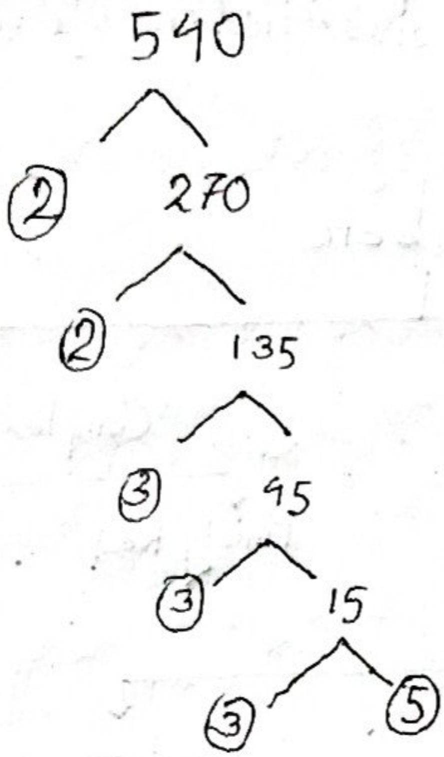
MAT-III

1. Write down the classification of number system.

- Number
  - Real Number
    - Rational
      - Integer
        - Negative
        - Zero
        - Positive/ Natural Number
          - Prime
          - Neither prime nor Composite
          - Composite
      - Fraction
        - Proper
        - Impoper
        - Mixed
    - Irrational
  - Complex Number
    - Purely Real
    - Purely Imaginary

2. Find the prime factorization of 540 using tree.



$\therefore 540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$

$= 2^2 . 3^3 . 5$

(Ans)

3. Find out the all factors of 540.

$540 = 1 \times 540$

$= 2 \times 270$

$= 3 \times 180$

$= 4 \times 135$

$= 5 \times 108$

$= 6 \times 90$

$= 9 \times 60$

$= 10 \times 54$

$= 12 \times 45$

$= 15 \times 36$

$= 18 \times 30$

$= 20 \times 27$

$\therefore$ The factors of 540 are =

1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36,

45, 54, 60, 90, 108, 135, 180, 270, 540.

4. What is the GCD & LCM of 240 & 540.

$240 = 2.120 = 2.2.60 = 2^3.30 = 2^4.3.5$

$540 = 2 \cdot 270 = 2^2.135 = 2^2.3.45 = 2^2.3^2.15$
$= 2^2.3^3.5$

$\therefore$ GCD of 240 & 540 $= 2^2.3.5 = 60$

$\therefore$ LCM of 240 & 540 $= 2^4.3^3.5 = 2160$

(Ans)

5. Find the H.C.F & L.C.M of 42, 63 & 140.

$42 = 2\times21 = 2\times3\times7 = 2^1\times3^1\times7^1$

$63 = 3\times21 = 3\times3\times7 = 3^2\times7^1$

$140 = 2\times70 = 2\times2\times35 = 2^2\times5^1\times7^1$

$\therefore$ LCM of (42, 63 and 140) $= 2^2.3^2.5.7 = 1260$

$\therefore$ HCF of (42, 63 and 140) $= 7$

(Ans)

6. Find the H.C.F & L.C.M of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ & $\frac{10}{27}$.

Calculation for Numerators

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

$10 = 2^1 \times 5^1$

$L.C.M\ (2,8,16,10) = 2^4 \times 5 = 80$

$H.C.F\ (2,8,16,10) = 2$

Calculation for Denominators

$3 = 3^1$

$9 = 3^2$

$81 = 3^4$

$27 = 3^3$

$\cancel{L.C.M\ (3,8,16,10}$

$L.C.M\ (3,9,81,27) = 3^4 = 81$

$H.C.F\ (3,9,81,27) = 3$

$$H.C.F \text{ of } \frac{2}{3}, \frac{8}{9}, \frac{16}{81} \& \frac{10}{27} = \frac{H.C.F\ (2,8,16,10)}{L.C.M\ (3,9,81,27)}$$

$$= \frac{2}{81}$$

$$L.C.M \text{ of } \frac{2}{3}, \frac{8}{9}, \frac{16}{81} \& \frac{10}{27} = \frac{L.C.M\ (2,8,16,10)}{H.C.F\ (3,9,81,27)}$$

$$= \frac{80}{3}$$

(Ans)

7. Find the modulus and Argument of $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$ and also its polar, exponential form.

$\Rightarrow \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$

$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$

$= \frac{1+2\sqrt{3}i-3}{1^2-(\sqrt{3}i)^2}$

$= \frac{-2+2\sqrt{3}i}{1+3}$

$= \frac{-2+2\sqrt{3}i}{4}$

$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

Let $z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

$\therefore |z| = \sqrt{(-\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}$

$= \sqrt{\frac{1}{4} + \frac{3}{4}}$

$= 1$

$\therefore$ Modulus of Z is $=1$

and Argument of z is

$\theta = \pi - \tan^{-1}\left|\frac{\sqrt{3}/2}{1/2}\right|$

$= \pi - \pi/3$

$= \frac{2\pi}{3}$

$\therefore$ Polar form $= \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$\therefore$ Exponential form is $= re^{i\theta}$

$= 1.e^{i\frac{2\pi}{3}}$

$= e^{\frac{2\pi}{3}i}$

(Ans)

8. Evaluate $\sqrt{-16} \times \sqrt{-4}$ & $\frac{\sqrt{-16}}{\sqrt{-4}}$

$\sqrt{-16} \times \sqrt{-4}$

$= 4i \times 2i$

$= 8i^2$

$= -8$

$\frac{\sqrt{-16}}{\sqrt{-4}}$

$= \frac{4i}{2i}$

$= 2$

(Ans)

9. Evaluate Modulus & Argument of $8z - z^2$ by replacing $z = 2+i$

Here, ~~H~~ $8z - z^2$

$= 8(2+i) - (2+i)^2$

$= 16 + 8i - 4 - 4i - i^2$

$= 13 + 4i$

Modulus $= \sqrt{13^2 + 4^2} = \sqrt{185}$

Argument $= \tan^{-1}\left(\frac{4}{13}\right) = 17.10°$

(Ans)

10. Express $1+i\sqrt{3}$ in the form of $r(\cos\theta + i\sin\theta)$.

$\Rightarrow$ $1+i\sqrt{3}$

$r = \sqrt{1^2+(\sqrt{3})^2}$

$= \sqrt{4} = 2$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$= \frac{\pi}{3}$

$\therefore 1+i\sqrt{3} = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

(Ans)