MD. Merajul Alam Meraj
ID- 221-15-5632 (U)

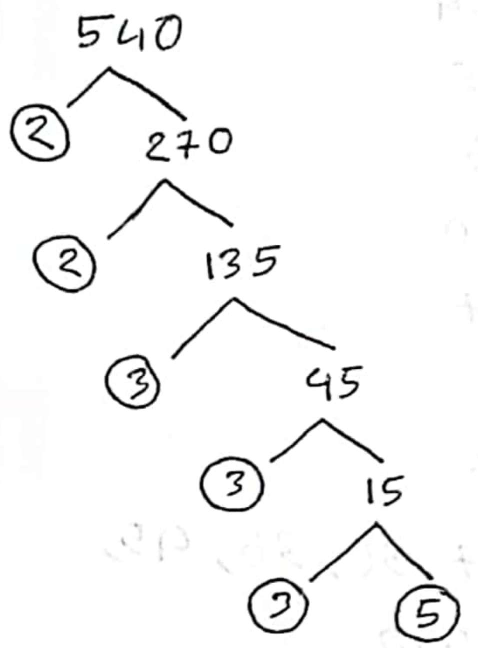
Ans.01:

Classification of Number System.

- Number
  - Real Number
    - Retional Number
      - Integer
        - Negative
        - Zero
        - Positive/ Natural Number
          - Prime
          - Neither prime nor Composite
          - Composite
      - Fraction
        - Proper
        - Improper
        - Mixed
    - Irretional Number
  - Complex Number
    - Pure Real
    - Imaginary

Ans 02:

Prime Factorization of 540 with ~~Thae~~ Tree Structure



So, The Prime Factorization of

540 is $= 2^2 . 3^3 . 5$

Ans

Ans. 03

As we know, the Prime Factorization of

540 is $= 2^2 . 3^3 . 5$

So, the total number of factors of

540 is $= (2+1)(3+1)(1+1)$

$= 3.4.2$

$= 24$

Calculation for all factors

$540 = 1 \times 540$
$= 2 \times 270$
$= 3 \times 180$
$= 4 \times 135$
$= 5 \times 108$
$= 6 \times 90$
$= 9 \times 60$
$= 10 \times 54$
$= 12 \times 45$
$= 15 \times 36$
$= 18 \times 30$
$= 20 \times 27$

So, the factors of 540 are
1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45,
54, 60, 90, 108, 135, 180, 270, 540

Ans. 04:

GCD and LCM of 240 and 540:

$$\begin{array}{r|l} 2 & 240 \\ \hline 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 540 \\ \hline 2 & 270 \\ \hline 3 & 135 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

The Prime Factorization of 240 is $= 2^4 \times 3 \times 5$

The Prime Factorization of 540 is $= 2^2 \times 3^3 \times 5$

Therefore, $LCM(240, 540) = 2^4 \times 3^3 \times 5 = 2160$

and $GCD(240, 540) = 2^2 \times 3 \times 5 = 60$

Ans.

Ans 05

Prime Factorization of 42, 63, 140 are.

$42 = 2 \times 21 = 2 \times 3 \times 7$

$63 = 3 \times 21 = 3 \times 3 \times 7 = 3^2 \times 7$

$140 = 2 \times 70 = 2 \times 2 \times 35 = 2^2 \times 5 \times 7$

$\therefore$ LCM of $(42, 63, 140) = 2^2 . 3^2 . 5 . 7 = 1260$

and HCF of $(42, 63, 140) = 7$

Ans.

Ans. 06

Finding the LCM and HCF of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$

Calculation of Numerator

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

~~18 18~~ $10 = 2 \times 5$

LCM of Numerator $= 2^4 \times 5$

$= 80$

HCF of Numerator $= 2$

Calculation of Denominators

$3 = 3^1$

$9 = 3^2$

$81 = 3^4$

$27 = 3^3$

LCM of Denominator $= 3^4 = 81$

HCF of Denominator $= 3$

So,

HCF of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{2}{81}$

LCM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{80}{3}$ Ans.

Ans. 07

Finding modulus, argument and polar:

$$\cancel{z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i} = \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)} = \frac{(1-\sqrt{3}i)^2}{(1)^2-(\sqrt{3}i)^2}}$$

$$z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i} = \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)} = \frac{(1+\sqrt{3}i)^2}{(1)^2-(\sqrt{3}i)^2}$$

$$= \frac{1^2+2.1.\sqrt{3}i+(\sqrt{3}i)^2}{(1)^2-(\sqrt{3}i)^2} = \frac{1+2\sqrt{3}i+3i^2}{1-3i}$$

$$= \frac{1+2\sqrt{3}i-3}{1+3} \quad [\because i^2=-1]$$

$$= \frac{-2+2\sqrt{3}i}{4}$$

$$= -\frac{2}{4} + \frac{2\sqrt{3}i}{4}$$

$$= -\frac{1}{2} + \frac{\sqrt{3}i}{2}$$

Where,

$$x = -\frac{1}{2} \quad \text{and} \quad y = \frac{\sqrt{3}}{2}$$

$$\therefore r = \sqrt{x^2 + y^2}$$

$$= \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \sqrt{\frac{1}{4} + \frac{3}{4}}$$

$$= \sqrt{1} = 1 \quad \underline{\text{Ans.}}$$

$$\therefore \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$= \pi - \tan^{-1}\left(\frac{y}{x}\right)$$

$$= \pi - \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right)$$

$$= \pi - \tan^{-1}(\sqrt{3})$$

$$= \pi - \tan^{-1}.\tan\frac{\pi}{3}$$

$$= \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

So, the polar form is, $z = r(\cos\theta + i\sin\theta)$

$$= 1\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

$$= \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$$

and exponential form, $z = re^{i\theta}$

$$= e^{i\frac{2\pi}{3}}$$

Ans.

Ans. 08:

Evaluate $\sqrt{-16} \times \sqrt{-4}$ and $\dfrac{\sqrt{-16}}{\sqrt{-4}}$

$\therefore \sqrt{-16} \times \sqrt{-4}$ and $\dfrac{\sqrt{-16}}{\sqrt{-4}}$

$= \sqrt{16}\,i \times \sqrt{4}\,i$

$$= \frac{\sqrt{16}\,i}{\sqrt{4}\,i}$$

$= 4i \times 2i$

$= 8i$

$$= \frac{4i}{2i}$$

$= -8$

$$= 2$$

Ans. 09:

Here given that,

$$z = 2+i$$

$$\therefore 8z - z^2 = 8(2+i) - (2+i)^2$$

$$= 16 + 8i - (4 + 4i + i^2)$$

$$= 16 + 8i - 4 - 4i - i^2$$

$$= 16 + 8i - 4 - 4i + 1 \quad [\because i^2 = -1]$$

$$= 16 + 8i - 4 - 4i + 1$$

$$= 13 + 4i$$

$$\therefore \text{Modulus}, r = \sqrt{x^2 + y^2} = \sqrt{(13)^2 + (4)^2}$$

$$= \sqrt{169 + 16}$$

$$= \sqrt{185}$$

$$\text{Argument}, \theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{4}{13}\right)$$

$$= 17.102$$

Ans

Ans. 10:

Here, $z = x + iy$

$= 1 + i\sqrt{3}$

$\therefore x = 1$

$y = \sqrt{3}$

$\therefore$ Modulus, $r = \sqrt{x^2+y^2}$

$= \sqrt{(1)^2+(\sqrt{3})^2}$

$= \sqrt{1+3}$

$= 2$

Argument, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$= \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$= \tan^{-1}\tan\frac{\pi}{3}$

$= \frac{\pi}{3}$

Therefore, $r(\cos\theta + i\sin\theta)$ form is,

$= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

Ans.