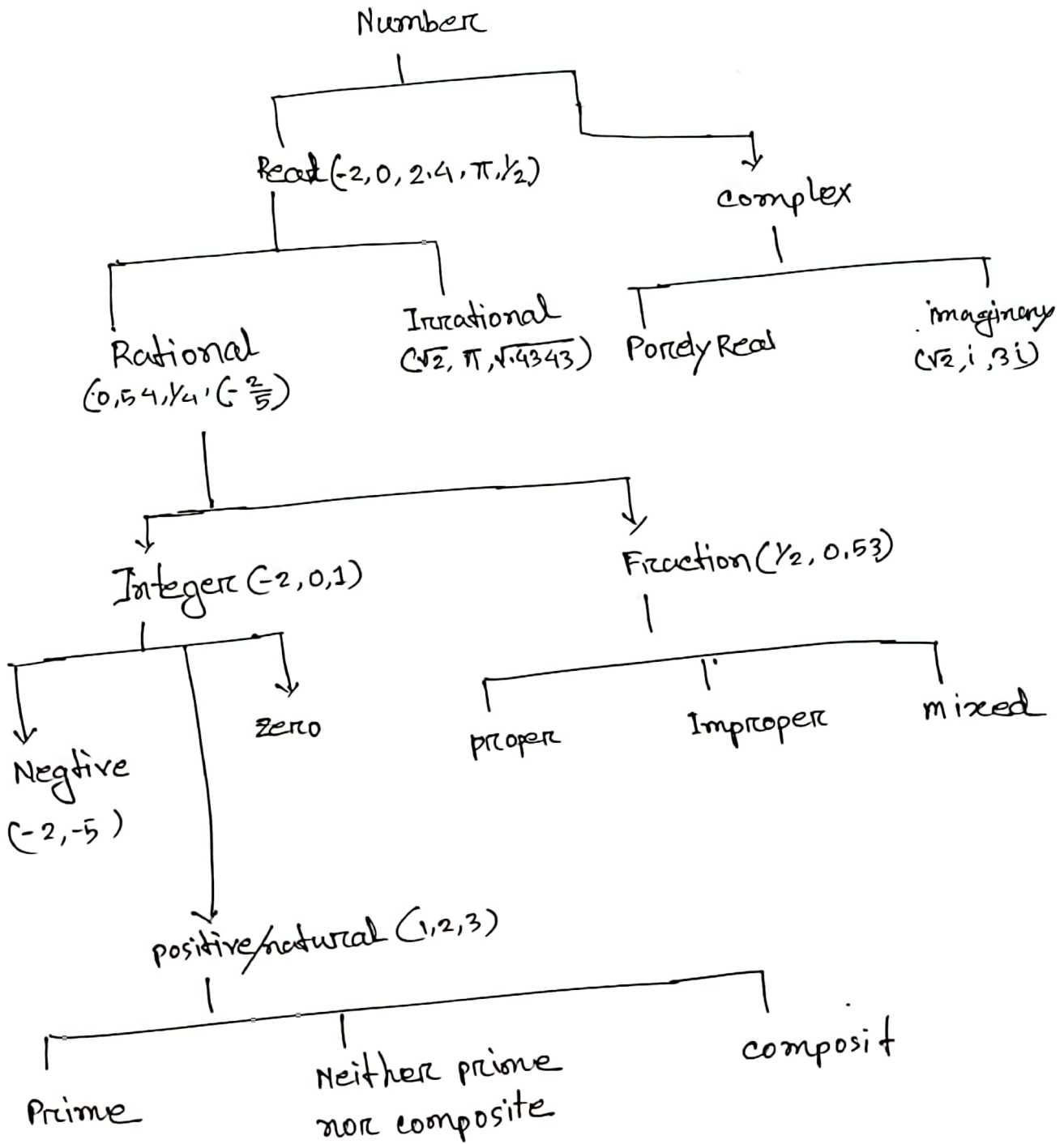


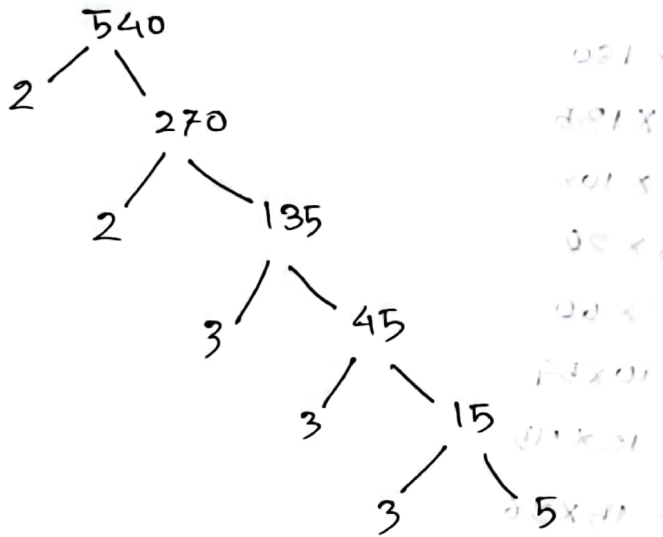
TAKT TAJWAR
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1.

classification of number system:

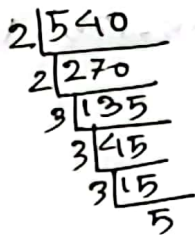


2. Tree diagram:



Prime factorization of 540 = $2^2 \cdot 3^3 \cdot 5^1$

3.



$\therefore 540 = 2^2 \cdot 3^3 \cdot 5^2$ [only prime]

$$\begin{aligned} \therefore \text{All factors of } 540 &= (2+1) \cdot (3+1) \cdot (1+1) \\ &= 3 \cdot 4 \cdot 2 \\ &= 24 \end{aligned}$$

Here,

$$540 = 1 \times 540$$

$$= 2 \times 270$$

$$= 3 \times 180$$

$$= 4 \times 135$$

$$= 5 \times 108$$

$$= 6 \times 90$$

$$= 9 \times 60$$

$$= 10 \times 54$$

$$= 12 \times 45$$

$$= 15 \times 36$$

$$= 18 \times 30$$

$$= 20 \times 27$$

\therefore all factor of 540 = 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 12, 27, 30

36, 45, 54, 60, 90, 108, 135, 180, 270, 540

4.

$$\begin{array}{r} 2 \overline{) 240} \\ 2 \overline{) 120} \\ 2 \overline{) 60} \\ 2 \overline{) 30} \\ 3 \overline{) 15} \\ 5 \end{array}$$

$$\begin{array}{r} 2 \overline{) 540} \\ 2 \overline{) 270} \\ 3 \overline{) 135} \\ 3 \overline{) 45} \\ 3 \overline{) 15} \\ 5 \end{array}$$

\therefore Prime factorization of $240 = 2^4 \cdot 3^1 \cdot 5^1$

and $540 = 2^2 \cdot 3^3 \cdot 5^1$

$$\therefore \text{H.C.D.} = 2^2 \cdot 3^1 \cdot 5^1 = 60$$

$$\text{L.C.M.} = 2^4 \cdot 3^3 \cdot 5^1 = 2160$$

5.

$$\begin{array}{r} 2 \overline{) 42} \\ 3 \overline{) 21} \\ 7 \end{array}$$

$$\begin{array}{r} 3 \overline{) 63} \\ 3 \overline{) 21} \\ 7 \end{array}$$

$$\begin{array}{r} 2 \overline{) 140} \\ 2 \overline{) 70} \\ 5 \overline{) 35} \\ 7 \end{array}$$

$$42 = 2 \times 3 \times 7$$

$$63 = 3 \times 3 \times 7$$

$$140 = 2 \times 2 \times 5 \times 7$$

$$\therefore \text{H.C.F.} = 7$$

$$\text{L.C.M.} = 2 \times 2 \times 3 \times 3 \times 7 = 252$$

$$6. \frac{2}{3}$$

$$6. \frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27}$$

Factorizations of Numerators:

$$2 = 2^1$$

$$8 = 2^3$$

$$16 = 2^4$$

$$10 = 2 \cdot 5$$

$$\text{HCF of numerators} = 2$$

$$\text{LCM of numerators} = 2^4 \cdot 5 = 80$$

Factorization of Denominators:

$$3 = 3^1$$

$$9 = 3^2$$

$$81 = 3^4$$

$$27 = 3^3$$

$$\text{HCF of denominators} = 3$$

$$\text{LCM of denominators} = 81$$

$$\therefore \text{HCF of } \frac{2}{3}, \frac{8}{9}, \frac{16}{81} \text{ and } \frac{10}{27} = \frac{\text{HCF}(2, 8, 16, 10)}{\text{LCM}(3, 9, 81, 27)}$$

$$= \frac{2}{81}$$

$$\therefore \text{LCM of } \frac{2}{3}, \frac{8}{9}, \frac{16}{81} \text{ and } \frac{10}{27} = \frac{\text{LCM}(3, 9, 81, 27)}{\text{HCF}(2, 8, 16, 10)}$$

$$= \frac{81}{2}$$

2. We have, $\frac{1 + \sqrt{3}i}{1 - \sqrt{3}i}$

$$= \frac{(1 + \sqrt{3}i)(1 + \sqrt{3}i)}{(1 - \sqrt{3}i)(1 + \sqrt{3}i)}$$

$$= \frac{(1 + \sqrt{3}i)^2}{1 - (\sqrt{3}i)^2}$$

$$= \frac{1 + 2\sqrt{3}i + 3i^2}{1 + 3}$$

$$= \frac{-2 + 2\sqrt{3}i}{4}$$

$$= \frac{2(-1 + \sqrt{3}i)}{4}$$

$$= \frac{-1 + \sqrt{3}i}{2}$$

$$= -\frac{1}{2} + \frac{\sqrt{3}i}{2}$$

$$= x + iy \text{ [where } x = -\frac{1}{2} \text{ and } y = \frac{\sqrt{3}i}{2}]$$

now, $r = \sqrt{x^2 + y^2}$

$$= \sqrt{\frac{1}{4} + \frac{3}{4}}$$

$$= 1$$

and $\theta = \pi - \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right)$

$$= \pi - \tan^{-1}\sqrt{3}$$

$$= \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

so polar form is, $z = r(\cos\theta + i\sin\theta)$

$$= 1 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)$$

$$= \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)$$

and exponential form is $z = e^{i \cdot \frac{2\pi}{3}}$

$$\begin{aligned}
 8. \quad & \sqrt{-16} \times \sqrt{-4} \\
 & = \sqrt{16i^2} \times \sqrt{4i^2} \quad [i^2 = -1] \\
 & = \sqrt{4^2 i^2} \times \sqrt{2^2 i^2} \\
 & = 4i \times 2i \\
 & = 8i^2 \\
 & = -8
 \end{aligned}$$

again,

$$\begin{aligned}
 & \frac{\sqrt{-16}}{\sqrt{-4}} \\
 & = \frac{\sqrt{16i^2}}{\sqrt{4i^2}} \\
 & = \frac{-4i}{2i} \\
 & = 2
 \end{aligned}$$

1. $\sqrt{-16} = 4i$ & $-4i$ to be considered

1. $\sqrt{-4} = 2i$ & $-2i$ to be considered

2. $\sqrt{-16} = 4i$ & $-4i$ to be considered

$\frac{4i}{2i} = 2$

1. $\sqrt{-16} = 4i$ & $-4i$ to be considered

2. $\sqrt{-4} = 2i$ & $-2i$ to be considered

3. $\sqrt{-16} = 4i$ & $-4i$ to be considered

4. $\sqrt{-4} = 2i$ & $-2i$ to be considered

5. $\sqrt{-16} = 4i$ & $-4i$ to be considered

6. $\sqrt{-4} = 2i$ & $-2i$ to be considered

7. $\sqrt{-16} = 4i$ & $-4i$ to be considered

8. $\sqrt{-4} = 2i$ & $-2i$ to be considered

9. $\sqrt{-16} = 4i$ & $-4i$ to be considered

10. $\sqrt{-4} = 2i$ & $-2i$ to be considered

11. $\sqrt{-16} = 4i$ & $-4i$ to be considered

12. $\sqrt{-4} = 2i$ & $-2i$ to be considered

13. $\sqrt{-16} = 4i$ & $-4i$ to be considered

14. $\sqrt{-4} = 2i$ & $-2i$ to be considered

15. $\sqrt{-16} = 4i$ & $-4i$ to be considered

16. $\sqrt{-4} = 2i$ & $-2i$ to be considered

17. $\sqrt{-16} = 4i$ & $-4i$ to be considered

18. $\sqrt{-4} = 2i$ & $-2i$ to be considered

$$\begin{aligned}
 9. \quad & 8z - z^2 \\
 & = 8(2+i) - (2+i)^2 \quad [z=2+i] \\
 & = 16+8i - 4-4i+1 \\
 & = 12+4i+1 \\
 & = 13+4i \\
 & = x+iy \quad [\text{where } x=13 \text{ and } y=4]
 \end{aligned}$$

modulus,

$$\begin{aligned}
 r & = \sqrt{13^2+4^2} \\
 & = \sqrt{185}
 \end{aligned}$$

argument,

$$\begin{aligned}
 \theta & = \tan^{-1} \frac{4}{13} \\
 & = 17.102
 \end{aligned}$$

10.

$$1+i\sqrt{3}$$

$$\text{let, } z=1+i\sqrt{3}$$

$$z=x+iy$$

$$r = \sqrt{1+(\sqrt{3})^2}$$

$$= 2$$

$$\therefore r=2$$

$$\text{Argument of } z = \tan^{-1} \left| \frac{y}{x} \right|$$

$$= \tan^{-1} \left| \frac{\sqrt{3}}{1} \right|$$

$$= \tan^{-1} \tan \frac{\pi}{3}$$

$$= \frac{\pi}{3}$$

$$\therefore \pi(\cos\theta + i\sin\theta) \text{ from 19}$$

$$= 2(\cos\theta + i\sin\theta)$$

$$= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$