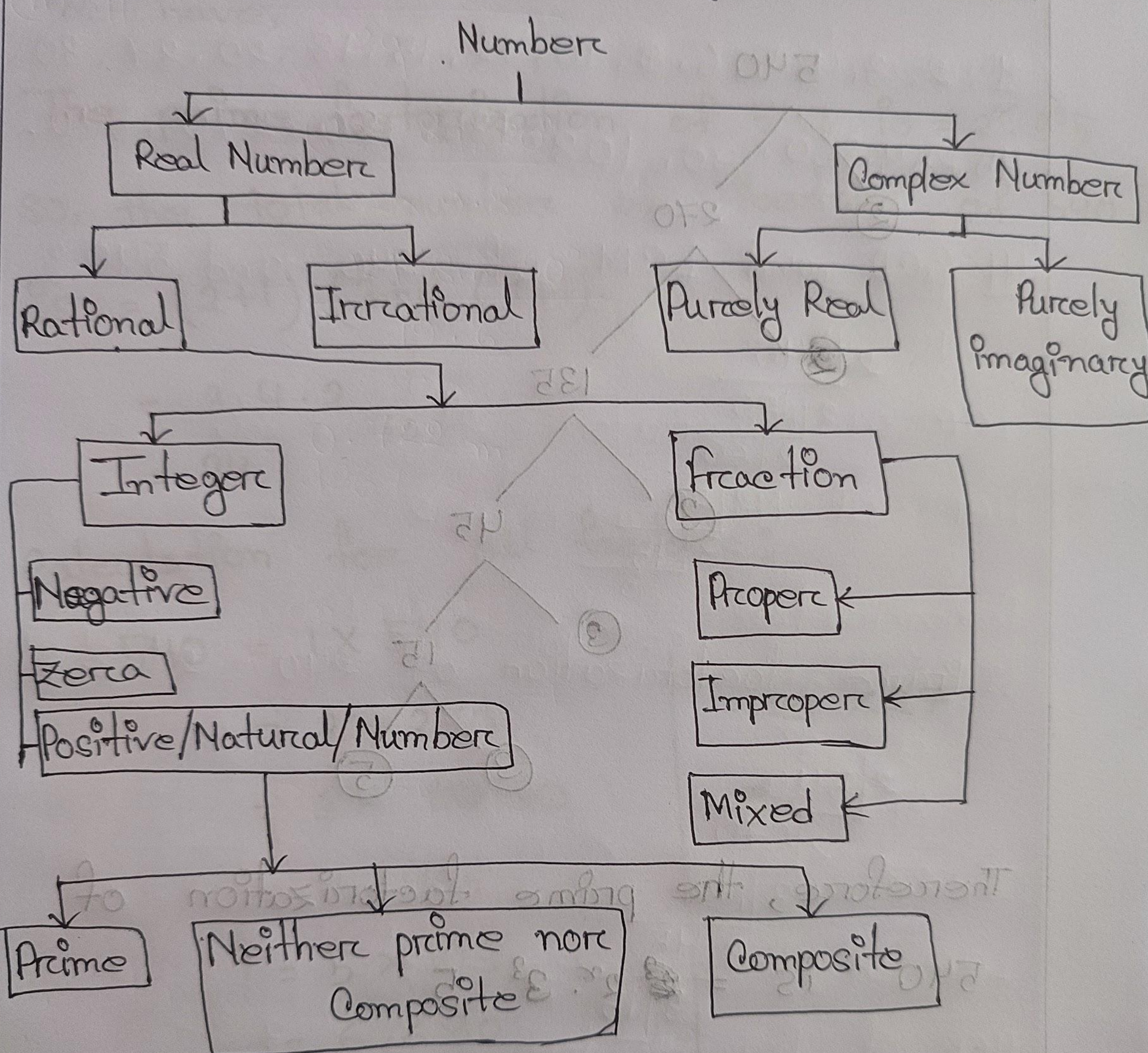
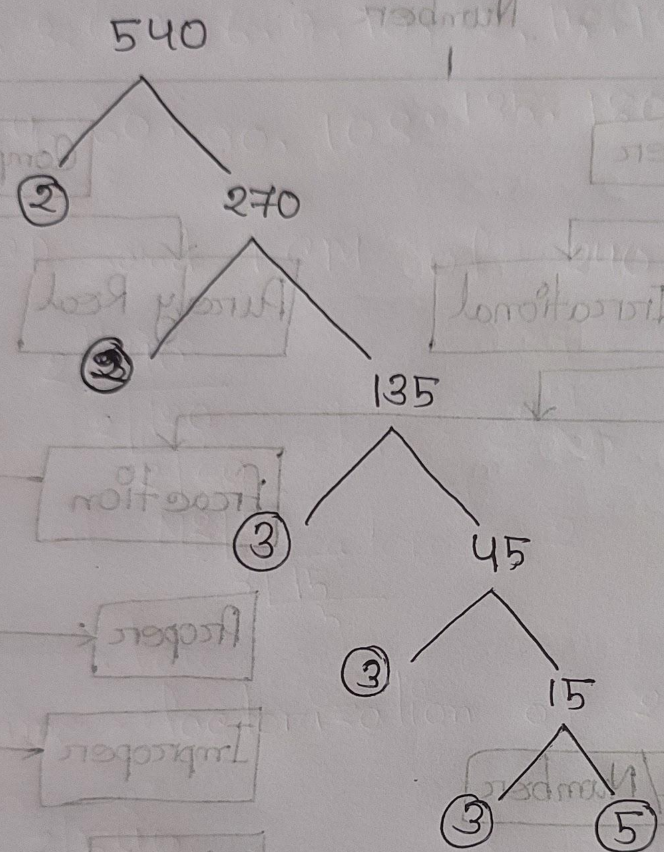
1. Classification of number system:



2| Prime factorization of 540:



Therefore, the prime factorization of

540 is $= 2^2 \cdot 3^3 \cdot 5$

3] From no. 2, ~~We~~

We have,

The prime factorization of 540 is $= 2^2 \cdot 3^3 5$

so, the total number of factors of 540

is $= (2+1)(3+1)(1+1)$

$= 3 \cdot 4 \cdot 2$

$= 24$

Calculation for all factors:

$540 = 1 \times 540$

$= 2 \times 270$

$= 3 \times 180$

$= 4 \times 135$

$= 5 \times 108$

$= 6 \times 90$

$= 9 \times 60$

$= 10 \times 54$

$= 12 \times 45$

$= 15 \times 36$

$= 18 \times 30$

$= 20 \times 27$

The factors of 540 are

1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540.

4) GCD and LCM of 240 and 540:

$$\begin{array}{r|l} 2 & 240 \\ \hline 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

$\therefore$ Prime factorization of $240 = 2^4 \cdot 3^1 \cdot 5^1$

$$\begin{array}{r|l} 2 & 540 \\ \hline 2 & 270 \\ \hline 3 & 135 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

$\therefore$ Prime factorization of $540 = 2^2 \cdot 3^3 \cdot 5^1$

GCD of $(240, 540) = 2^2 \times 3 \times 5$

$= 4 \times 3 \times 5$

$= 60$

LCM of $(240, 540) = 2^4 \times 3^3 \times 5$

$= 16 \times 27 \times 5$

$= 2160$

5| H.C.F and L.C.M of 42, 63 and 140:

$$\begin{array}{r|l} 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 2 & 140 \\ \hline 2 & 70 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

$= 2 \times 3 \times 7$ $\qquad = 3^2 \times 7$ $\qquad = 2^2 \times 5 \times 7$

H.C.F of $(42, 63, 140) = 7$

L.C.M of $(42, 63, 140) = 2^2 \times 3^2 \times 7 \times 5$

$= 1260$

6) HCF and LCM of $\frac{2}{3}$, $\frac{8}{9}$, $\frac{16}{81}$ and $\frac{10}{27}$:

Numerator Side,

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

$10 = 2 \times 5$

LCM of Numerator $= 2^4 \times 5 = 80$

HCF of Numerator $= 2$

Denominator Side,

$3 = 3^1$

$9 = 3^2$

$81 = 3^4$

$27 = 3^3$

LCM of Denominator $= 3^4 = 81$

HCF of Denominator $= 3^1 = 3$

Therefore,

The LCM of $\left(\frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27}\right) = \frac{80}{3}$

The HCF of $\left(\frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27}\right) = \frac{2}{81}$

7| Finding Modulus, Argument and Polar, Exponential form of $z = \frac{1+\sqrt{3}\,i}{1-\sqrt{3}\,i}$:

$$z = \frac{1+\sqrt{3}\,i}{1-\sqrt{3}\,i}$$

$$= \frac{(1+\sqrt{3}\,i)(1+\sqrt{3}\,i)}{(1-\sqrt{3}\,i)(1+\sqrt{3}\,i)}$$

$$= \frac{(1+\sqrt{3}\,i)^2}{1^2-(\sqrt{3}\,i)^2}$$

$$= \frac{1+2\sqrt{3}\,i+(\sqrt{3}\,i)^2}{1-3i^2}$$

$$= \frac{1+2\sqrt{3}\,i+3i^2}{1+3}$$

$$= \frac{1+2\sqrt{3}\,i-3}{4}$$

$$= \frac{-2+2\sqrt{3}\,i}{4}$$

$$= -\frac{1}{2}+\frac{\sqrt{3}\,i}{2}$$

so, $x = -\frac{1}{2}$ and $y = \frac{\sqrt{3}}{2}$

$\therefore$ Modulas, $|z| = \sqrt{x^2 + y^2}$

$= \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \sqrt{\frac{1}{4} + \frac{3}{4}}$

$= \sqrt{\frac{4}{4}}$

$= 1$

$\therefore$ Argument, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$= \pi - \tan^{-1}\left(\frac{y}{x}\right)$

$= \pi - \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right)$

$= \pi - \tan^{-1}\left(\frac{\sqrt{3}}{2} \times -\frac{2}{1}\right)$

$= \pi - \tan^{-1}(-\sqrt{3})$

$= \pi - \tan^{-1}(\sqrt{3})$

$= \pi - \cancel{\tan} \ 60°$

$= \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

Polar form,

$$z = r(\cos\theta + i\sin\theta)$$

$$= 1\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

$$= \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$$

Exponensial form,

$$z = re^{i\theta}$$

$$= 1 \cdot e^{i\frac{2\pi}{3}}$$

$$= e^{\frac{2\pi}{3}i}$$

8] Evaluate:

$$\sqrt{-16} \times \sqrt{-4}$$

$$= i\sqrt{16} \times i\sqrt{4}$$

$$= 4i \times 2i$$

$$= 8i^2$$

$$= 8 \times (-1)$$

$$= -8$$

And

$$\frac{\sqrt{-16}}{\sqrt{-4}}$$

$$= \frac{i\sqrt{16}}{i\sqrt{4}}$$

$$= \frac{4i}{2i}$$

$$= 2.$$

9) Evaluate modulus and Argument:

$$8z - z^2$$

$$= 8(2+i) - (2+i)^2 \qquad [z = 2+i]$$

$$= 16 + 8i - 2^2 - 2\cdot 2\cdot i - i^2$$

$$= 16 + 8i - 4 - 4i + 1$$

$$= 12 + 4i + 1$$

$$= 13 + 4i$$

So, $x = 13$, $y = 4$

Therefore,

Modulus; $|z| = \sqrt{x^2 + y^2}$

$$= \sqrt{(13)^2 + (4)^2}$$

$$= \sqrt{169 + 16}$$

$$= \sqrt{185}$$

$\therefore$ Argument, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$= \tan^{-1} \frac{4}{13}$

10| We have,

$1 + \sqrt{3}\,i$

So, $x = 1$, $y = \sqrt{3}$

Argument, $\theta = \tan^{-1}\left|\frac{y}{x}\right|$

$= \tan^{-1}\left|\frac{\sqrt{3}}{1}\right|$

$= \tan^{-1}\sqrt{3}$

$= \tan^{-1} 60$

$= \frac{\pi}{3}$

~~So, Modulus, $|z| = r =$~~

Modulus, $|z| = r = \sqrt{x^2+y^2}$

$= \sqrt{(1)^2 + (\sqrt{3})^2}$

$= \sqrt{1+3} = \sqrt{4} = 2$

So,

$$z = r\,(\cos\theta + i\sin\theta)$$

$$= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$