

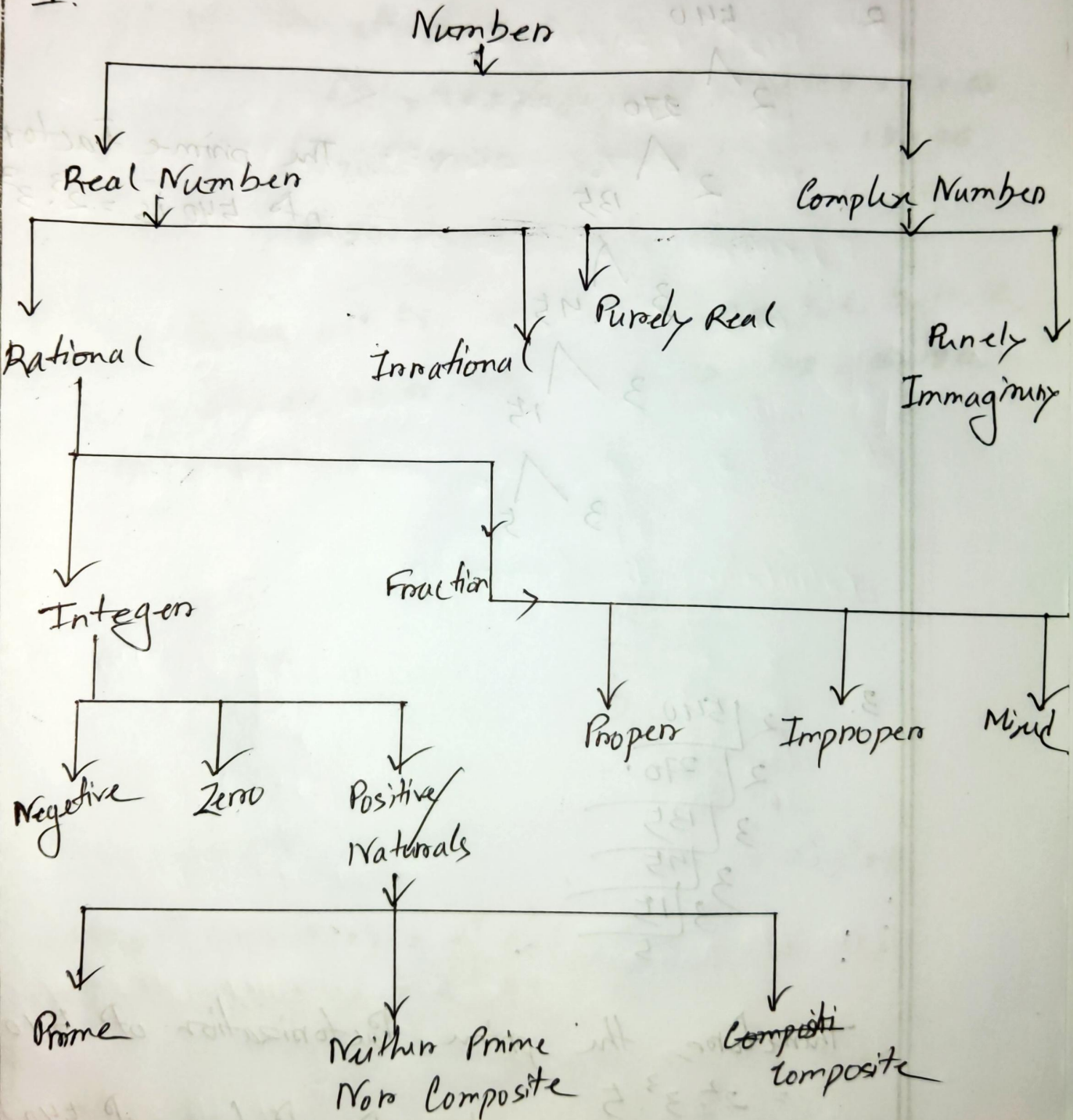
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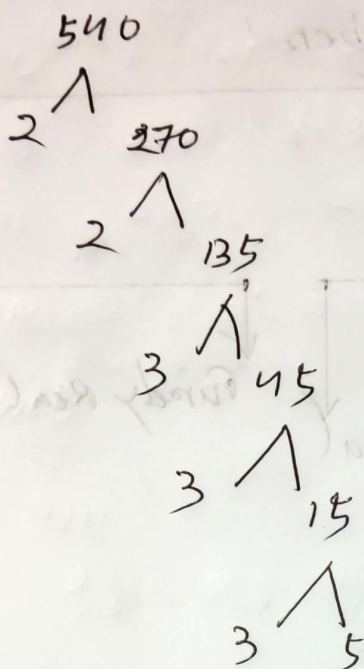
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1.

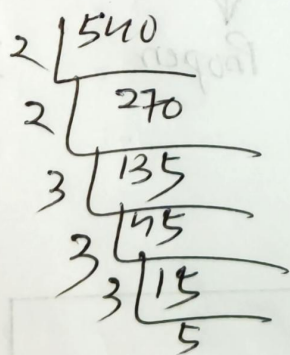


2.



\therefore The prime factorization of 540 is $= 2^3 \cdot 3^3 \cdot 5$

3.



Therefore, the prime factorization of 540 is

$$= 2^3 \cdot 3^3 \cdot 5$$

So, the total number of factors of 540 is

$$= (2+1)(3+1)(1+1)$$

$$= 2 \cdot 4 \cdot 2$$

$$= 24$$

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Calculating for all factors:

$$\begin{aligned}
 540 &= 1 \times 540 = 2 \times 270 \neq 3 \times 180 \neq 4 \times 135 = 5 \times 108 \\
 &= 6 \times 90 = 9 \times 60 = 10 \times 54 \neq 12 \times 45 = 15 \times 36 \\
 &= 18 \times 30 = 20 \times 27
 \end{aligned}$$

The factors of 540 are = 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540

4.

$$\begin{array}{r}
 2 \overline{) 240} \\
 \underline{2 \overline{) 120}} \\
 \quad 2 \overline{) 60} \\
 \quad \quad 2 \overline{) 30} \\
 \quad \quad \quad 3 \overline{) 15} \\
 \quad \quad \quad \quad 5
 \end{array}$$

$$\therefore 240 = 2^4 \times 3 \times 5$$

$$\begin{array}{r}
 2 \overline{) 540} \\
 \underline{2 \overline{) 270}} \\
 \quad 3 \overline{) 135} \\
 \quad \quad 3 \overline{) 45} \\
 \quad \quad \quad 3 \overline{) 15} \\
 \quad \quad \quad \quad 5
 \end{array}$$

$$\therefore 540 = 2^2 \times 3^3 \times 5$$

$$\text{LCM of } (240, 540) = 2^4 \cdot 3^3 \cdot 5 = 2160$$

$$\text{GCD of } (240, 540) = 2^2 \cdot 3 \cdot 5 = 60$$

5 $42 = 2 \times 21 = 2 \times 3 \times 7$

$$63 = 3 \times 21 = 3 \times 3 \times 7 = 3^2 \times 7$$

$$140 = 2 \times 70 = 2 \times 2 \times 35 = 2^2 \times 5 \times 7$$

$$\therefore \text{LCM of } = 2^2 \cdot 3^2 \cdot 5 \cdot 7 = 1260$$

$$\therefore \text{GCD} = 7$$

6 $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$

Calculating the Numerators,

$$2 = 2^1$$

$$8 = 2^3$$

$$16 = 2^4$$

$$10 = 2 \times 5$$

Calculating the Denominators

$$3 = 3^1$$

$$9 = 3^2$$

$$81 = 3^4$$

$$27 = 3^3$$

$$\text{LCM of Numerators} = 2^4 \times 5 = 80$$

$$\text{LCM of Denominator} = 3^4 = 81$$

$$\text{GCD of Numerator} = 2$$

$$\text{GCD of Denominator} = 3$$

$$\text{HCF of } \frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27} = \frac{2}{81}$$

$$\text{LCM of } \frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27} = \frac{80}{3}$$

$$\begin{aligned}
 z &= \frac{1 + \sqrt{3}i}{1 - \sqrt{3}i} = \frac{(1 + \sqrt{3}i)(1 + \sqrt{3}i)}{(1 - \sqrt{3}i)(1 + \sqrt{3}i)} = \frac{(1 + \sqrt{3}i)^2}{1 - (\sqrt{3}i)^2} \\
 &= \frac{1^2 + 2 \cdot 1 \cdot \sqrt{3}i + (\sqrt{3}i)^2}{1 - 3i^2} = \frac{1 + 2\sqrt{3}i + 3i^2}{1 + 3} \\
 &= \frac{-2 + 2\sqrt{3}i}{4} = \frac{-2}{4} + \frac{2\sqrt{3}i}{4} = -\frac{1}{2} + \frac{\sqrt{3}i}{2}
 \end{aligned}$$

$$\therefore x = -\frac{1}{2}, \quad y = \frac{\sqrt{3}}{2}$$

$$r = \sqrt{x^2 + y^2} = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{1} = 1$$

$$\begin{aligned}
 \theta &= \pi - \tan^{-1} \left(\frac{y}{x} \right) \\
 &= \pi - \tan^{-1} \left(\frac{\sqrt{3}/2}{-1/2} \right) \\
 &= \pi - \tan^{-1} (\sqrt{3}) \\
 &= \pi - \tan^{-1} \tan \frac{\pi}{3} \\
 &= \pi - \frac{\pi}{3} \\
 &= \frac{2\pi}{3}
 \end{aligned}$$

the polar form of $z = r(\cos \theta + i \sin \theta)$

$$z = 1 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)$$

$$z = \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}$$

Exponential form $z = e^{i \cdot \frac{2\pi}{3}}$

8) Now,

$$\begin{aligned} & \sqrt{-16} \times \sqrt{-4} \\ &= \sqrt{16i} \times \sqrt{4i} \\ &= 4i \times 2i \\ &= 8i^2 \\ &= -8 \end{aligned}$$

and,

$$\begin{aligned} & \frac{\sqrt{-16}}{\sqrt{-4}} \\ &= \frac{4i}{2i} \\ &= 2 \end{aligned}$$

9) $z = 2 + i$

$$\begin{aligned} 8z - z^2 &= 8(2+i) - (2+i)^2 \\ &= 16 + 8i - 4 - 4i + i^2 \\ &= 13 + 4i \end{aligned}$$

$$\begin{aligned} \text{modulus } |z| &= \sqrt{x^2 + y^2} = \sqrt{(13)^2 + (4)^2} \\ &= \sqrt{185} \end{aligned}$$

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$$\begin{aligned} \text{Argument, } \theta &= \tan^{-1} \left(\frac{y}{x} \right) \\ &= \tan^{-1} \left(\frac{4}{13} \right) \\ &= \approx 17.1 \end{aligned}$$

10/ $z = x + iy$
 $= 1 + i\sqrt{3}$

$$\begin{aligned} x &= 1 \\ y &= \sqrt{3} \end{aligned}$$

$$\text{Modulus, } r = \sqrt{x^2 + y^2} = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = 2$$

$$\begin{aligned} \text{Argument, } \theta &= \tan^{-1} \left(\frac{y}{x} \right) \\ &= \tan^{-1} \left(\frac{\sqrt{3}}{1} \right) \\ &= \tan^{-1} \tan \frac{\pi}{3} \\ &= \frac{\pi}{3} \end{aligned}$$

Therefore, $z = (\cos \theta + i \sin \theta)$ form is

$$= 2 \cdot \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$