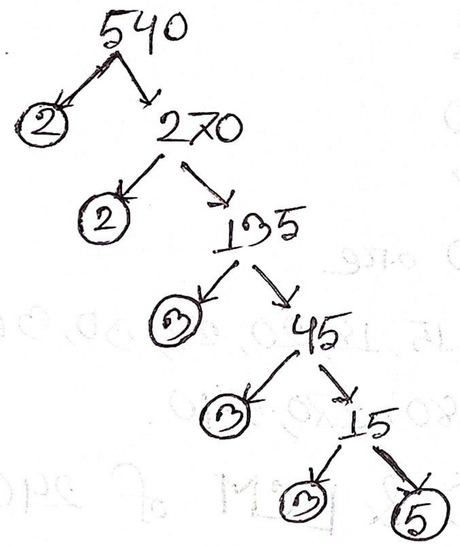
1/ Write down the classification of number system.

⇒ Number System: A number is a way to represent arithmetic value, count or measure of a particular quantity.

2/ Find the prime factorization of 540 using tree

⇒



Therefore, the prime factorization of 540 is = $2^2 \cdot 3^3 \cdot 5$

3/ Find out the all factors of 540.

$\Rightarrow$ $540 = 1 \times 540$

$= 2 \times 270$

$= 3 \times 180$

$= 4 \times 135$

$= 5 \times 108$

$= 6 \times 90$

$= 9 \times 60$

$= 10 \times 54$

$= 12 \times 45$

$= 15 \times 36$

$= 18 \times 30$

$= 20 \times 27$

The factors of 540 are

1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540.

4/ What is the GCD & LCM of 240 & 540.

$\Rightarrow$ $240 = 2 \times 120 = 2 \times 2 \times 60 = 2^2 \times 2 \times 30 = 2^3 \times 2 \times 15 = 2^4 \times 3 \times 5$

$540 = 2 \times 270 = 2 \times 2 \times 135 = 2^2 \times 3 \times 45 = 2^2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 3 \times 5$

$= 2^2 \times 3^3 \times 5$

$\therefore$ LCM $(240, 540) = 2^4 \times 3^3 \times 5 = 2160$

& GCD $(240, 540) = 2 \times 3 \times 5 = 30$

5/ Find the H.C.F & L.C.M of 42, 63 & 140.

⇒ Given numbers are 42, 63 & 140

$42 = 2\times21 = 2\times3\times7$

$63 = 3\times21 = 3\times3\times7 = 3^2\times7$

$140 = 2\times70 = 2\times2\times35 = 2^2\times5\times7$

$\therefore$ LCM (42, 63 & 140) $= 2^2\times3^2\times5\times7 = 1260$

& HCF (42, 63 & 140) $= 7$

6/ Find the HCF & LCM of $\frac{2}{3}$, $\frac{8}{9}$, $\frac{16}{81}$ and $\frac{10}{27}$

⇒ Calculation for Numerators

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

$10 = 2\times5$

LCM (2, 8, 16, 10) $= 2^4\times5 = 80$

HCF (2, 8, 16, 10) $= 2$

Calculation for Denominators

$3 = 3^1$

$9 = 3^2$

$81 = 3^4$

$27 = 3^3$

LCM (3, 9, 81, 27) $= 3^4 = 81$

HCF (3, 9, 81, 27) $= 3$

LCM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{\text{LCM of } 2, 8, 16, 10}{\text{HCF of } 3, 9, 81, 27} = \frac{80}{3}$

HCF of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{\text{HCF of } 2, 8, 16, 10}{\text{LCM of } 3, 9, 81, 27} = \frac{2}{81}$

7/ Find the modulus and Argument of $\frac{1+\sqrt{3}i}{1-\sqrt{3}i}$ and also its polar, exponential form.

$$\Rightarrow \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$$

$$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$$

$$= \frac{(1+\sqrt{3}i)^2}{(1)^2-(\sqrt{3}i)^2}$$

$$= \frac{1+2\sqrt{3}i+3i^2}{1+3}$$

$$= \frac{1+2\sqrt{3}i-3}{4}$$

$$= \frac{-2+2\sqrt{3}i}{4}$$

$$= \frac{2(-1+\sqrt{3}i)}{4}$$

$$= \frac{1}{2}(-1+\sqrt{3}i)$$

$$= -\frac{1}{2}+\frac{\sqrt{3}}{2}i$$

$\therefore x = -\frac{1}{2}, \; y = \frac{\sqrt{3}}{2}$

Modulus(r) $= \sqrt{\left(-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{3}{4}} = \sqrt{\frac{4}{4}} = 1$

Argument(θ) $= 180^\circ - \tan^{-1}\left|\frac{y}{x}\right| = 180^\circ - \tan^{-1}(\sqrt{3})$

$= 180^\circ - 60^\circ = 120^\circ / \frac{2\pi}{3}$

Polar form $z = r(\cos\theta + i\sin\theta) = 1\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$\cancel{(\cos 120^\circ + i\sin 120^\circ)} = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$

Exponential form.

$$z = re^{i\theta}$$

$$= 1 \cdot e^{i \cdot \frac{2\pi}{3}}$$

$$= e^{\frac{2\pi}{3} i}$$

8/ Evaluate $\sqrt{-16} \times \sqrt{-4}$ & $\frac{\sqrt{-16}}{\sqrt{-4}}$

$\Rightarrow$ Here,

$$\sqrt{-16} = \sqrt{-1} \times \sqrt{16}$$

$$= 4i$$

$$\sqrt{-4} = \sqrt{-1} \times \sqrt{4}$$

$$= 2i$$

$$\therefore \sqrt{-16} \times \sqrt{-4} = 4i \times 2i$$

$$= 8i^2$$

$$= -8$$

$$\& \frac{\sqrt{-16}}{\sqrt{-4}} = \frac{4i}{2i} = 2$$

2/ Evaluate Modulus & Argument of $8z - z^2$ by replacing $z = 2+i$

$\Rightarrow$ Here, $z = 2+i$

$\therefore 8z - z^2 = 8(2+i) - (2+i)^2$

$= 16 + 8i - 4 - 4i + 1$

$= 13 + 4i$

$\therefore x = 13,\ y = 4$

Modulus, $r = \sqrt{13^2 + 4^2} = \sqrt{185}$

Argument, $\theta = \tan^{-1}\left(\frac{4}{13}\right)$

$= 17.10^\circ$

10/ Express $1 + i\sqrt{3}$ in the form of $r(\cos\theta + i\sin\theta)$

$\Rightarrow$ $1 + i\sqrt{3}$

$\therefore x = 1,\ y = \sqrt{3}$

$r = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$

$\therefore 1 + i\sqrt{3} = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$