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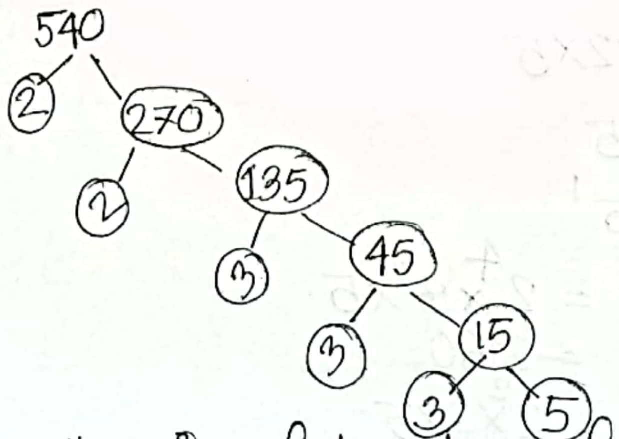
① Classification of number system:

- Number
  - Real number
    - Rational
      - Integer
        - Negative
        - Zero
        - Positive
          - Prime
          - Neither Prime non composite
          - composite
      - Fraction
        - Proper
        - Improper
        - Mixed
    - Irrational
  - complex number
    - Purely Real
    - Purely Imaginary

② Find the Prime factorization of 540 using tree method



∴ Therefore, the prime factorization of 540 is = $2^2 \cdot 3^3 \cdot 5^1$

③ Find all the factors of 540

$$540 = 1 \times 540$$
$$= 2 \times 270$$
$$= 4 \times 135$$
$$= 5 \times 108$$
$$= 6 \times 90$$
$$= 9 \times 60$$
$$= 12 \times 45$$
$$= 15 \times 36$$
$$= 18 \times 30$$
$$= 20 \times 27$$

All the factors of 540 are :- 1, 2, 4, 5, 6, 9, 12, 15, 18, 20, 27, 30, 36, 45, 60, 90, 108, 135, 270, 540.

④ $240 = 2 \times 120$
$$= 2 \times 2 \times 60$$
$$= 2 \times 2 \times 2 \times 2 \times 5$$
$$= 2^4 \times 3 \times 5$$

$540 = 2^2 \times 3^3 \times 5^1$

$L.CM\ (240, 540) = 2^4 \times 3^3 \times 5$
$= 2160$

$G.C.D\ (240, 540) = 2^2 \times 3 \times 5$
$= 60$

(5) ~~42 = ...~~

(5) $42 = 2 \times 3 \times 7$

$63 = 3 \times 3 \times 7 = 3^2 \times 7$

$140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$

$L.C.M\,(42, 63, 140) = 2^2 \times 3^2 \times 7 \times 5$

$= 1260$

$H.C.F\,(42, 63, 140) = 7$

(6) calculation for Numerators

$2 = 2^1$ $\qquad L.CM(2, 8, 10, 16) = 2^4 . 5 = 80$

$8 = 2^3$ $\qquad H.C.F = 2^1 = 2$

$16 = 2^4$

$10 = 2.5$

calculation for Denominators

$3 = 3^1$ $\qquad L.CM\,(\ldots 27, 81) = 3^4 = 81$

$9 = 3.3$ $\qquad H.C.F = 3^1 = 3$

$81 = 3.3.3.3$ $\qquad 27 = 3.9 = 3.3.3 = 3^3$

$= 3^4$

Therefore, L.CM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{80}{3}$

H.C.F of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{2}{81}$

(x) Finding modulus, Argument and Polar:-

$$Z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$$

$$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}\cdot i)}{(1+\sqrt{3}i)(1+\sqrt{3}i)}$$

$$= \frac{(1+\sqrt{3}i)^2}{1^2-(\sqrt{3}i)^2}$$

$$= \frac{1+2\sqrt{3}i-3}{1+3}$$

$$= \frac{-2+2\sqrt{3}i-3}{1+3}$$

$$= \frac{2(-1+\sqrt{3}i)}{2\cdot 2}$$

$$= \frac{-1+\sqrt{3}i}{2}$$

$$= -\frac{1}{2}+\frac{\sqrt{3}}{2}i$$

$$= -\frac{1}{2}+\frac{\sqrt{3}}{2}i$$

Let $z = -\frac{1}{2}+\frac{\sqrt{3}}{2}i$

$$|Z| = \sqrt{\left(-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$Z = \sqrt{\frac{1}{4}+\frac{3}{4}}$$

$$= 1$$

Modulus of z is $= 1$

And Argument of Z will,

$$\theta = \pi - \tan^{-1}\left|\frac{\frac{\sqrt{3}}{2}}{1/2}\right|$$

$$= \pi - \tan^{-1}(\sqrt{3})^2$$

$$= \pi - \pi/3$$

$$= \frac{2\pi}{3}$$

Polar form $= -\frac{1}{2}+\frac{\sqrt{3}}{2}i$

Exponential form is $z = re^{i\theta}$

$$= 1\cdot e^{i\cdot\frac{2\pi}{3}}$$

$$= e^{\frac{2\pi}{3}i}$$

⑧ Given

$\sqrt{-16} \times \sqrt{-4}$

$= \sqrt{16i} \times \sqrt{4i}$

$= 4i \times 2i$

$= 8i^2 = 8$

Again,

$\frac{\sqrt{-16}}{\sqrt{-4}}$

$= \sqrt{\frac{-16}{-4}}$

$= \sqrt{-4}$

$= \sqrt{2 \times 2 \times i^2} = -2$

⑨ Given,

$z = 2+i$

$8z - z^2 = 8(2+i) - (2+i)^2$

$= 16+8i - (4+4i+i^2)$

$= 16+4i$

Modulus $r = \sqrt{(13)^2 + (4)^2}$

$= \sqrt{185}$

$\theta = \tan^{-1}\frac{4}{13}$

$= 17.102$ Ans.

(10) Let, $z = 1+i\sqrt{3}$

$z = x+iy$

$|z| = \sqrt{x^2+y^2}$

we know, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Modulus of $z = \sqrt{(1)^2+(\sqrt{3})^2} = \sqrt{1+3} = 2$

~~r z = 2~~

$r = 2$

Again,

Argument of $z = \tan^{-1}\left|\frac{y}{x}\right|$

$= \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$= \tan^{-1}\left(\tan\frac{\pi}{3}\right)$

$= \pi/3$

Therefore, $r(\cos\theta + i\sin\theta)$ form is $= 2\left(\cos\pi/3 + i\sin\frac{\pi}{3}\right)$