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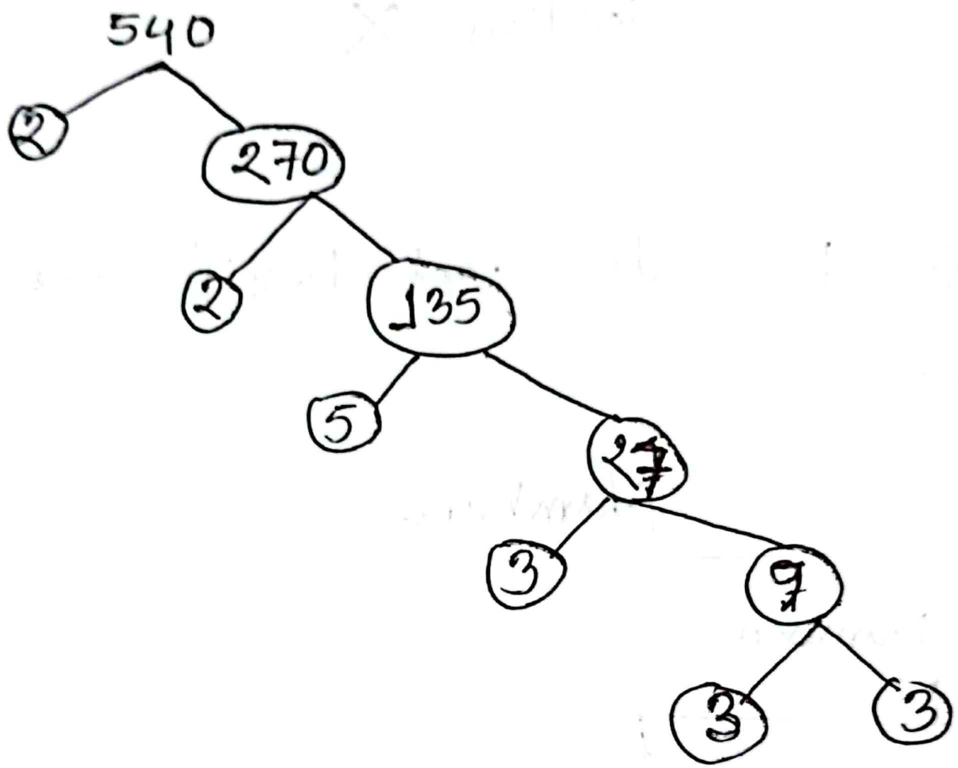
Section: X

1. Write down the ~~cat~~ classification of number system?

⟹ Number

- Real Number
  - Rational
    - Integer
      - Negative
      - Zero
      - Positive/Natural Number
        - Prime
        - Neither prime nor Composite
        - Composite
    - Fraction
      - Proper
      - Improper
      - Mixed
  - Irrational
- Complex Number
  - Purely Real
  - Purely Imaginary

2. Find the prime factorization of 540 using tree



$540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$

$= 2^2 3^3 5$

(Ans)

3. Find out the all factors of 540.

$$540 = 1 \times 540$$
$$= 2 \times 270$$
$$= 3 \times 180$$
$$= 4 \times 135$$
$$= 5 \times 108$$
$$= 6 \times 90$$
$$= 9 \times 60$$
$$= 10 \times 54$$
$$= 12 \times 45$$
$$= 15 \times 36$$
$$= 18 \times 30$$
$$= 20 \times 27$$

The factors of 540 are = 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540.

(Ans)

4. What is the GCD & LCM of 240 & 540.

~~240 =~~

$$\begin{array}{r|l} 2 & 240 \\ \hline 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 540 \\ \hline 2 & 270 \\ \hline 5 & 135 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$240 = 2^4 \cdot 3 \cdot 5$

$540 = 2^2 \cdot 3^3 \cdot 5$

$GCD = 2^2 \cdot 3 \cdot 5 = 60$

$LCM = 2^4 \cdot 3^3 \cdot 5 = 2160$

(Ans.)

5. Find the H.C.F & L.C.M of 42, 63 & 140 ?

$$\begin{array}{r|l} 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 2 & 140 \\ \hline 2 & 70 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

$42 = 2 \times 3 \times 7$

$63 = 3 \times 3 \times 7 = 3^2 \times 7$

$140 = 2^2 \times 5 \times 7$

~~H.C.F = [illegible]~~

$H.C.F = 7$

$L.C.M = 2^2 \times 3^2 \times \cancel{7}\, 5 \times 7$

$= 1260$

(Ans)

6. Find the H.C.F & L.C.M of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$.

$\Rightarrow$ ~~Calcu~~ Calculation for Numerators

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

$10 = 2 \cdot 5$

$\text{L.C.M} (2, 8, 16, 10) = 2^4 \times 5$
$= 80$

$\text{H.C.F} (2, 8, 16, 10) = 2$.

Calculation for Denominators

$3 = 3^1$

$9 = 3^2$

$81 = 3^4$

$27 = 3^3$

$\text{L.C.M} (3, 9, 81, 27) = 3^4$
$= 81$

$\text{H.C.F} (3, 9, 81, 27) = 3$

$\therefore \text{L.C.M of } \frac{2}{3}, \frac{8}{9}, \frac{16}{81} \text{ and } \frac{10}{27} = \frac{\text{L.C.M of } (2, 8, 16, 10)}{\text{H.C.F of } (3, 9, 81, 27)}$

$= \frac{80}{3}$

$$\text{H.C.F of } \frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27} = \frac{\text{H.C.F of } (2,8,16,10)}{\text{L.C.M of } (3,9,81,27)}$$

$$= \frac{2}{81}$$

7. Find the modulus and Argument of $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$ and also its polar, exponential form.

⇒ ~~who~~ We have,

$$\frac{1+\sqrt{3}i}{1-\sqrt{3}i}$$

$$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1+\sqrt{3}i)(1-\sqrt{3}i)}$$

$$= \frac{1+2\sqrt{3}i-3}{(1)^2-(\sqrt{3}i)^2}$$

$$= \frac{-2+2\sqrt{3}i}{4}$$

$$= \frac{-1+\sqrt{3}i}{2}$$

$\therefore$ Polar Form $= -\frac{1}{2}+\frac{\sqrt{3}}{2}i$ (Ans).

Let $z = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$

$$|z| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$r = \sqrt{\frac{1}{4} + \frac{3}{4}}$$

$$= \sqrt{\frac{4}{4}}$$

$$= 1$$

$\therefore$ Modulus of z is $= 1$ (Ans)

and, Argument of z will

$$\theta = \pi - \tan^{-1}\left|\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right|$$

$$= \pi - \tan^{-1}(\sqrt{3})$$

$$= \pi - \frac{\pi}{3}$$

$$= \frac{3\pi - \pi}{3}$$

$$= \frac{2\pi}{3}$$ (Ans).

Exponential form is $z = re^{i\theta}$

$$= 1 \cdot e^{i\frac{2\pi}{3}}$$

$$= e^{\frac{2\pi}{3}i}$$ (Ans).

8. Evaluate $\sqrt{-16} \times \sqrt{-4}$ & $\frac{\sqrt{-16}}{\sqrt{-4}}$.

Let,

$\sqrt{-16} \times \sqrt{-4}$

$= 4i \times 2i$

$= 8i^2$

$= -8$ (Ans).

and

$\frac{\sqrt{-16}}{\sqrt{-4}}$

$= \frac{4i}{2i}$

$= 2$ (Ans)

9. Evaluate Modulus & Argument of. $8z - z^2$ by replacing $z = 2 + i$

$\Rightarrow$ we have, $8z - z^2$

$= 8(2+i) - (2+i)^2$ $[z = 2+i]$

$= 16 + 8i - 4 - 4i + 1$

$= 13 + 4i$

So, $x = 13$ and $y = 4$

Modulus $r = \sqrt{(13)^2 + (4)^2}$

$= \sqrt{169 + 16}$

$= \sqrt{185}$

Argument,

$\theta = \tan^{-1}\left|\frac{4}{13}\right|$

$= \tan^{-1}\frac{4}{13}$ Ans.

10. Express $1 + i\sqrt{3}$ in the form of $r(\cos\theta + i\sin\theta)$

$\Rightarrow$ Let, $z = 1 + i\sqrt{3}$

$z = x + iy$

$\therefore |z| = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\left|\frac{y}{x}\right|$

$\therefore$ Modulus of $z = \sqrt{(1)^2 + (\sqrt{3})^2}$

$= \sqrt{4}$

$= 2$

$\therefore r = 2$

Argument of $z = \tan^{-1}\left|\frac{y}{x}\right|$

$= \tan^{-1}\left|\frac{\sqrt{3}}{1}\right|$

$= \tan^{-1}\tan\frac{\pi}{3}$

$= \frac{\pi}{3}$ (Ans).

Therefore,

$r(\cos\theta + i\sin\theta)$ form is $= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

Ans.