Sub: Complex Number 2nd

Day: WEDNESD

Time: 8PM Date: 09/02/2022

MD. Abdur Rahim
ID: 221-15-5915

Exercise :

1. Write down the classification of number system.
2. Find the prime factorization of 540 using tree.
3. Find out the all factors of 540.
4. What is the GCD & LCM of 240 & 540.
5. Find the H.C.F & L.C.M of 42, 63 & 140.
6. Find the H.C.F & L.C.M of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$.
7. Find the modulus and Argument of $= \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$ and also it's polar, exponential form.
8. Evaluate $\sqrt{-16} \times \sqrt{-4}$ & $\frac{\sqrt{-16}}{\sqrt{-4}}$.
9. Evaluate modulus & Argument of $8_z - z^2$ by replacing $z = 2+i$.
10. Express $1+i\sqrt{3}$ in the from of $r(\cos\theta + i\sin\theta)$.

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Solution

1) Classification of number system.

- Number
  - Real Number
    - Rational
      - Intreger
        - Negative
        - Zero
        - Positive
          - Prime
          - Neither prime ~~non complex~~ nor composite
          - Composite
      - Fraction
        - Proper
        - Mixed
        - Imprope
    - Irrational
  - Complex Number
    - Pure Real
    - Pure Imaginary

2) Prime factorization of 540 with tree structure.

- 540
  - 2
  - 270
    - 2
    - 135
      - 3
      - 45
        - 3
        - 15
          - 3
          - 5

$\therefore$ prime factorization of $540 : 2^2 \cdot 3^3 \cdot 5^1$ Ans:

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(3)

$$\begin{array}{r|l} 2 & 540 \\ \hline 2 & 270 \\ \hline 3 & 135 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

∴ prime factorization 540 is: $2^2 . 3^3 . 5^1$

So, the total number of factors of 540 is $\Rightarrow (2+1)(3+1)(1+1)$

$\Rightarrow 3.4.2$

$\Rightarrow 24$

calculating for all factors

$540 \Rightarrow 1 \times 540$

$\Rightarrow 2 \times 270$

$\Rightarrow 3 \times 180$

$\Rightarrow 4 \times 135$

$\Rightarrow 5 \times 108$

$\Rightarrow 6 \times 90$

$\Rightarrow 9 \times 60$

$\Rightarrow 10 \times 54$

$\Rightarrow 12 \times 45$

$\Rightarrow 15 \times 36$

$\Rightarrow 18 \times 30$

$\Rightarrow 20 \times 27$

∴ All factors of 540 :$\Rightarrow$ 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540. Ans:

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④ GCD & LCM of 240 and 540:

$$\begin{array}{r|l} 2 & 240 \\ \hline 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 540 \\ \hline 2 & 270 \\ \hline 3 & 135 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

$\therefore$ Prime factorization of $240 = 2^4 . 3^1 . 5^1$
and of $540 = 2^2 . 3^3 . 5^1$

$\therefore$ GCD $\Rightarrow 2^2 . 3^1 . 5^1 = 60$.

$\therefore$ LCM $\Rightarrow 2^4 . 3^3 . 5^1 = 2160$. Ans:

⑤

$$\begin{array}{r|l} 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 2 & 140 \\ \hline 2 & 70 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

$42 = 2\times3\times7$

$63 = 3\times3\times7$

$140 = 2\times2\times5\times7$

$\therefore$ H.C.F $= 7$

$\therefore$ L.C.F $\Rightarrow 2^2 . 3^2 . 5 . 7 = 1260$ Ans:

⑥ Find the HCF & LCM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ & $\frac{10}{27}$

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calculating for Numerator

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

$10 = 2.5$

$\therefore LCM(2,8,16,10) \Rightarrow 2^4.5 = 80$

$\therefore HCF(2,8,16,10) \Rightarrow 2^1 = 2$

calculating for Denominator

$3 = 3^1$

$9 = 3^2$

$81 = 3^4$

$27 = 3^3$

$\therefore LCM(3,9,81,27) \Rightarrow 3^4 = 81$

$\therefore HCF(3,9,81,27) \Rightarrow 3^1 = 3$

$\therefore LCM(2/3, 8/9, \frac{16}{81}$ & $10/27) \Rightarrow \frac{\text{LCM of } 2,8,16,10}{\text{HCF of } 3,9,81,27}$

$\Rightarrow \frac{80}{3}$

$\therefore HCF(2/3, 8/9, 16/81$ & $10/27) \Rightarrow \frac{\text{HCF of } 2,8,16,10}{\text{LCM of } 3,9,81,27}$

$\Rightarrow \frac{2}{81}$ Ans:

7)

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Finding Modulus, Argument and polar:

$$z \Rightarrow \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$$

$$\Rightarrow \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$$

$$\Rightarrow \frac{1^2+\sqrt{3}i+\sqrt{3}i+3i^2}{1-3i^2}$$

$$\Rightarrow \frac{1+2\sqrt{3}i+3(-1)}{1-3(-1)}$$

$$\Rightarrow \frac{1+2\sqrt{3}i-3}{1+3}$$

$$\Rightarrow \frac{2\sqrt{3}i-2}{4}$$

$$\Rightarrow \frac{\sqrt{3}}{2}i-\frac{1}{2}$$

$$\Rightarrow -\frac{1}{2}+\frac{\sqrt{3}}{2}i$$

$\Rightarrow x+iy$ [where $x=-\frac{1}{2}$ & $y=\frac{\sqrt{3}}{2}$]

now, $r=\sqrt{x^2+y^2}$

$$\Rightarrow \sqrt{\frac{1}{4}+\frac{3}{4}}=1$$

and

$$\theta = \pi - \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right)$$

$$= \pi - \tan^{-1}(-\sqrt{3})$$

$$= \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

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So,
Polar form is, $z \Rightarrow r(\cos\theta + i\sin\theta)$

$$\Rightarrow 1.\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

$$\Rightarrow \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

$\therefore$ Exponential form is $z \Rightarrow re^{i\theta}$

$$\Rightarrow 1e^{i\frac{2\pi}{3}}$$

$$\Rightarrow e^{\frac{2\pi}{3}i}$$ Ans:

8) $\sqrt{-16} \times \sqrt{-4}$

$\Rightarrow \sqrt{16i^2} \times \sqrt{4i^2}$ $[i^2 = -1]$

$\Rightarrow \sqrt{4^2 \cdot i^2} \times \sqrt{2^2 \cdot i^2}$

$\Rightarrow 4i \times 2i \Rightarrow 8i^2 = -8$

And, $\frac{\sqrt{-16}}{\sqrt{-4}}$

$\Rightarrow \frac{\sqrt{16i^2}}{\sqrt{4i^2}}$

$= \frac{4i}{2i}$

$= 2$ Ans:

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(9) $8z - z^2$

$\Rightarrow 8(2+i) - (2+i)^2$ [$z = 2+i$]

$\Rightarrow 16 + 8i - 4 - 4i + 1$

$\Rightarrow 12 + 4i + 1$

$\Rightarrow 13 + 4i$

$\Rightarrow x + iy$ [where $x = 13$ & $y = 4$]

$\therefore$ modulus $r \Rightarrow \sqrt{x^2+y^2} \Rightarrow \sqrt{13^2+4^2} = \sqrt{185}$

$\therefore$ Argument $\theta \Rightarrow \tan^{-1}\frac{4}{13}$

$= 17.102$ Ans:

(10) $1 + i\sqrt{3}$

$\therefore$ modulus $r \Rightarrow \sqrt{x^2+y^2}$

$\cancel{= \sqrt{1^2 + \sqrt{3}^2}}$

$\Rightarrow \sqrt{1^2 + 3^2} \Rightarrow \sqrt{1+3} = 2$

$\therefore$ Argument $\theta \Rightarrow \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$\Rightarrow \tan^{-1}(\sqrt{3})$

$\Rightarrow \frac{\pi}{3}$

$\therefore r(\cos\theta + i\sin\theta)$ form is

$= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ Ans: