1. Classification of Number system.

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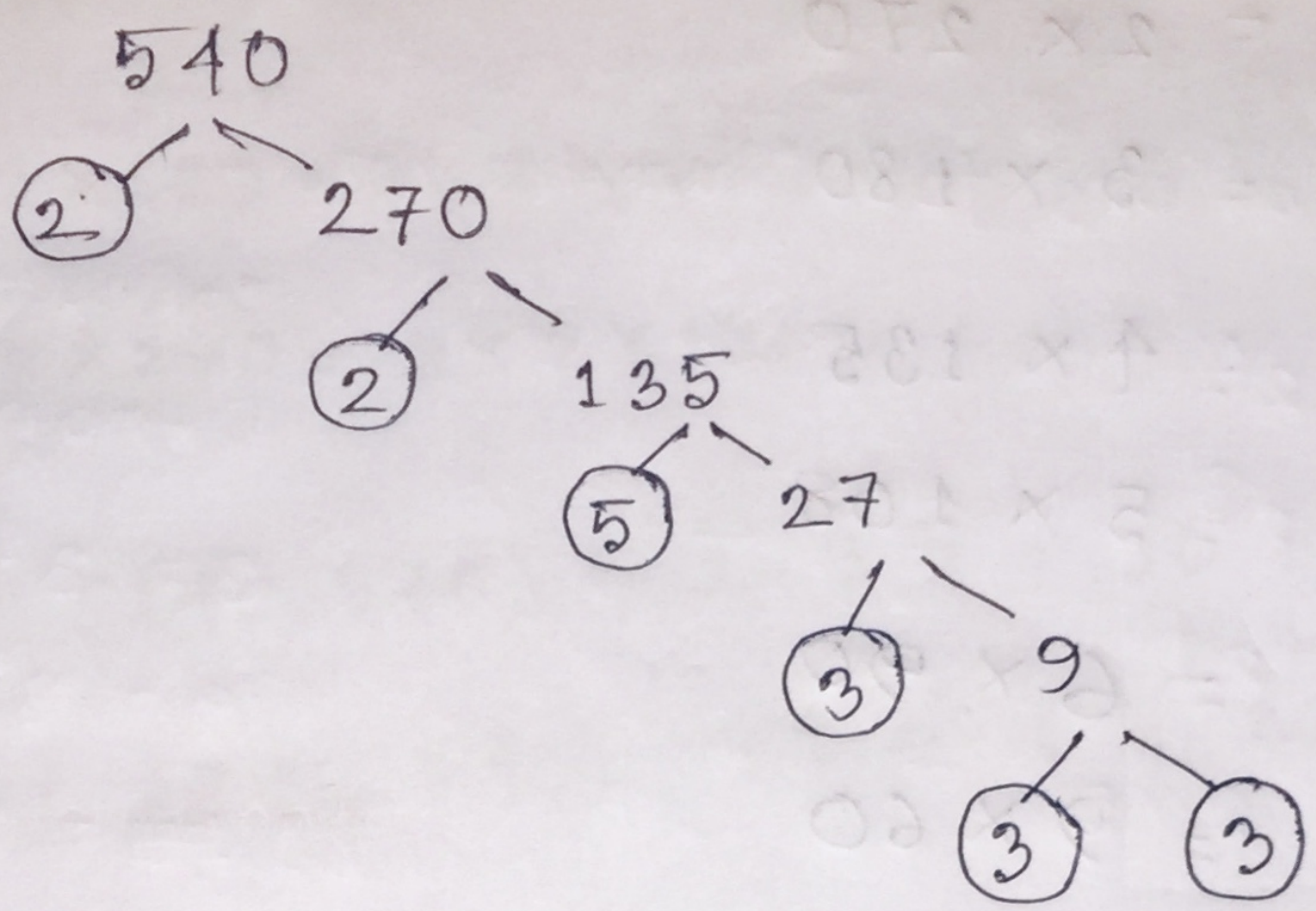
- Number
  - Real Number
    - Rational
      - Integer
        - Negative
        - Zero
        - Positive/ Natural N.
          - Prime
          - Neither prime nor composite
          - Composite.
      - Fraction
        - proper
        - Improper
        - Mixed
    - Irrational
  - Complex Number
    - Purely real
    - Purely imaginary

2. Find the prime factorization of 540 using tree



The prime factorization of 540 is $= 2^2 \cdot 3^3 \cdot 5$

3. Findout all the factors of 540.

$$
\begin{array}{r|l}
2 & 540 \\
\hline
2 & 270 \\
\hline
5 & 135 \\
\hline
3 & 27 \\
\hline
3 & 9 \\
\hline
 & 3
\end{array}
$$

Prime factorization of $540 = 2^2 \cdot 3^3 \cdot 5$.

Total numbers of factors $= (2+1)(3+1)(1+1)$

$= 3 \times 4 \times 2$

$= 24$.

$$
\begin{aligned}
540 &= 1 \times 540 \\
&= 2 \times 270 \\
&= 3 \times 180 \\
&= 4 \times 135 \\
&= 5 \times 108 \\
&= 6 \times 90 \\
&= 9 \times 60 \\
&= 10 \times 54 \\
&= 12 \times 45 \\
&= 15 \times 36 \\
&= 18 \times 30 \\
&= 20 \times 27
\end{aligned}
$$

All the factors are- 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270. 540.

4. What is the GCD & LCM of 240 & 540.

$240 = 2 \times 120 = 2 \times 2 \times 60 = 2 \times 2 \times 2 \times 2 \times 3 \cdot 5 = 2^4 \cdot 3 \cdot 5$

$540 = 2 \times 270 = 2 \times 2 \times 135 = 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 2^2 \cdot 3^3 \cdot 5$

$\therefore$ GCD of $(240, 540) = 2 \cdot 3 \cdot 5 = 30$.

$\therefore$ LCM of $(240, 540) = 2^4 \cdot 3^3 \cdot 5 = 2160$ (Ans)

5. Find the HCF & LCM of (42, 63 & 140)

$42 = 6 \times 7 = 2 \cdot 3 \cdot 7$

$63 = 7 \times 9 = 3^2 \cdot 7$

$140 = 2 \times 2 \times 5 \times 7 = 2^2 \cdot 5 \cdot 7$.

HCF of (42, 63 & 140) $= \cancel{2}\ 7$.

LCM of (42, 63 & 140) $= 2^2 \cdot 3^2 \cdot 5 \cdot 7$

$= 1260$ (Ans)

6. Find the HCF & LCM of $\frac{2}{3}$, $\frac{8}{9}$, $\frac{16}{81}$ and $\frac{10}{27}$.

Calculations for Numerator

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

$10 = 2 \times 5$

Calculations for Denominator

$3 = 3^1$

$9 = 3^2$

$81 \cancel{16} = 3^4$

$27 = 3^3$

LCM of $(2, 8, 16, 10) = 2^4 \times 5 = 80$

HCF of $(2, 8, 16, 10) = 2$

LCM of $(3, 9, 81, 27) = 3^4 = 81$

HCF of $(3, 9, 81, 27) = 3$

$\therefore$ LCM of $\left(\frac{2}{3}, \frac{8}{9}, \frac{16}{81} \text{ and } \frac{10}{27}\right) = \frac{\text{LCM of } (2,8,16,10)}{\text{HCF of } (3,9,81,27)}$

$= \frac{80}{3}$

$\therefore$ HCF of $\left(\frac{2}{3}, \frac{8}{9}, \frac{16}{81} \text{ and } \frac{10}{27}\right) = \frac{\text{HCF of } (2,8,16,10)}{\text{LCM of } (3,9,81,27)}$

$= \frac{\cancel{3}\ 2}{81}$

(Ans)

7. Find the modulus and Argument of $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$ and also it's polar, exponential form.

Given that,

$$z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$$

$$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$$

$$= \frac{1+\sqrt{3}i+\sqrt{3}i+(\sqrt{3}i)^2}{(1)^2-(\sqrt{3}i)^2}$$

$$= \frac{1+2\sqrt{3}i+3i^2}{1-3i^2}$$

$$= \frac{2\sqrt{3}i-2}{1+3}$$

$$= \frac{2(\sqrt{3}i-1)}{4}$$

$$= \frac{\sqrt{3}i-1}{2}$$

$$= -\frac{1}{2}+i\frac{\sqrt{3}}{2}$$

modulas of z, $|z| = \sqrt{\left(-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$

$$= \sqrt{\frac{1}{4}+\frac{3}{4}} = \sqrt{\frac{4}{4}} = \pm 1.$$

and the argument of z,

$$\theta = \pi - \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right)$$

$$= \pi - \tan^{-1}(-\sqrt{3})$$

$$= \pi - \left(-\frac{\pi}{3}\right)$$

$$= \pi + \frac{\pi}{3}$$

$$= \frac{4\pi}{3}$$

The polar form of z is,

$$z = r(\cos\theta + i\sin\theta)$$

$$= 1\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$$

Exponent form of z is,

$$r = e^{i\theta}$$

$$= e^{i\frac{4\pi}{3}}.$$

8. Evaluate $\sqrt{-16} \times \sqrt{-4}$ & $\frac{\sqrt{-16}}{\sqrt{-4}}$.

a) $\sqrt{-16} \times \sqrt{-4}$

$= \sqrt{16} \times \sqrt{-1} \times \sqrt{4} \times \sqrt{-1}$

$= 4i \times 2i$

$= 8i^2$

$= -8.$

b) $\frac{\sqrt{-16}}{\sqrt{-4}}$

$= \frac{\sqrt{16} \times \sqrt{-1}}{\sqrt{4} \times \sqrt{-1}}$

$= \frac{4}{2}$

$= 2.$

9. Evaluate Modulus & Arguement of $8z - z^2$ by replacing $z = 2+i$.

Given that, $8z - z^2$

$= 8(2+i) - (2+i)^2$

$= 16 + 8i - 4 - 4i - i^2$

$= 16 + 8i - 4 - 4i + 1$

$= 12i - 13.$

$= (-13) + (12)i$

modulas of $|8z - z^2| = \sqrt{(-13)^2 + (12)^2} = \sqrt{169 + 144}$

$= \sqrt{313}.$

Argument of $8z - z^2$, $\theta = \pi - \tan^{-1}\left(\frac{12}{-13}\right)$.

(Ans)

10. Express $1+i\sqrt{3}$ in the form of $r(\cos\theta + i\sin\theta)$

Given that,

$$1 + i\sqrt{3}$$

$$r = \sqrt{(1)^2 + (\sqrt{3})^2}$$
$$= \sqrt{4}$$
$$= 2$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$$
$$= \tan^{-1}(\sqrt{3})$$
$$= \frac{\pi}{3}$$

In the form of $r(\cos\theta + i\sin\theta)$

$$= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

(Ans)