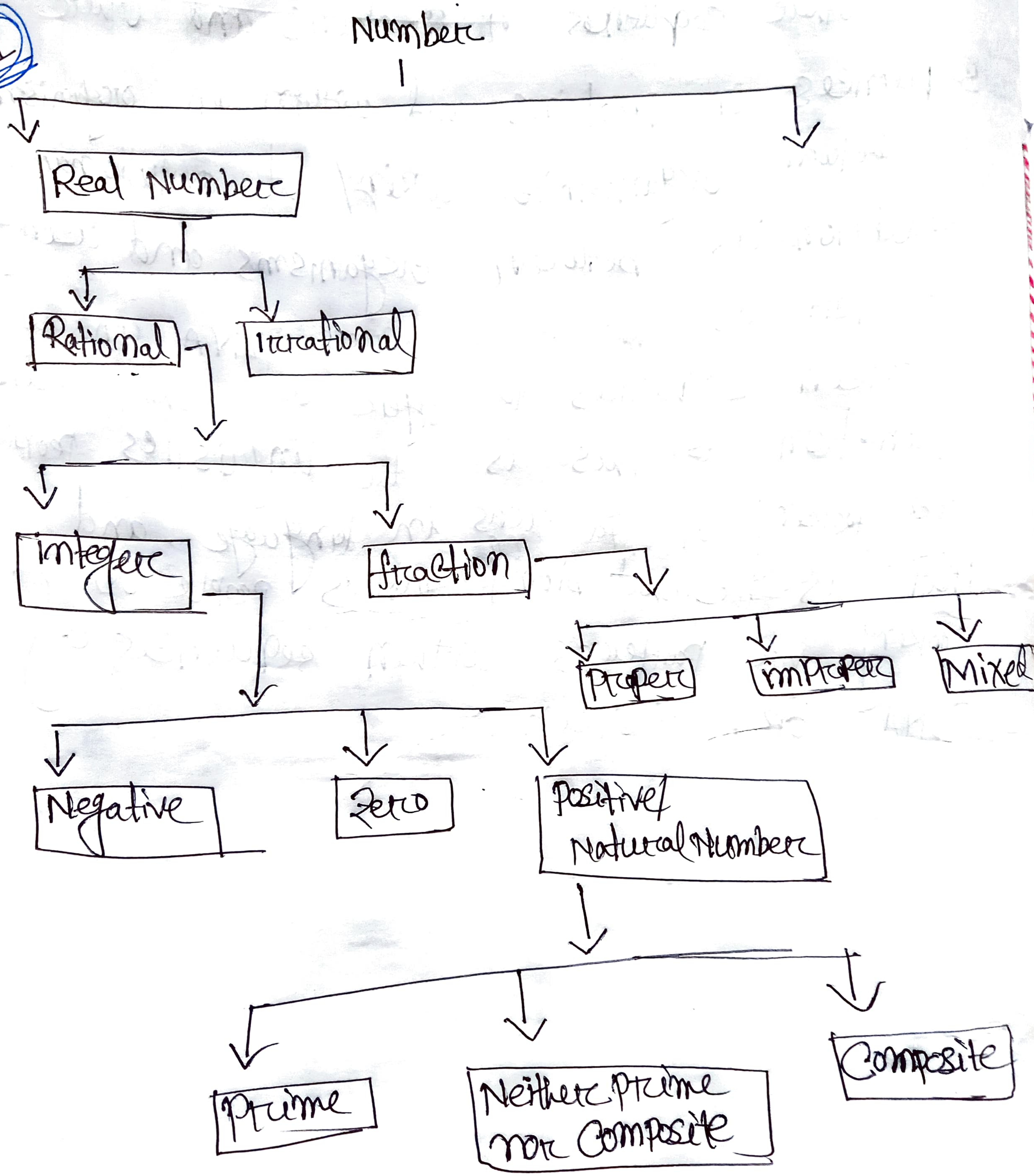
Mathematics

1



(2)

$$\begin{array}{r|l} 2 & 540 \\ \hline 2 & 270 \\ \hline 3 & 135 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

Therrefore the prrime factrcozation of 540 is $= 2^2 \times 3^3 \times 5$

(3)

$$\begin{array}{r|l} 2 & 540 \\ \hline 2 & 270 \\ \hline 3 & 135 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

Therrefore, the prrime factorrization of 540 is $= 2^2 \times 3^3 \times 5$

So the total numberr of factorrs is $= (2+1)(3+1)(1+1)$

$= 3 \times 4 \times 2$

$= 24$

calculation for all factors

$540 = 1 \times 540$
$= 2 \times 270$
$= 3 \times 180$
$= 4 \times 135$
$= 5 \times 108$
$= 6 \times 90$
$= 9 \times 60$
$= 10 \times 54$
$= 12 \times 45$
$= 15 \times 36$
$= 18 \times 30$
$= 20 \times 27$
~~$= 27 \times 20$~~

The factors of 1600 are :

1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540

4) $240 = 2 \times 120$
$= 2 \times 2 \times 60$
$= 2 \times 2 \times 2 \times 30$
$= 2 \times 2 \times 2 \times 2 \times 15$
$= 2 \times 2 \times 2 \times 2 \times 3 \times 5$
$= 2^4 \times 3 \times 5$

$540 = 2 \times 270$
$= 2 \times 2 \times 135$
$= 2 \times 2 \times 3 \times 45$
$= 2 \times 2 \times 3 \times 3 \times 15$
$= 2 \times 2 \times 3 \times 3 \times 3 \times 5$
$= 2^2 \times 3^3 \times 5$

$\therefore$ GCD $(240, 540) = 2^2 \times 3 \times 5$
$= 60$

$\therefore$ LCM $(240, 540) = 2^4 \times 3^3 \times 5 = 2160$

5) $42 = 2 \times 21$
$= 2 \times 3 \times 7$

$63 = 3 \times 21$
$= 3 \times 3 \times 7 = 3^2 \times 7$

$140 = 2 \times 70$
$= 2 \times 2 \times 35$
$= 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$

HCF $(42, 63, 140) = 7$

LCM $(42, 63, 140) = 2^2 \times 3^2 \times 5 \times 7$
$= 1260$

(6) $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27}$

Calculation for Numerators

$2 = 2 \times 1$

$8 = 2^3$

$16 = 2^4$

$10 = 2 \times 5$

$LCM(2, 8, 16, 10) = 2^4 \times 5 = 16 \times 5 = 80$

$HCF(2, 8, 16, 10) = 2$

Calculation for ~~Demon~~ Denominators

$3 = 3 \times 1$

$9 = 3^2$

$81 = 3^4$

$27 = 3^3$

$LCM(3, 9, 81, 27) = 3^4 = 81$

$HCF(3, 9, 81, 27) = 3$

$LCM \text{ of } \frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27} = \frac{LCM(2,8,16,10)}{HCF(3,9,81,27)} = \frac{80}{3}$

$HCF \text{ of } \frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27} = \frac{HCF(2,8,16,10)}{LCM(3,9,81,27)} = \frac{2}{81}$

7

$$z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$$

$$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$$

$$= \frac{(1+\sqrt{3}i)^2}{1-(\sqrt{3}i)^2}$$

$$= \frac{1+2\sqrt{3}i+3i^2}{1+3}$$

$$= \frac{1+2\sqrt{3}i-3}{4}$$

$$= \frac{2\sqrt{3}i-2}{4}$$

$$= \frac{2(\sqrt{3}i-1)}{4}$$

$$= \frac{\sqrt{3}i-1}{2}$$

$$= -\frac{1}{2}+\frac{\sqrt{3}}{2}i$$

$\therefore$ Polar form $-\frac{1}{2}+\frac{\sqrt{3}}{2}i$

$$|z| = \sqrt{\left(-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \sqrt{\frac{1}{4}+\frac{3}{4}}$$

$$= \sqrt{\frac{4}{4}}$$

$$= 1$$

And Argument of z will

$$\theta = \pi - \tan^{-1}\left|\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right|$$

$$= \pi - \tan^{-1}(\sqrt{3})$$

$$= \pi - \tan^{-1}\tan\frac{\pi}{3}$$

$$= \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

Exponetial form is $z = re^{i\theta}$

$= 1 \cdot e^{i\frac{2\pi}{3}}$

$= e^{\frac{2\pi}{3}i}$

(8) $\sqrt{-16} \times \sqrt{-4}$

$= \sqrt{16}\, i \times \sqrt{4}\, i$

$= 4i \times 2i$

$= 8i^2$

$= -8$ Ans

and :

$\dfrac{\sqrt{-16}}{\sqrt{-4}}$

$= \dfrac{\sqrt{16}\, i}{\sqrt{4}\, i}$

$= \dfrac{4}{2}$

$= 2$ A

(9) Here given $z = 2+i$

$\therefore 8z - z^2 = 8(2+i) - (2+i)^2$

$= 16 + 8i - (4 + 4i + i^2)$

$= 16 + 8i - (4 + 4i - 1)$

$= 16 + 8i - 4 - 4i + 1$

$= 13 + 4i$

Modulus $r = \sqrt{(13)^2 + (4)^2}$

$= \sqrt{169 + 16}$

$= \sqrt{185}$

Argument $\theta = \tan^{-1} \left| \frac{y}{x} \right|$

$= \tan^{-1} \left| \frac{4}{13} \right|$

$= 17.102$

10) Let $z = 1 + i\sqrt{3}$, $z = x + iy$, $|z| = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1} \left| \frac{y}{x} \right|$

$\therefore$ modulus of $z = \sqrt{(1)^2 + (\sqrt{3})^2}$

$= \sqrt{1+3}$

$= 2$

$r = 2$

Argument of $z = \tan^{-1} \left| \frac{y}{x} \right|$

$= \tan^{-1} \left| \frac{\sqrt{3}}{1} \right|$

$= \tan^{-1} \times \tan \frac{\pi}{3}$

$= \frac{\pi}{3}$

Therefore, $r(\cos\theta + i\sin\theta)$ form is

$= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$