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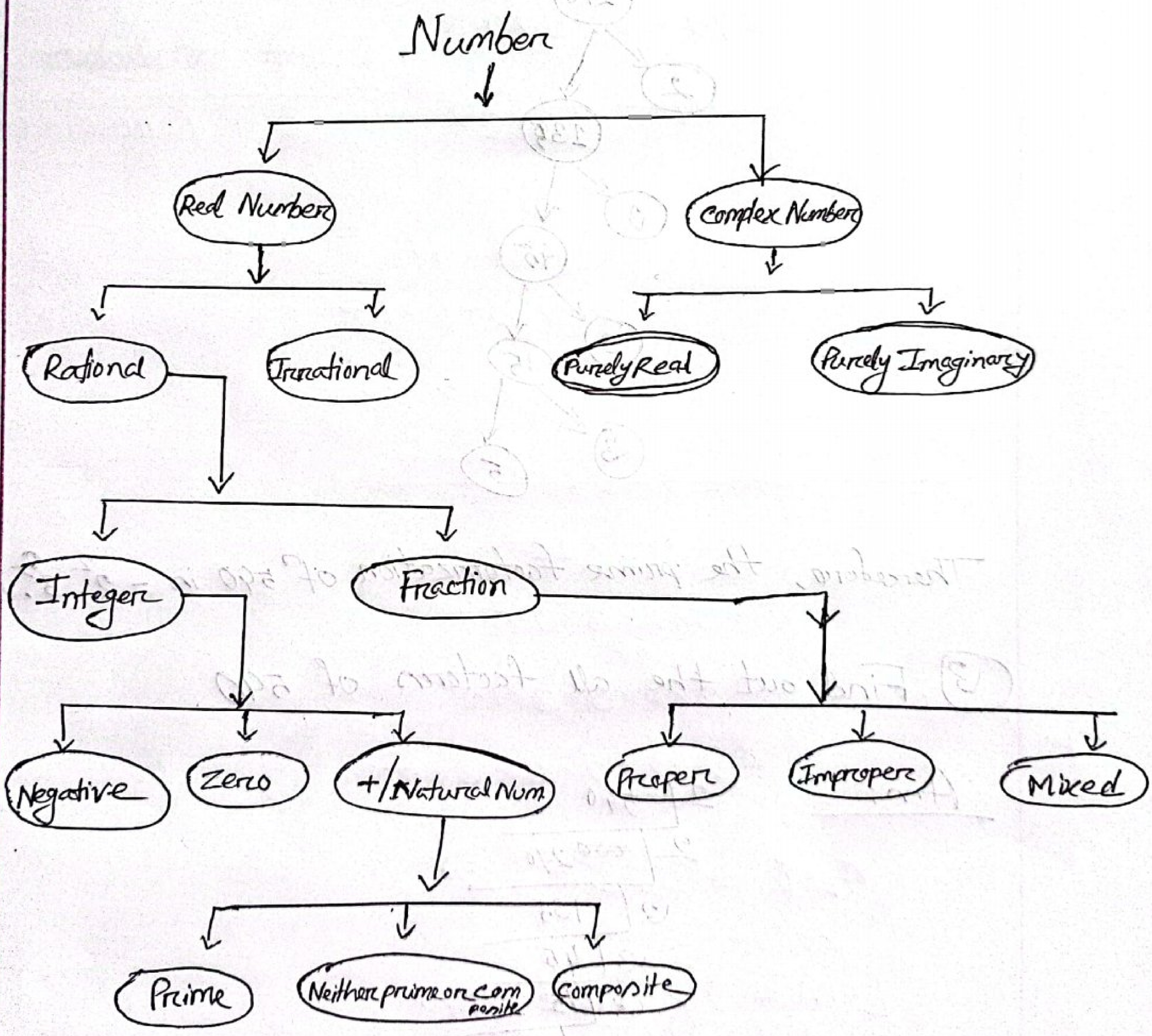
Math Home work:

(1) Write down the classification of Number system.

Ans:



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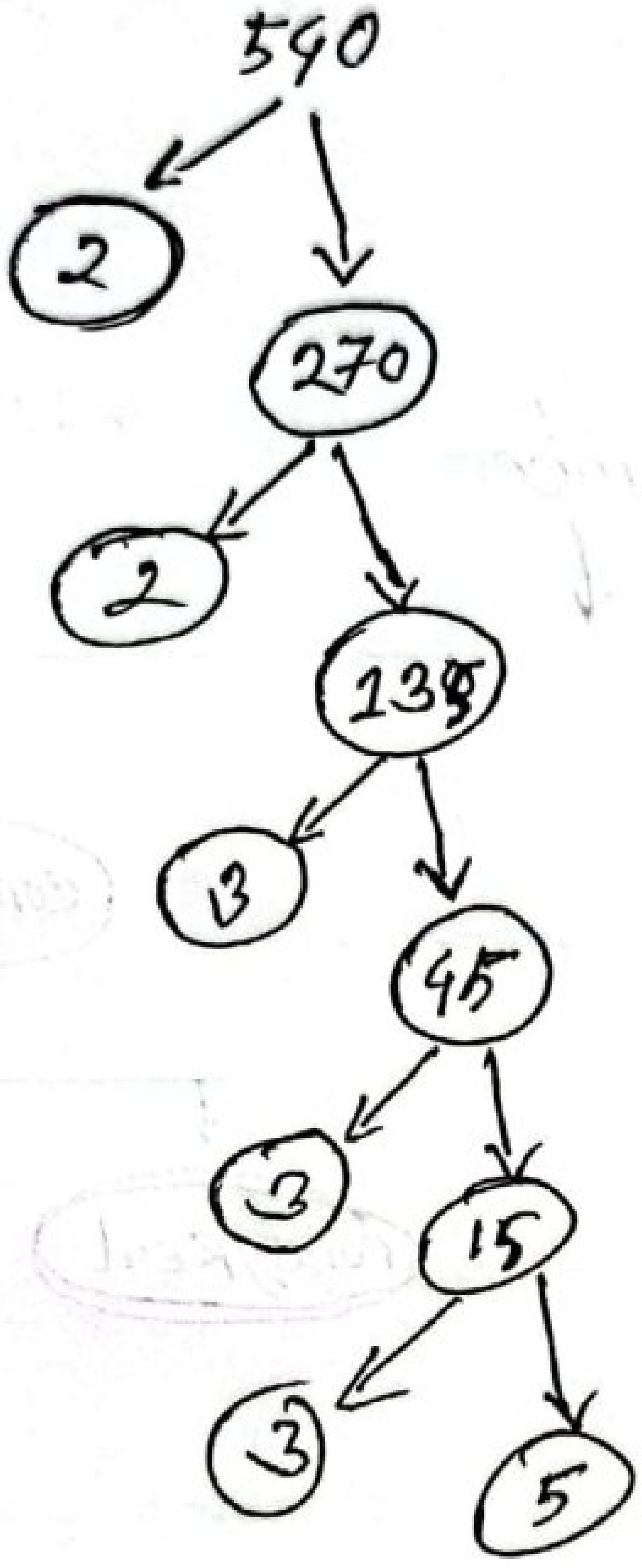
(2) Finding the prime factorization of 540 using tree:



Therefore, the prime factorization of 540 is $= 2^2 \cdot 3^3 \cdot 5$

(3) Find out the all factors of 540.

Ans:

2 | 540
2 | ~~000~~ 270
3 | 135
3 | 45
3 | 15
  | 5

Therefore, the prime factorization of 540 is $= 2^2 \cdot 3^3 \cdot 5$

So, the total number of factors of 540 is

$(2+1) \cdot (3+1) \cdot (1+1) = 24$

Calculation for all factors:

$540 = 1 \times 540$

$= 2 \times 270$

$= 4 \times 135$

$= 3 \times 180$

$= 6 \times \cancel{60}\ 90$

$= 12 \times 45$

$= 15 \times 36$

$\cancel{= 30 \times}$

$= 18 \times 30$

$= 10 \times 54$

$= 20 \times 27$

$= 9 \times 60$

$= 5 \times 108$

Factors: 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540

(9) What in the GCD and LCM of 240 and 540

~~240~~

Ans:

$\cancel{240 = 2 \times 120}$

$240 = 2 \times 120$

$= 2 \times 2 \times 60$

$= 2 \times 2 \times 2 \times 30$

$= 2 \times 2 \times 2 \times 2 \times 15$

$= 2 \times 2 \times 2 \times 2 \times 3 \times 5$

$540 = 2 \times 270$

$= 2 \times 2 \times 135$

$= 2 \times 2 \times 3 \times 45$

$= 2 \times 2 \times 3 \times 3 \times 15$

$= 2 \times 2 \times 3 \times 3 \times 3 \times 5$

$\therefore GCD = 2^2 . 3 . 5 = 60$

$\therefore LCM = 2^4 . 3^3 . 5 = 2160$

(5) Find the H.C.F and L.C.M of 42, 63, 140

<u>Ans:</u>

$42 = 2 \times 21$

$= 2 \times 3 \times 7$

$63 = 3 \times 21$

$= 3 \times 3 \times 7$

$140 = 2 \times 70$

$= 2 \times 2 \times 35$

$= 2 \times 2 \times 5 \times 7$

$\therefore HCF = \cancel{2 \cdot 7 = 14}\ 7$

$\therefore L.C.M. = 2^2 \cdot 3^2 \cdot 5 \cdot 7 = 1260$

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(6) Find the H.C.F and L.C.M of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$

Ans:

~~Wk~~ We know,

$$\cancel{\text{LCM}}\ \text{HCF of fractions} = \frac{\text{HCF of Numerators}}{\text{LCM of Denominators}}$$

$$\text{and LCF of fractions} = \frac{\text{LCM of Numerators}}{\text{HCF of Denominators}}$$

calculation for Numerators:

$2 = 2$

$8 = 2 \times 4$
$= 2 \times 2 \times 2$

$16 = 2 \times 8$
$= 2 \times 2 \times 4$
$= 2 \times 2 \times 2 \times 2$

$10 = 2 \times 5$

$\therefore$ H.C.F $= 2$

and L.C.F $= 2^4 \cdot 5 = 80$

Calculation for denominators:

$3 = 3$

$9 = 3 \times 3$

$81 = 3 \times 27$

$= 3 \times 3 \times 9$

$= 3 \times 3 \times 3 \times 3$

$27 = 3 \times 9$

$= 3 \times 3 \times 3$

$\therefore$ HCF $= 3$

LCF $= 3^4 = 81$

$\therefore$ HCF of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27} = \frac{2}{81}$

and, LCF of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27} = \frac{80}{3}$

(Ans:)

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(7) Find the modulus and Argument of

$$Z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$$ and also its polar, exponential form.

Ans:

$$Z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$$

$$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$$

$$= \frac{(1+\sqrt{3}i)^2}{1-(\sqrt{3}i)^2}$$

$$= \frac{1+2\sqrt{3}i+(\sqrt{3}i)^2}{1+3}$$

$$= \frac{1+2\sqrt{3}i-3}{4}$$

$$= \frac{-2+2\sqrt{3}i}{4}$$

$$= \frac{-2}{4} + \frac{2\sqrt{3}i}{4}$$

$$= -\frac{1}{2} + \frac{\sqrt{3}i}{4}$$

Where, $x = -\frac{1}{2}$ and $y = \frac{\sqrt{3}i}{4}$

Modulus $|z| = \sqrt{x^2+y^2}$

$= \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \sqrt{\frac{1}{4} + \frac{3}{4}}$

$= 1$

$\therefore$ Modulus of z is $= 1$

Argument of z will

$\theta = \pi - \tan^{-1}\left|\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right|$

$= \pi - \tan^{-1}(-\sqrt{3})$

$= \pi - \tan\sqrt{3}$

$= \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

$\therefore$ Argument of z is $= \frac{2\pi}{3}$

$\therefore$ Polar form $= -\frac{1}{2} + \frac{\sqrt{3}i}{2}$

and Exponential form of $z = re^{i\theta}$

$= 1e^{i\frac{2\pi}{3}}$

$= e^{\frac{2\pi}{3i}}$ (Ans)

(8) Evaluate $\sqrt{-16} \times \sqrt{-9}$ and $\frac{\sqrt{-16}}{\sqrt{-4}}$

Ans:

We have,

$\sqrt{-16} \times \sqrt{-9}$

$= \sqrt{16}\,i \times \sqrt{9}\,i$

$= 4i \times 2i$

$= 8i^2$

$= -8$

and,

$\frac{\sqrt{-16}}{\sqrt{-9}}$

$= \frac{\sqrt{16}\,i}{\sqrt{4}\,i}$

$= \frac{4i}{2i}$

$= 2$

(9) Evaluate Modulus and Argument of $8z - z^2$ by replacing $z = 2+i$

Ans:

We have,

$8z - z^2$

$= 8(2+i) - (2+i)^2$

$= 16 + 8i - \{(2)^2 + 2.2i + i^2\}$

$= 16 + 8i - \{4 + 4i + i^2\}$

$= 16 + 8i - \{4 + 4i - 1\}$

$= 16 + 8i - \{4i + 3\}$

$= 16+8i-9i-3$

$= 13+9i$

Modulus $= \sqrt{(9)^2+(13)^2}$

$= \sqrt{16+169}$

$= \sqrt{185}$

$\theta = \tan^{-1}\frac{9}{13}$

$= 17.102$ (Ans.)

(10) Express $1+i\sqrt{3}$ in the form of $r(\cos\theta + i\sin\theta)$

let; $z = 1+i\sqrt{3}$

$z = x+iy$

$|z| = \sqrt{x^2+y^2}$

We know, $\theta = \tan^{-1}\left|\frac{y}{x}\right|$

Modulus of $z = \sqrt{(1)^2+(\sqrt{3})^2}$

$= \sqrt{1+3}$

$= \sqrt{4}$

$\therefore r = 2$

Again,

$$\text{Argument of } z = \tan^{-1}\left|\frac{y}{x}\right|$$

$$= \tan^{-1}\left|\frac{\sqrt{3}}{1}\right|$$

$$= \tan^{-1} \cdot \tan\frac{\pi}{3}$$

$$= \frac{\pi}{3}$$

Therefore, $r(\cos\theta + i\sin\theta)$ form is $= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

(Ans.)