Assignment

Subject Code: MAT-111

Course Title: Basic Mathematics

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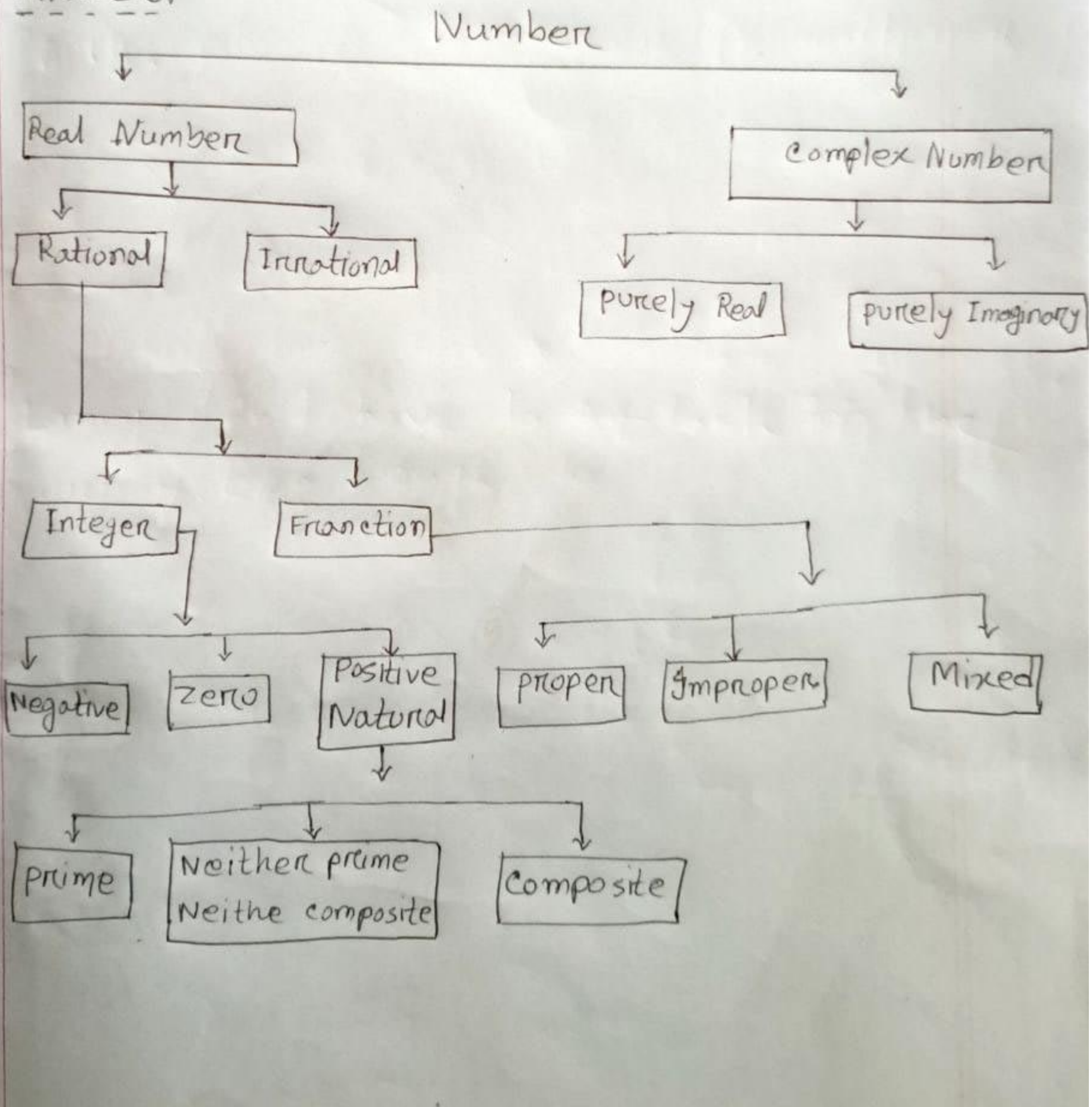
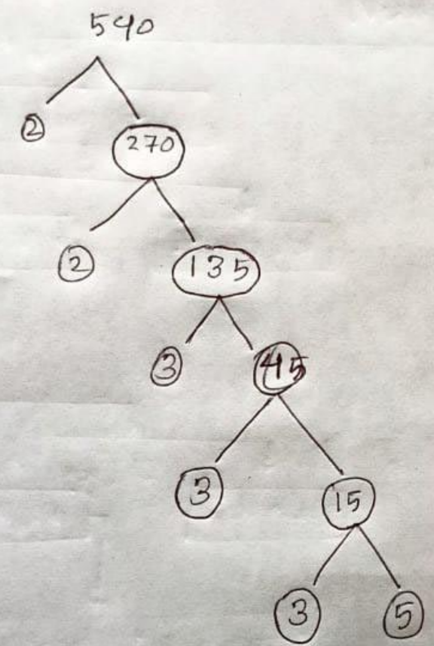
problem 1: write down the classification of number system.

Answer:



problem 2: Find the prime factorization of 540 using tree.

Answer:



The prime factoraization of 540 is $= 2^2 . 3^3 . 5$

problem 3: Find out the all factors of 540.

Answer: From problem ② we get,

The prime factoraization of 540 is $= 2^2 . 3^3 . 5$

So, the total number of factors of 540 is $= (2+1)(3+1)(1+1)$

$= 3.4.2$

$= 24$

$540 = 1 \times 540$
$= 2 \times 270$
$= 3 \times 180$
$= 4 \times 135$
$= 5 \times 108$
$= 6 \times 90$
$= 9 \times 60$
$= 10 \times 54$
$= 12 \times 45$
$= 15 \times 36$
$= 18 \times 30$
$= 20 \times 27$

The all factors of 540 are 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540

Problem 4: What is the GCD & LCM of 240 & 540.

Answer:

$$\begin{array}{r|l} 2 & 240 \\ \hline 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 540 \\ \hline 2 & 270 \\ \hline 3 & 135 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

$\therefore 240 = 2^4 . 3 . 5$ $\qquad$ $\therefore 540 = 2^2 . 3^3 . 5$

LCM of 240 & 540 $= 2^4 . 3^3 . 5 = 2160$

GCD of 240 & 540 $= 2^2 . 3 . 5 = 60$

Problem 5: Find the HCF & LCM of 42, 63 & 140.

Answer:

$$\begin{array}{r|l} 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 2 & 140 \\ \hline 2 & 70 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

$\therefore 42 = 2 \times 3 \times 7$ $\qquad$ $\therefore 63 = 3^2 \times 7$ $\qquad$ $\therefore 140 = 2^2 \times 5 \times 7$

LCM of $(42, 63 \,\&\, 140) = 2^2 . 3^2 . 5 . 7 = 1260$

HCF of $(42, 63 \,\&\, 140) = 7$

Problem 6: Find the HCF & LCM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ & $\frac{10}{27}$

Answer:

Calculation of Numerator

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

$10 = 2.5$

LCM of Numerator $= 2^4 . 5 = 80$

HCF of Numerator $= 2$

Calculation of Denumerator

$3 = 3^1$

$9 = 3^3$

$81 = 3^4$

$27 = 3^3$

LCM of Denumerator $= 3^4 = 81$

HCF of Denumerator $= 3$

$\therefore$ LCM of $\left(\frac{2}{3}, \frac{8}{9}, \frac{16}{81} \& \frac{10}{27}\right) = \frac{80}{3}$

$\therefore$ HCF of $\left(\frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27}\right) = \frac{2}{81}$

Problem 7: Find the modulus and Argument of $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$ and also its polar, exponential form.

Answer: we have,

$$z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$$

$$= \frac{(1+\sqrt{3}i)^2}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$$

$$= \frac{1+2\sqrt{3}i+(\sqrt{3})^2 i^2}{1-(\sqrt{3})^2 \cdot i^2}$$

$$= \frac{1+2\sqrt{3}i-3}{1+3}$$

$$= \frac{2\sqrt{3}i-2}{4}$$

$$= \frac{2\sqrt{3}i}{4} - \frac{2}{4}$$

$$= \frac{\sqrt{3}i}{2} - \frac{1}{2}$$

So polar form $= -\frac{1}{2} + \frac{\sqrt{3}i}{2}$

Now, $z = -\frac{1}{2} + \frac{\sqrt{3}i}{2}$

$$\therefore |z| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \sqrt{\frac{1}{4} + \frac{3}{4}}$$

$$= \sqrt{\frac{1+3}{4}}$$

$$= \sqrt{\frac{4}{4}}$$

$$= 1$$

So modulus of $z = 1$

And Argument of $z = \theta = \pi - \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right)$

$$= \pi - \tan^{-1}\left(\frac{\sqrt{3}}{2} \times \frac{2}{1}\right)$$

$$= \pi - \tan^{-1}\sqrt{3}$$

$$= \pi - \tan^{-1}\tan\frac{\pi}{3}$$

$$= \pi - \cancel{\tan}\,\frac{\pi}{3}$$

$$= \frac{3\pi - \pi}{3}$$

$$= \frac{2\pi}{3}$$

Exponential form of $z = r.e^{i\theta}$

$$= 1.e^{\frac{2\pi}{3i}}$$

$$= e^{\frac{2\pi}{3}i}$$

problem 8: Evaluate $\sqrt{-16} \times \sqrt{-4}$ & $\frac{\sqrt{-16}}{\sqrt{-4}}$

Answer: $\sqrt{-16} \times \sqrt{-4}$

$$= 4i \times 2i$$

$$= 8i^2$$

$$= -8$$

$$\frac{\sqrt{-16}}{\sqrt{-4}}$$

$$= \frac{4i}{2i}$$

$$= 2$$

problem 9: Evaluate Modulus & Argument of $8z - z^2$ by replacing $z = 2+i$

Answer: Here given that,

$$z = 2+i$$

$$\therefore 8z - z^2 = 8(2+i) - (2-i)^2$$

$$= 16+8i-(4-2.2.i.i^2)$$

$$= 16+8i-4-4i-i^2$$

$$= 16+8i-4-4i-i$$

$$= 13+4i$$

Modulus,

$$r = \sqrt{x^2+y^2}$$

$$= \sqrt{(13)^2+(4)^2}$$

$$= \sqrt{169+16}$$

$$= \sqrt{185}$$

Argument,

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$= \tan^{-1}\frac{4}{13}$$

$$= 17.10^\circ$$

Problem 10: Express $1+\sqrt{3}i$ in the form of $r(\cos\theta + \sin\theta)$

Answer: Here,

$$z = 1 + \sqrt{3}i$$

$\therefore$ Modulus, $r = \sqrt{1^2 + (\sqrt{3})^2}$

$$= \sqrt{1+3}$$

$$= \sqrt{4}$$

$$= 2$$

$\therefore$ Argument $\theta = \tan^{-1}\frac{\sqrt{3}}{1}$

$$= \tan^{-1}\left(\tan\frac{\pi}{3}\right)$$

$$= \frac{\pi}{3}$$

Therefore, $r(\cos\theta + i\sin\theta)$ form is

$$= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$