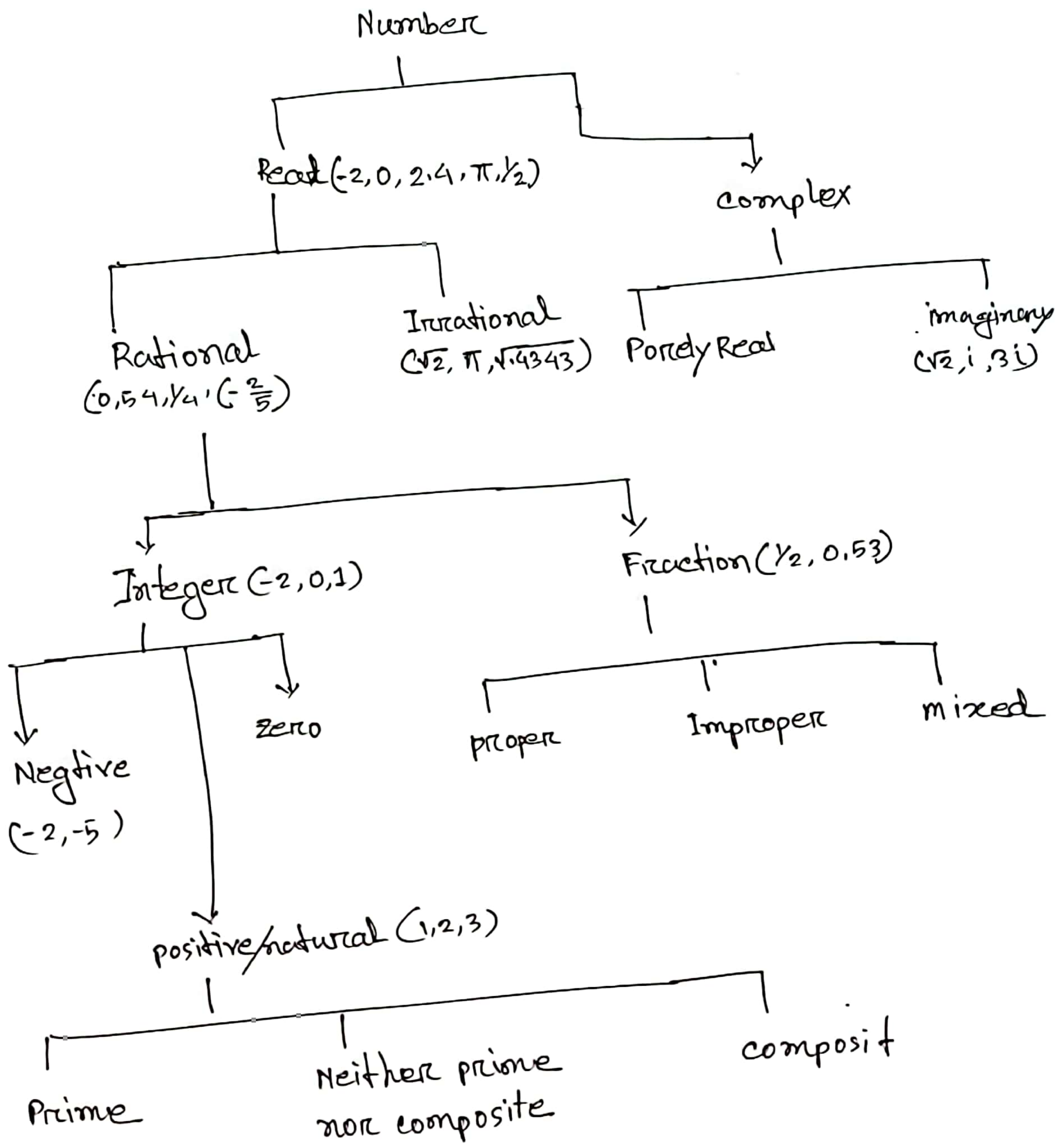
TAKT TAJWAR

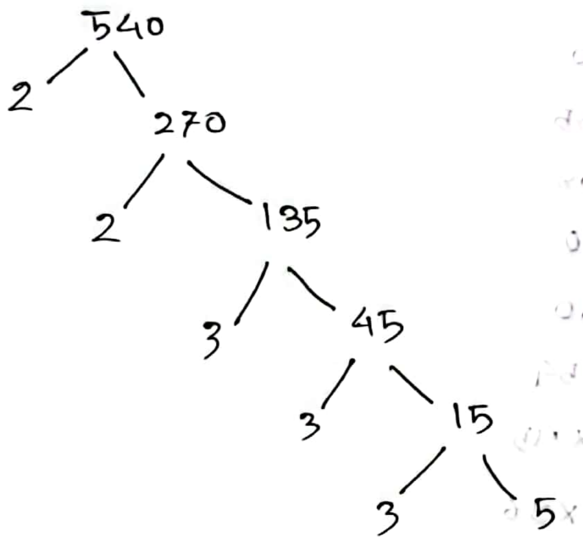
ID: 221-15-5789

1.

classification of number system:



2. Tree diagram:



Prime factorization of 540 $= 2^2 \cdot 3^3 \cdot 5^1$

3.

$2\,|\,540$
$2\,|\,270$
$3\,|\,135$
$3\,|\,45$
$3\,|\,15$
$\quad 5$

$\therefore 540 = 2^2 \cdot 3^3 \cdot 5^1$ [only prime]

$\therefore$ All factors of 540 $= (2+1)\cdot(3+1)(1+1)$

$= 3\cdot 4\cdot 2$

$= 24$

Here,

$540 = 1 \times 540$
$= 2 \times 270$
$= 3 \times 180$
$= 4 \times 135$
$= 5 \times 108$
$= 6 \times 90$
$= 9 \times 60$
$= 10 \times 54$
$= 12 \times 45$
$= 15 \times 36$
$= 18 \times 30$
$= 20 \times 27$

$\therefore$ all factor of 540 = 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 12, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540

4. .

$$\begin{array}{r|l} 2 & 240 \\ 2 & 120 \\ 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ & 5 \end{array} \qquad \begin{array}{r|l} 2 & 540 \\ 2 & 270 \\ 3 & 135 \\ 3 & 45 \\ 3 & 15 \\ & 5 \end{array}$$

$\therefore$ Prime factorization of $240 = 2^4 . 3^1 . 5^1$

and $540 = 2^2 . 3^3 . 5^1$

$\therefore$ GCD $= 2^2 . 3^1 . 5^1 = 60$

LCM $= 2^4 . 3^3 . 5^1 = 2160$

5. $$\begin{array}{r|l} 2 & 42 \\ 3 & 21 \\ & 7 \end{array} \qquad \begin{array}{r|l} 3 & 63 \\ 3 & 21 \\ & 7 \end{array} \qquad \begin{array}{r|l} 2 & 140 \\ 2 & 70 \\ 5 & 35 \\ & 7 \end{array}$$

$42 = 2 \times 3 \times 7$

$63 = 3 \times 3 \times 7$

$140 = 2 \times 2 \times 5 \times 7$

$\therefore$ HCF $= 7$

LCM $= 2 \times 2 \times 3 \times 3 \times 7 = 252$

6. $\frac{2}{3}$.

6. $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27}$

Factorizations of Numerators:

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

$10 = 2^1 \cdot 5$

HCF of numerators = 2

LCM of numerators = $2^4 \cdot 5 = 80$

Factorization of Denominators:

$3 = 3^1$

$9 = 3^2$

$81 = 3^4$

$27 = 3^3$

HCF of denominators = 3

LCM of denominators = 81

$\therefore$ HCF of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$ $= \frac{\text{HCF}(2,8,16,10)}{\text{LCM}(3,9,81,27)}$

$= \frac{2}{81}$

$\therefore$ LCM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$ $= \frac{\text{LCM}(\cancel{3,9}, 2,8,16,10)}{\text{HCF}(3,9,81,27)}$

$= \frac{80}{3}$

Ans.

7. We have, $\frac{1+\sqrt{3}i}{1-\sqrt{3}i}$

$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$

$= \frac{(1+\sqrt{3}i)^2}{1^2-(\sqrt{3}i)^2}$

$= \frac{1^2+2\cdot\sqrt{3}i+3i^2}{1+3}$

$= \frac{-2+2\sqrt{3}i}{4}$

$= \frac{2(-1+\sqrt{3}i)}{4}$

$= \frac{-1+\sqrt{3}i}{2}$

$= -\frac{1}{2} + \frac{\sqrt{3}i}{2}$

$= x+iy$ [where $x = -\frac{1}{2}$ and $y = \frac{\sqrt{3}i}{2}$]

now, $r = \sqrt{x^2+y^2}$

$= \sqrt{\frac{1}{4}+\frac{3}{4}}$

$= 1$

and $\theta = \pi - \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right)$

$= \pi - \tan^{-1}\sqrt{3}$

$= \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

so polar from is, $z = r(\cos\theta + i\sin\theta)$

$= 1\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$= \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

and exponential from is $z = e^{i\frac{2\pi}{3}}$

8. $\sqrt{-16} \times \sqrt{-4}$

$= \sqrt{16i^2} \times \sqrt{4i^2}$ $[i^2 = -1]$

$= \sqrt{4^2 i^2} \times \sqrt{2^2 i^2}$

$= 4i \times 2i$.

$= 8i^2$.

$= -8$

again, $\dfrac{\sqrt{-16}}{\sqrt{-4}}$

$= \dfrac{\sqrt{16i^2}}{\sqrt{4i^2}}$

$= \dfrac{4i}{2i}$

$= 2$

9.

$8z - z^2$

$= 8(2+i) - (2+i)^2$ [$z = 2+i$]

$= 16 + 8i - 4 - 4i + 1$

$= 12 + 4i + 1$

$= 13 + 4i$

$= x + iy$ [where $x = 13$ and $y = 4$]

modulus,

$r = \sqrt{13^2 + 4^2}$

$= \sqrt{185}$

argument,

$\theta = \tan^{-1} \frac{4}{13}$

$= 17.102$

10.

$1 + i\sqrt{3}$

Let, $z = 1 + i\sqrt{3}$

$z = x + iy$

$r = \sqrt{1^2 + (\sqrt{3})^2}$

$= 2$

$\therefore r = 2$

Argument of $z = \tan^{-1}\left|\frac{y}{x}\right|$

$= \tan^{-1}\left|\frac{\sqrt{3}}{1}\right|$

$= \tan^{-1} \tan \frac{\pi}{3}$

$= \frac{\pi}{3}$

$\therefore r(\cos\theta + i\sin\theta)$ from is

$$= 2(\cos\theta + i\sin\theta)$$

$$= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$