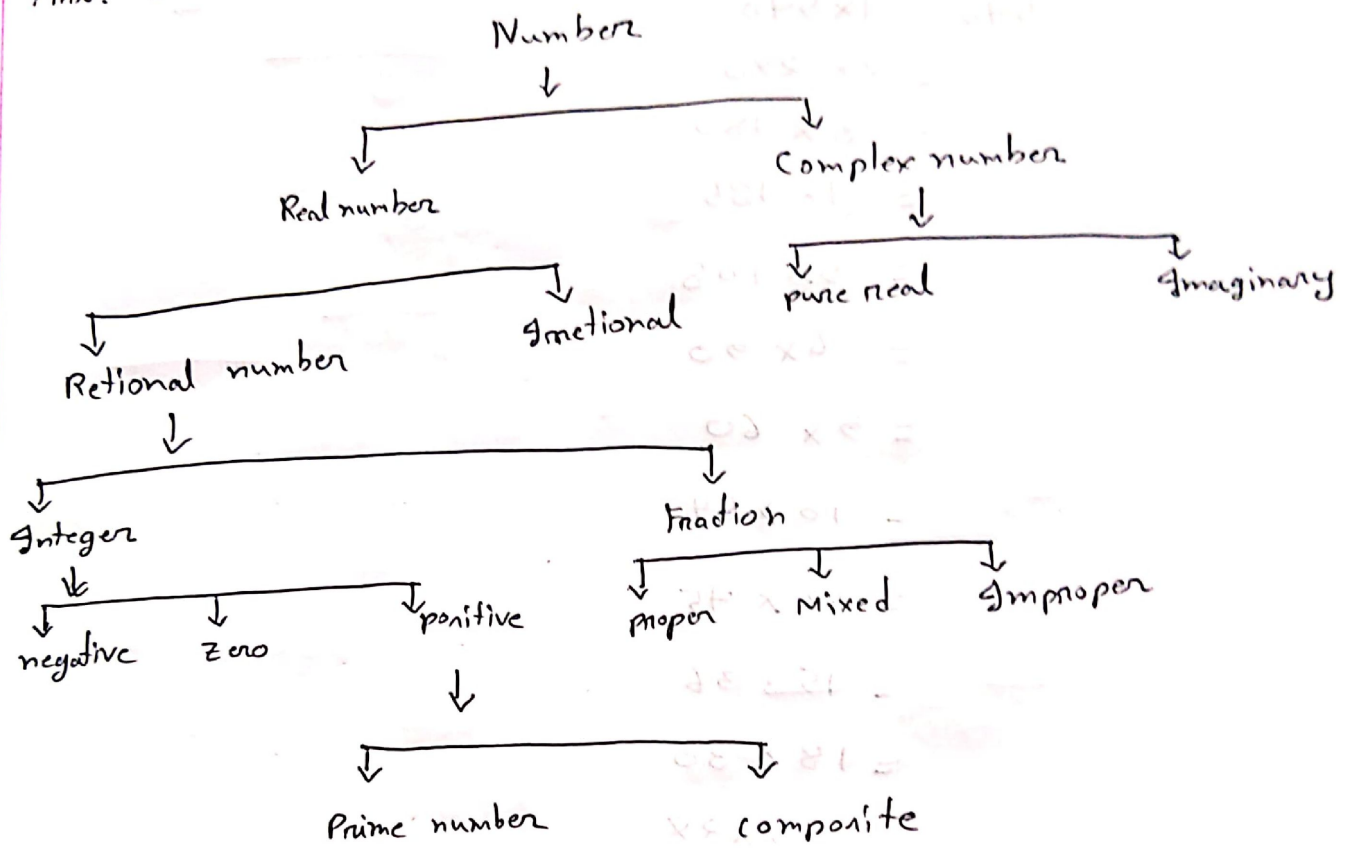
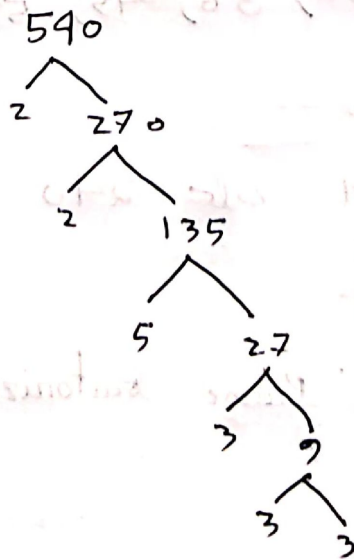


Ans: ① Classification of Number System



② Prime factorization of 540 with tree structure.



So, Prime factorization 540 is = $2^2 \cdot 3^3 \cdot 5$

③ Find the all factors of 540 are -

$$\begin{aligned} 540 &= 1 \times 540 \\ &= 2 \times 270 \\ &= 3 \times 180 \\ &= 4 \times 135 \\ &= 5 \times 108 \\ &= 6 \times 90 \\ &= 9 \times 60 \\ &= 10 \times 54 \\ &= 12 \times 45 \\ &= 15 \times 36 \\ &= 18 \times 30 \\ &= 20 \times 27 \end{aligned}$$

Therefore all factors are, 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540.

④ GCD and LCM of 240 and 540!

$$\begin{array}{r} 2 \overline{) 240} \\ \underline{2 \quad} 120 \\ 2 \overline{) 120} \\ \underline{2 \quad} 60 \\ 2 \overline{) 60} \\ \underline{2 \quad} 30 \\ 2 \overline{) 30} \\ \underline{3 \quad} 15 \\ 3 \overline{) 15} \\ \underline{3 \quad} 5 \end{array}$$

∴ Prime factorization of 240 is $2^4 \cdot 3 \cdot 5$

$$\begin{array}{r}
 2 \overline{) 540} \\
 \underline{270} \\
 2 \overline{) 270} \\
 \underline{135} \\
 2 \overline{) 135} \\
 \underline{67.5} \\
 3 \overline{) 27} \\
 \underline{9} \\
 3 \overline{) 9} \\
 \underline{3}
 \end{array}$$

Prime factorization = $2^2 \cdot 3^3 \cdot 5$

Therefore

$$\begin{aligned}
 \text{GCD} &= 2^2 \cdot 3 \cdot 5 = 60 \\
 \text{LCM} &= 2^2 \cdot 3^3 \cdot 5 = 2160 \quad \underline{\text{Ans}}
 \end{aligned}$$

⑤ HCF and LCM of 42, 63, 140

$$42 = 2 \times 21 = 2 \times 3 \times 7$$

$$63 = 7 \times 9 = 7 \times 3 \times 3$$

$$140 = 2 \times 70 = 2 \times 2 \times 35 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$$

Therefore

$$\begin{aligned}
 \text{HCF} &= 7 \\
 \text{LCM} &= 2^2 \times 7 \times 5 \times 3 \\
 &= 1260
 \end{aligned}$$

⑥ Finding the LCM and HCF of $\frac{2}{3}$, $\frac{8}{9}$, $\frac{16}{81}$ and $\frac{10}{27}$

Calculations of Numerators

$$2 = 2^1$$

$$8 = 2^3$$

$$16 = 2^4$$

$$10 = 2 \times 5$$

$$\text{LCM} = 2^4 \cdot 5 = 80$$

$$\text{HCF} = 2$$

Calculation of Denominators

$$3 = 3^1$$

$$9 = 3^2$$

$$81 = 3^4$$

$$27 = 3^3$$

$$\text{LCM} = 3^4 = 81$$

$$\text{HCF} = 3$$

Therefore, LCM of $\frac{2}{3}$, $\frac{8}{9}$, $\frac{16}{81}$ and $\frac{10}{27}$

$$= \frac{90}{3}$$

HCF of $\frac{2}{3}$, $\frac{8}{9}$, $\frac{16}{81}$ and $\frac{10}{27}$

$$= \frac{2}{81} \quad \underline{\text{Ans}}$$

⑦ Finding Modulus, Argument and Polar:

$$z = \frac{1 + \sqrt{3}i}{1 - \sqrt{3}i}$$

$$= \frac{(1 + \sqrt{3}i)(1 + \sqrt{3}i)}{(1 - \sqrt{3}i)(1 + \sqrt{3}i)}$$

$$= \frac{(1 + \sqrt{3}i)^2}{i^2 + (\sqrt{3})^2}$$

$$= \frac{1^2 + 2\sqrt{3}i + (\sqrt{3}i)^2}{9}$$

$$= \frac{1 + 2\sqrt{3}i - 3}{9}$$

$$= \frac{-2 + 2\sqrt{3}i}{9}$$

$$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$\text{So, } x = -\frac{1}{2}, \quad y = \frac{\sqrt{3}}{2}$$

\therefore Modulus $|z| = \sqrt{x^2 + y^2}$ Argument $\theta = \tan^{-1}\left(\frac{y}{x}\right)$
 $= \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$ $= \pi - \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right)$
 $= \sqrt{\frac{1}{4} + \frac{3}{4}}$ $= \pi - \tan^{-1}(-\sqrt{3})$
 $= 1$ $= \pi - \tan^{-1}(\sqrt{3})$
 $= \pi - 60^\circ$
 $= \pi - \frac{\pi}{3}$
 $= \frac{2\pi}{3}$

Polar form,

$$z = r(\cos \theta + i \sin \theta)$$

$$= 1\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$$

$$= \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$$

8) Evaluate :-

$$\sqrt{-16} \times \sqrt{-9}$$

$$= i\sqrt{16} \times i\sqrt{9}$$

$$= 4i \times 3i$$

$$= 12i^2$$

$$= -12$$

and,

$$\frac{\sqrt{-16}}{\sqrt{-9}}$$

$$\left(\frac{i\sqrt{16}}{i\sqrt{9}}\right) = \frac{4i}{3i}$$

$$= \frac{4}{3}$$

9) Evaluate Modulus and ~~Argument~~ Argument :-

$$\text{Let } z = 8z - z^2 \quad [z = 2+i]$$

$$= 8(2+i) - (2+i)^2$$

$$= 16 + 8i - 4^2 + 4i + (-i)^2$$

$$= 12 + 9i + 1$$

$$= 13 + 9i$$

$$\text{So, } x = 13, \quad y = 9$$

$$\text{Modulus } |z| = \sqrt{x^2 + y^2}$$

$$= \sqrt{13^2 + 9^2}$$

$$= \sqrt{185}$$

Argument

$$\theta = \tan^{-1} \left| \frac{y}{x} \right|$$

$$= \tan^{-1} \frac{9}{13}$$

10) Express $z = r(\cos \theta + i \sin \theta)$ from $1 + i\sqrt{3}$

$$\text{So } x = 1, \quad y = \sqrt{3}$$

$$\text{We know } \theta = \tan^{-1} \left(\frac{y}{x} \right)$$

$$= \tan^{-1} \left(\frac{\sqrt{3}}{1} \right)$$

$$= \tan^{-1} \sqrt{3}$$

$$= 60^\circ$$

$$= \frac{\pi}{3}$$

∴ Therefore,

$$\text{So, } z = r(\cos \theta + i \sin \theta) \\ = 1 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

$$\text{from } 1 + i\sqrt{3} = \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) e$$