Exercise on Number System

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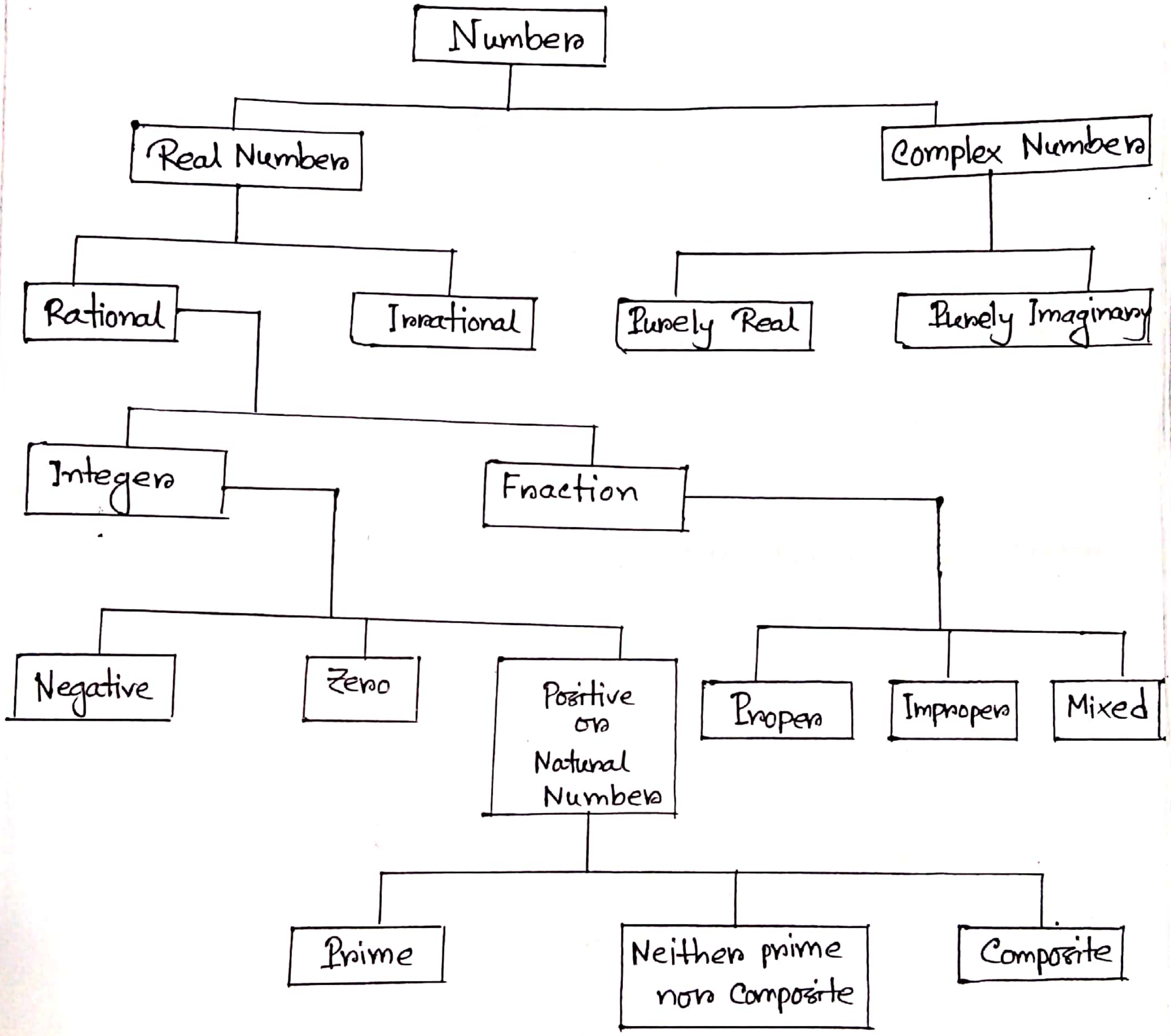
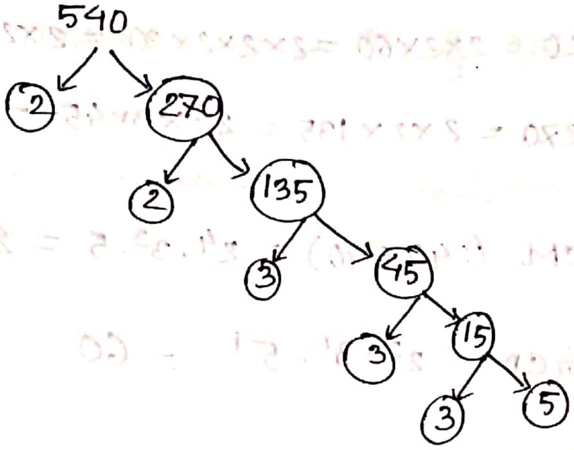
sec - V

1 : Write down the classification of number system :



(2) Find the prime factorization of 540 using tree :-




$\therefore$ The prime factorization of 540 is $= 2^2 \cdot 3^3 \cdot 5^1$

(3) Find out the all factors of 540 :

The total number of factors of 540 $= (2+1)(3+1)(1+1)$

$= 3 \cdot 4 \cdot 2$

$= 24$

Calculation for all factors

$540 = 1 \times 540$

$= 2 \times 270$

$= 3 \times 180$

$= 4 \times 135$

$= 5 \times 108$

$= 6 \times 90$

$= 9 \times 60$

$= 10 \times 54$

$= 12 \times 45$

$= 15 \times 36$

$= 18 \times 30$

$= 20 \times 27$

The factors of 540 are : 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540

(4) What is the GCD and LCM of 240 and 540 ?

$240 = 2\times120 = 2\times2\times60 = 2\times2\times2\times30 = 2\times2\times2\times2\times15 = 2^4\times3\times5$

$540 = 2\times270 = 2\times2\times135 = 2^2\times3\times45 = 2^2\times3\times3\times15 = 2^2\times3^3\times5$

$\therefore$ LCM $(240, 540) = 2^4 . 3^3 . 5 = 2160$

$\therefore$ GCD $= 2^2 . 3^1 . 5^1 = 60$

(5) Find the HCF & L.CM of 42, 63 & 140

$42 = 2\times21 = 2\times3\times7$

$63 = 7\times9 = 7\times3\times3 = 7\times3^2$

$140 = 2\times70 = 2\times2\times35 = 2\times2\times5\times7 = 2^2\times5\times7$

$\therefore$ HCF $= 7$

LCM $= 2^2\times3^2\times5\times7 = 1260$

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(6) Find the HCF & LCM of $2/3$, $8/9$, $16/81$ and $10/27$

Given fractions are $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27}$

Calculation of Numerators

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

$10 = 2 \times 5$

$LCM = 2^4 \cdot 5 = 80$

$HCF = 2^1 = 2$

Calculation of Denominators

$3 = 3^1$

$9 = 3^2$

$81 = 3^4$

$27 = 3^3$

$LCM = 3^4 = 81$

$HCF = 3^1 = 3$

$\therefore$ LCM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27} = \frac{80}{3}$

HCF of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27} = \frac{2}{81}$

(7) Find the modulus and Argument of $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$ and also its polar, exponential form.

Given that, $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$

$$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$$

$$= \frac{1+2\cdot\sqrt{3}i+3(-1)}{1-3(-1)}$$

$$= \frac{1+2\sqrt{3}i-3}{4}$$

$$= \frac{2\sqrt{3}i-2}{4}$$

$$= \frac{\sqrt{3}i}{2} - \frac{1}{2}$$

$$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$\therefore\ r = |z| = \sqrt{(-\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} = \sqrt{\frac{1}{4}+\frac{3}{4}} = 1$

$\text{Arg}(z) = \pi - \tan^{-1}\left(\frac{\sqrt{3}/2}{-1/2}\right) = \pi - \tan^{-1}|-\sqrt{3}| = \frac{2\pi}{3}$

polar form of $z = r(\cos\theta + i\sin\theta)$

$$= 1\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

$$= \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$$

Exponential form of

$z = r\cdot e^{i\theta}$

$= 1\cdot e^{i\frac{2\pi}{3}} = e^{\frac{2\pi}{3}i}$

(8) Evaluate $\sqrt{-16} \times \sqrt{-4}$ & $\frac{\sqrt{-16}}{\sqrt{-4}}$

Ans: Now,

$\sqrt{-16} \times \sqrt{-4}$

$= \sqrt{16i} \times \sqrt{4i}$

$= 4i \times 2i$

$= 8i^2$

$= -8$

and, $\frac{\sqrt{-16}}{\sqrt{-4}}$

$= \frac{4i}{2i}$

$= 2$

(9) Evaluate Modulus and Argument of $8z - z^2$ by replacing $z = 2 + i$

$\therefore 8z - z^2$

$= 8(2+i) - (2+i)^2$

$= 16 + 8i - (4 + 4i + i^2)$

$= 16 + 8i - 4 - 4i - i^2$

$= 16 - 4 + 1 + 4i$

$= 13 + 4i$

Modulus, r $= \sqrt{(13)^2 + (4)^2}$

$= \sqrt{185}$

$Arg(\theta) = \tan^{-1}\left(\frac{4}{13}\right)$

$= 17.102$

(10) Express $1 + i\sqrt{3}$ in the form of $r(\cos\theta + i\sin\theta)$.

Let $z = 1 + i\sqrt{3}$ $\quad z = x + iy \quad |z| = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\left|\frac{y}{x}\right|$

$\therefore$ Modules of $z = \sqrt{(1)^2 + (\sqrt{3})^2}$

$= 2$

$\therefore r = 2$

Argument of $z = \tan^{-1}\left(\frac{y}{x}\right)$

$= \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$= \tan^{-1}.\tan\frac{\pi}{3}$

$= \frac{\pi}{3}$

Therefore $r(\cos\theta + i\sin\theta)$ form is $= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$