① Write down the classification of number system...

Number
- Real Number
  - Rational
    - Integer
    - Fraction
  - Irrational
- Complex number
  - Purely Real
  - Purely imagin[ary]

② Find the Prime factorization of 540 using tree method

540
- ②
- 270
  - ②
  - 135
    - ③
    - 45
      - ⑤
      - 9
        - ③
        - ③

Prime factorization of 540 $= 2\times 2\times 3\times 5\times 3\times 3 = 2^2 . 3^3 . 5^1$

$= 2^2 . 3^3 . 5^1$

③ Find out the all factors of 540

$$
\begin{array}{r|l}
2 & 540 \\
2 & 270 \\
3 & 135 \\
5 & 45 \\
3 & 9 \\
 & 3
\end{array}
$$

Prime factorization of $540 = 2 \times 2 \times 3 \times 5 \times 3 \times 3$

$= 2^2 \cdot 3^3 \cdot 5^1$

total number of factor is $= (n+1) \cdot (n+1) \cdot (n+1)$

$= (2+1) \cdot (3+1) \cdot (1+1)$

$= 3 \cdot 4 \cdot 2$

$= 24$

Now...

$540 = 1 \times 540$

$= 2 \times 270$

$= 4 \times 135$

$= 12 \times 45$

$= 108 \times 5$

$= 54 \times 10$

$= 27 \times 20$

$= 3 \times 180$

$= 6 \times 90$

$= \cancel{12}\, 60 \times 10$

$= 18 \times 30$

$= 15 \times 36$

all the factors of 540 are: 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540,

(4) What is the G.C.D and L.C.M of 240 and 540.

$$\begin{array}{r|l} 2 & 240 \\ \hline 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 540 \\ \hline 3 & 270 \\ \hline 3 & 90 \\ \hline 3 & 30 \\ \hline 2 & 10 \\ \hline & 5 \end{array}$$

$= 2 \times 2 \times 2 \times 2 \times 3 \times 5$

$= 2^4 . 3 . 5^1$

$= 2 \times 3 \times 3 \times 3 \times 2 \times 5$

$= 2^2 . 3^3 . 5^1$

The GCD of 240 and 540 is: $2^2 . 3^1 . 5^1$

$= 4 . 3 . 5$

$= 60$

The LCM of 240 and 540 is: $2^4 . 3^3 . 5^1$

$= 16 . 27 . 5$

$= \cancel{2400}\ 2160$

Ancor

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⑤ What is the H.C.F and L.C.M of 42, 63, 140

$$\begin{array}{r|l} 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 2 & 140 \\ \hline 2 & 70 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

$= 2\times3\times7$

$= 3\times3\times7$

$= 3^2\times7$

$= 2\times2\times5\times7$

$= 2^2\times5\times7$

$42 = 2\times3\times7$

$63 = 3^2\times7$

$140 = 2^2\times5\times7$

H.C.F of 42, 63 and 140 is: 7

L.C.M " 42, 63 and 140 is: $2^2\cdot3^2\cdot5\cdot7 = 1260$

⑥ Find the H.C.F and L.C.M of $\frac{2}{3}$, $\frac{8}{9}$, $\frac{16}{81}$ and $\frac{10}{27}$

Factorization of numerators:

$2 = 2^1$

$8 = 2\times4 = 2\times2\times2 = 2^3$

$16 = 2\times8 = 2\times2\times4 = 2\times2\times2\times2 = 2^4$

$10 = 2\times5 = 2^1\cdot5^1$

H.C.F of Numerators is $= 2^1$

L.C.M of " is $= 2^4\cdot5 = 16\times5 = 80$

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Fraclorization of Denominators:

$3 = 3^1$

$9 = 3 \times 3 = 3^2$

$81 = 3 \times 27 = 3 \times 3 \times 9 = 3 \times 3 \times 3 \times 3 = 3^4$

$27 = 3 \times 9 = 3 \times 3 \times 3 = 3^3$

HCF of Denominators is: 3

LCM " " is: $3^4 = 81$

now, we know that.

H.C.F of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$ is $= \frac{\text{HCF of Numerators}}{\text{LCM of Denominators}}$

$= \frac{2^1}{3^4} = \frac{2}{81}$

L.C.M of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$ is $= \frac{\text{LCM of Numerators}}{\text{HCF of Denominators}}$

$= \frac{2^4 \cdot 5}{3}$

$= \frac{80}{3}$

⑦ Find the modulus and Argument of $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$, and also it's Polar, exponential form

$$z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i} = \frac{(1+\sqrt{3}i)^2}{(1-\sqrt{3}i)\cdot(1+\sqrt{3}i)} = \frac{1^2+2\cdot 1\cdot\sqrt{3}i+(\sqrt{3}i)^2}{1^2-(\sqrt{3}i)^2}$$

$$= \frac{1+2\sqrt{3}i+3i^2}{1-3i^2} = \frac{1+2\sqrt{3}i-3}{1-(-3)} = \frac{2\sqrt{3}i-2}{4}$$

$$= \frac{2\sqrt{3}i}{4} - \frac{2}{4} = \frac{\sqrt{3}i}{2} - \frac{1}{2} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

Where, $a = -\frac{1}{2}$ and $b = \frac{\sqrt{3}}{2}i$ (Ans)

⑧ Evaluate $\sqrt{-16}\times\sqrt{-4}$ and $\frac{\sqrt{-16}}{\sqrt{-4}}$

$\sqrt{-16}\times\sqrt{-4}$

we know that, $i^2 = -1$

$$\sqrt{16i^2}\times\sqrt{4i^2}$$

$$= \sqrt{4^2 i^2}\times\sqrt{2^2 i^2}$$

$$= 4i \times 2i$$

$$= 8i^2$$

$$= 8(-1) = -8$$

$$\frac{\sqrt{-16}}{\sqrt{-4}}$$

$$= \frac{\sqrt{16i^2}}{\sqrt{4i^2}}$$

$$= \frac{\sqrt{4^2 \cdot i^2}}{\sqrt{2^2 \cdot i^2}} = \frac{4i}{2i} = 2$$

$\therefore \sqrt{-16} \times \sqrt{-4} = -8$

$\therefore \frac{\sqrt{-16}}{\sqrt{-4}} = 2$

9. Evaluate Modulus and Argument of $8z - z^2$ by replacing, $z = 2+i$

$8z - z^2$

$= 8(2+i) - (2+i)^2$

$= 16 + 8i - (4 + 2 \cdot 2 \cdot i + i^2)$

$= 16 + 8i - (4 + 4i - 1)$

$= 16 + 8i - 4 - 4i + 1$

$= 13 + 4i$

where, $13 = a$ and $4 = b$, $z = a + bi$

we know that
modulus, ~~Argument~~, $r = \sqrt{a^2+b^2}$

$= \sqrt{13^2+4^2}$

$= \sqrt{169+16}$

$= \sqrt{185}$

Argument, $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$= \tan^{-1}\left(\frac{4}{13}\right)$

$= 17.10^\circ$

(10) Express $1+i\sqrt{3}$ in the form of $r(\cos\theta + i\sin\theta)$

$z = 1+i\sqrt{3}$

where, $1 = a$, $\sqrt{3} = b$

now,
modulus, $|z| = r = \sqrt{a^2+b^2}$

$= \sqrt{1^2+(\sqrt{3})^2}$

$= \sqrt{4}$

$= 2$

argument, $\theta = \tan^{-1}(b/a) = \tan^{-1}(\sqrt{3}/a) = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

$= 60^\circ$ or $\frac{\pi}{3}$

$r(\cos\theta + i\sin\theta)$

$= 2\left(\cos\theta \frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

⑦ Find the modulus and argument of $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$ and also its polar, exponential from.

Here,

$$z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$$

$$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)} = \frac{(1+\sqrt{3}i)^2}{1^2-(\sqrt{3})^2} = \frac{1+2\sqrt{3}i+(\sqrt{3}i)^2}{1-(\sqrt{3}i)^2}$$

$$= \frac{1+2\sqrt{3}i-3}{1-(-3)} = \frac{-2+2\sqrt{3}i}{4} = -\frac{2}{4} + \frac{2\sqrt{3}i}{4}$$

$$= \cancel{\quad} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$\therefore z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

where, $a = -\frac{1}{2}$ and, $b = \frac{\sqrt{3}}{2}$

Modulus, $r = \sqrt{a^2+b^2} = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{3}{4}}$

$$= \sqrt{\frac{4}{4}} = 1$$

modulus of z is, $r = 1$

Argument, $\theta = \pi - \tan^{-1}\left|\frac{\sqrt{3}/2}{1/2}\right|$

$$= \pi - \tan^{-1}(\sqrt{3})^{2} = \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

Polar form $= -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

Exponential form is, $z = re^{i\theta}$

$$= 1 \cdot e^{i\frac{2\pi}{3}}$$

$$= e^{\frac{2\pi}{3}i}$$