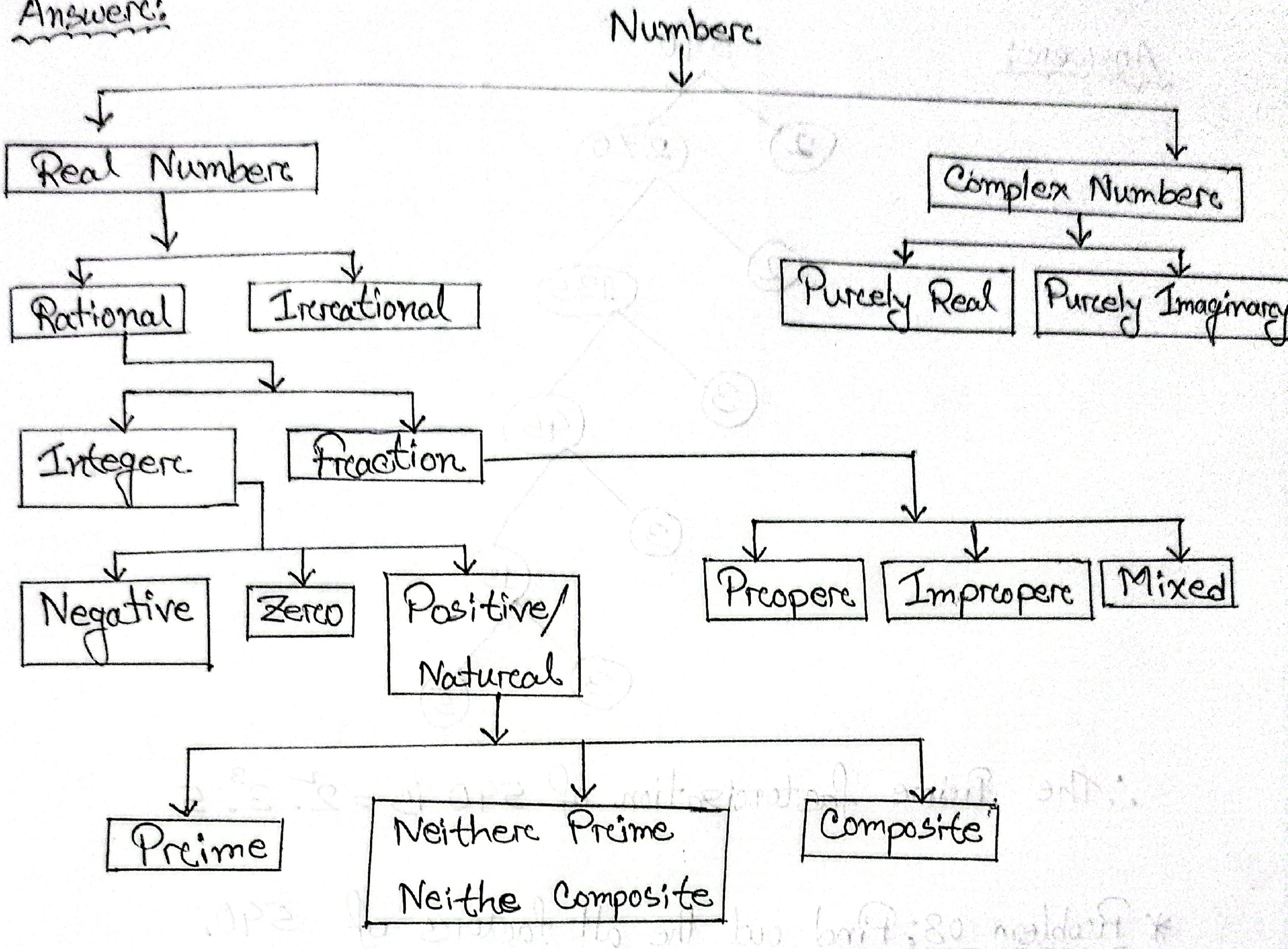
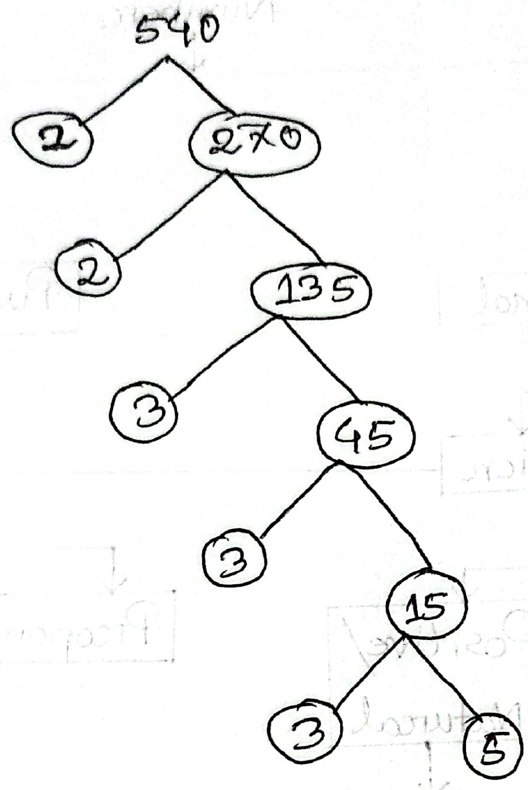
*Problem 01: Write down the classification of number system.

Answer:



* **Problem 02:** Find the prime factorization of 540 using tree.

Answer:



$\therefore$ The Prime factorization of 540 is $= 2^2 . 3^3 . 5$

* **Problem 03:** Find out the all factors of 540.

Answer: From Problem ② we get,

The prime factorization of 540 is $= 2^2 . 3^3 . 5$

So, the total number of factors of 540 is =

$(2+1)(3+1)(1+1)$

$= 3.4.2$

$= 24$

$540 = 1 \times 540$
$= 2 \times 270$
$= 3 \times 180$
$= 4 \times 135$
$= 5 \times 108$
$= 6 \times 90$
$= 9 \times 60$
$= 10 \times 54$
$= 12 \times 45$
$= 15 \times 36$
$= 18 \times 30$
$= 20 \times 27$

The all factors of 540 are 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540

***Problem 04:** What is the GCD & LCM of 240 of 540.

Answer:

$$\begin{array}{r|l} 2 & 240 \\ \hline 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 540 \\ \hline 2 & 270 \\ \hline 3 & 135 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

$\therefore 240 = 2^4 . 3 . 5$

$\therefore 540 = 2^2 . 3^3 . 5$

LCM of 240 & 540 $= 2^4 . 3^3 . 5 = 2160$

GCD of 240 & 540 $= 2^2 . 3 . 5 = 60$

*Problem 05: Find the HCF & LCM of 42, 63 & 140.

Answer:

$$\begin{array}{r|l} 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 2 & 140 \\ \hline 2 & 70 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

$\therefore 4 = 2 \times 3 \times 7$ | $\therefore 63 = 3^2 \times 7$ | $\therefore 140 = 2^2 \times 5 \times 7$

LCM of $(42, 63 \& 140) = 2^2 . 3^2 . 5 . 7 = 1260$

HCF of $(42, 63 \& 140) = 7$

*Problem 06: Find the HCF & LCM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81} \& \frac{10}{27}$

Answer:

Calculation of Numerator	Calculation of Denumerator
$2 = 2^1$	$3 = 3^1$
$8 = 2^3$	$9 = 3^3$
$16 = 2^4$	$81 = 3^4$
$10 = 2.5$	$27 = 3^3$

LCM of Numerator $= 2^4 . 5 = 80$ | LCM of Denumerator $= 3^4 = 81$

HCF of Numerator $= 2$ | HCF of Denumerator $= 3$

$$\therefore \text{LCM of} \left(\frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \& \frac{10}{27}\right) = \frac{80}{3}$$

$$\therefore \text{HCF of} \left(\frac{2}{3}, \frac{8}{9}, \frac{16}{81} \& \frac{10}{27}\right) = \frac{2}{81}$$

* **Problem 07:** Find the modulus and Argument of $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$ and also it's polar, exponential form.

Answer: We have,

$$z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$$

$$= \frac{(1+\sqrt{3}i)^2}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$$

$$= \frac{1+2\sqrt{3}i+(\sqrt{3})^2 i^2}{1-(\sqrt{3})^2 \cdot i^2}$$

$$= \frac{1+2\sqrt{3}i-3}{1+3}$$

$$= \frac{2\sqrt{3}i-2}{4}$$

$$= \frac{2\sqrt{3}i}{4} - \frac{2}{4}$$

$$= \frac{\sqrt{3}i}{2} - \frac{1}{2}$$

So, Polar form $= -\frac{1}{2} + \frac{\sqrt{3}i}{2}$

Now, $z = -\frac{1}{2} + \frac{\sqrt{3}i}{2}$

$$\therefore |z| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \sqrt{\frac{1}{4} + \frac{3}{4}}$$

$$= \sqrt{\frac{1+3}{4}}$$

$$= \sqrt{\frac{4}{4}}$$

$$= 1$$

So, modulus of $z = 1$

And Argument of $z = \theta = \pi - \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right)$

$= \pi - \tan^{-1}\left(\frac{\sqrt{3}}{2} \times \frac{2}{1}\right)$

$= \pi - \tan^{-1}\sqrt{3}$

$= \pi - \tan^{-1}\tan\frac{\pi}{3}$

$= \pi - \frac{\pi}{3}$

$= \frac{3\pi - \pi}{3}$

$= \frac{2\pi}{3}$

Exponential form of $z = r.e^{i\theta}$

$= 1.e^{\frac{2\pi}{3}i}$

$= e^{\frac{2\pi}{3}i}$

*Problem 08: Evaluate $\sqrt{-16} \times \sqrt{-4}$ & $\frac{\sqrt{-16}}{\sqrt{-4}}$

Answer:

$\sqrt{-16} \times \sqrt{-4}$

$= 4i \times 2i$

$= 8i^2$

$= -8$

$\frac{\sqrt{-16}}{\sqrt{-4}}$

$= \frac{4i}{2i}$

$= 2$

* **Problem 09:** Evaluate Modulus & Argument of $8z - z^2$ by replacing $z = 2+i$

Answer: Here given that,

$$z = 2+i$$

$$\therefore 8z - z^2 = 8(2+i) - (2+i)^2$$

$$= 16 + 8i - (4 - 2.2.i + i^2)$$

$$= 16 + 8i - 4 - 4i - i^2$$

$$= 16 + 8i - 4 - 4i + 1$$

$$= 13 + 4i$$

Modulus, $r = \sqrt{x^2 + y^2}$

$$= \sqrt{(13)^2 + (4)^2}$$

$$= \sqrt{169 + 16}$$

$$= \sqrt{185}$$

Argument, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$$= \tan^{-1} \frac{4}{13}$$

$$= 17.10^\circ$$

*Problem 10: Express $1+\sqrt{3}i$ in the form of $r(\cos\theta + i\sin\theta)$

Answer:

$$z = 1+\sqrt{3}i$$

$\therefore$ Modulus, $r = \sqrt{1^2+(\sqrt{3})^2}$

$$= \sqrt{1+3}$$

$$= \sqrt{4}$$

$$= 2$$

$\therefore$ Argument $\theta = \tan^{-1}\frac{\sqrt{3}}{1}$

$$= \tan^{-1}\left(\tan\frac{\pi}{3}\right)$$

$$= \frac{\pi}{3}$$

Therefore, $r(\cos\theta + i\sin\theta)$ form (Polar form) is

$$= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$