{3}i)}{4}$

$= \frac{-1+\sqrt{3}i}{2}$

$= -\frac{1}{2}+\frac{\sqrt{3}}{2}i$

Let $z = -\frac{1}{2}+\frac{\sqrt{3}}{2}i$

$|z| = \sqrt{(-\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}$

$r = \sqrt{\frac{1}{4}+\frac{3}{4}}$

$= \sqrt{\frac{4}{4}}$

$= 1$

Modulus of z is $= 1$

And Argument of z -

$\theta = \pi - \tan^{-1}\left|\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right|$

$= \pi - \tan^{-1}(\sqrt{3})^2$

$= \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

Polar Form $= -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

Exponential form is $z = re^{i\theta}$

$= 1 \cdot e^{i\frac{2\pi}{3}}$

$= e^{\frac{2\pi}{3}i}$

8] We have,

$\sqrt{-16} \times \sqrt{-4}$

$= \sqrt{16}i \times \sqrt{4}i$

$= 4i \times 2i$

$= 8i^2$

$= -8$

And $\frac{\sqrt{-16}}{\sqrt{-4}}$

$= \frac{4i}{2i}$

$= 2$

9] we have, $z = 2+i$

$\therefore 8z - z^2 = 8(2+i) - (2+i)^2$

$= 16 + 8i - (4 + 4i + i^2)$

$= 16 + 8i - 4 - 4i + 1$

$= 13 + 4i$

Modulus, $r = \sqrt{(13)^2 + (4)^2}$

$= \sqrt{169+16} = \sqrt{185}$

$\theta = \tan^{-1}\frac{4}{13}$

$= 17.102$

$\boxed{10}$ Let, $z = 1 + i\sqrt{3}$, $z = x + iy$

$|z| = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\left|\frac{y}{x}\right|$

Modulus of $z = \sqrt{(1)^2 + (\sqrt{3})^2}$

$= \sqrt{1+3}$

$= 2$

Argument of $z = \tan^{-1}\left|\frac{y}{x}\right|$

$= \tan^{-1}\left|\frac{\sqrt{3}}{1}\right|$

$= \tan^{-1}\tan\frac{\pi}{3}$

$= \frac{\pi}{3}$

Therefore, $r(\cos\theta + i\sin\theta)$ from is

$= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$