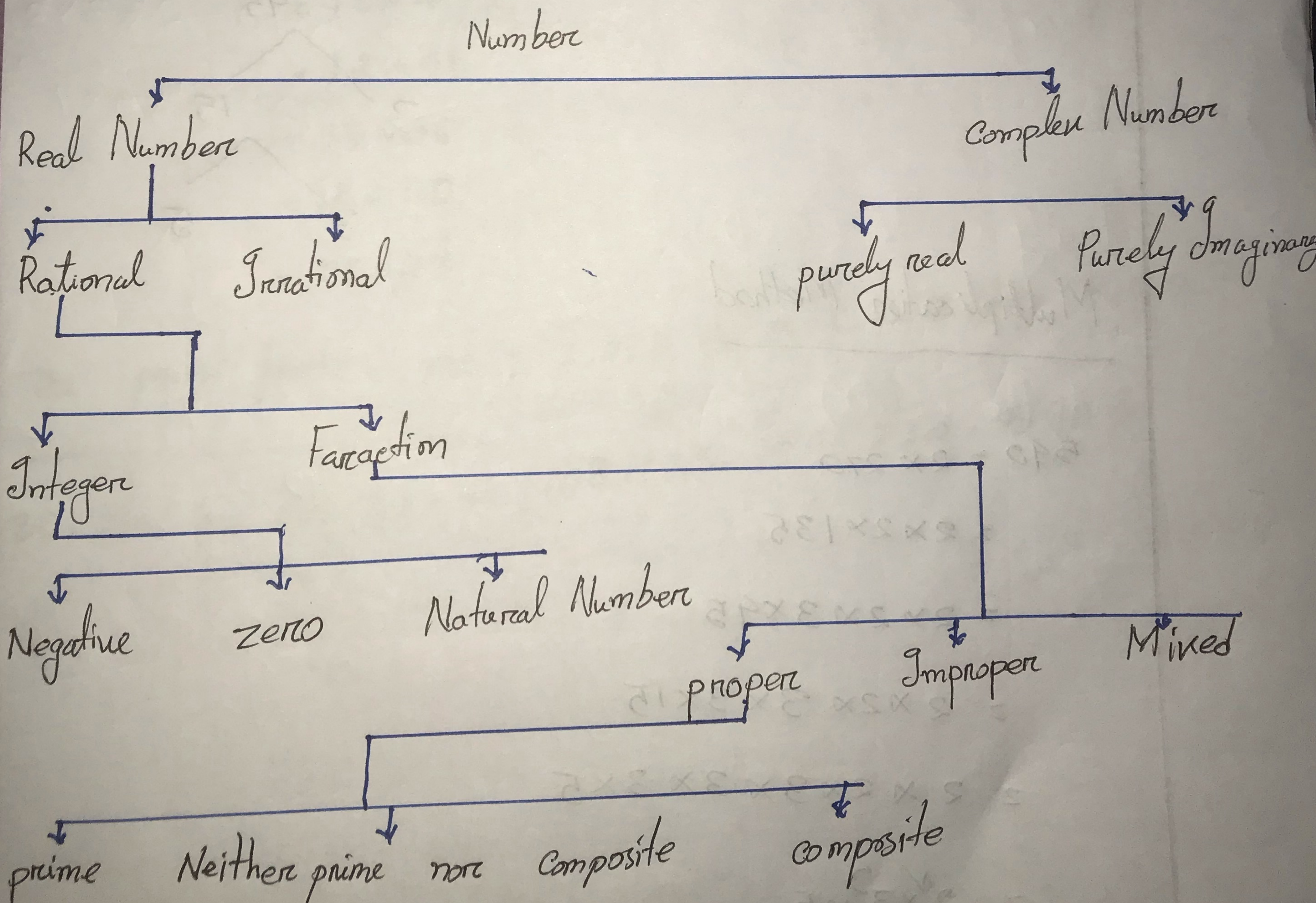
Name - Abdullah
Section - Y
ID - 221-15-5006

1. Solution:

Classification of Number system

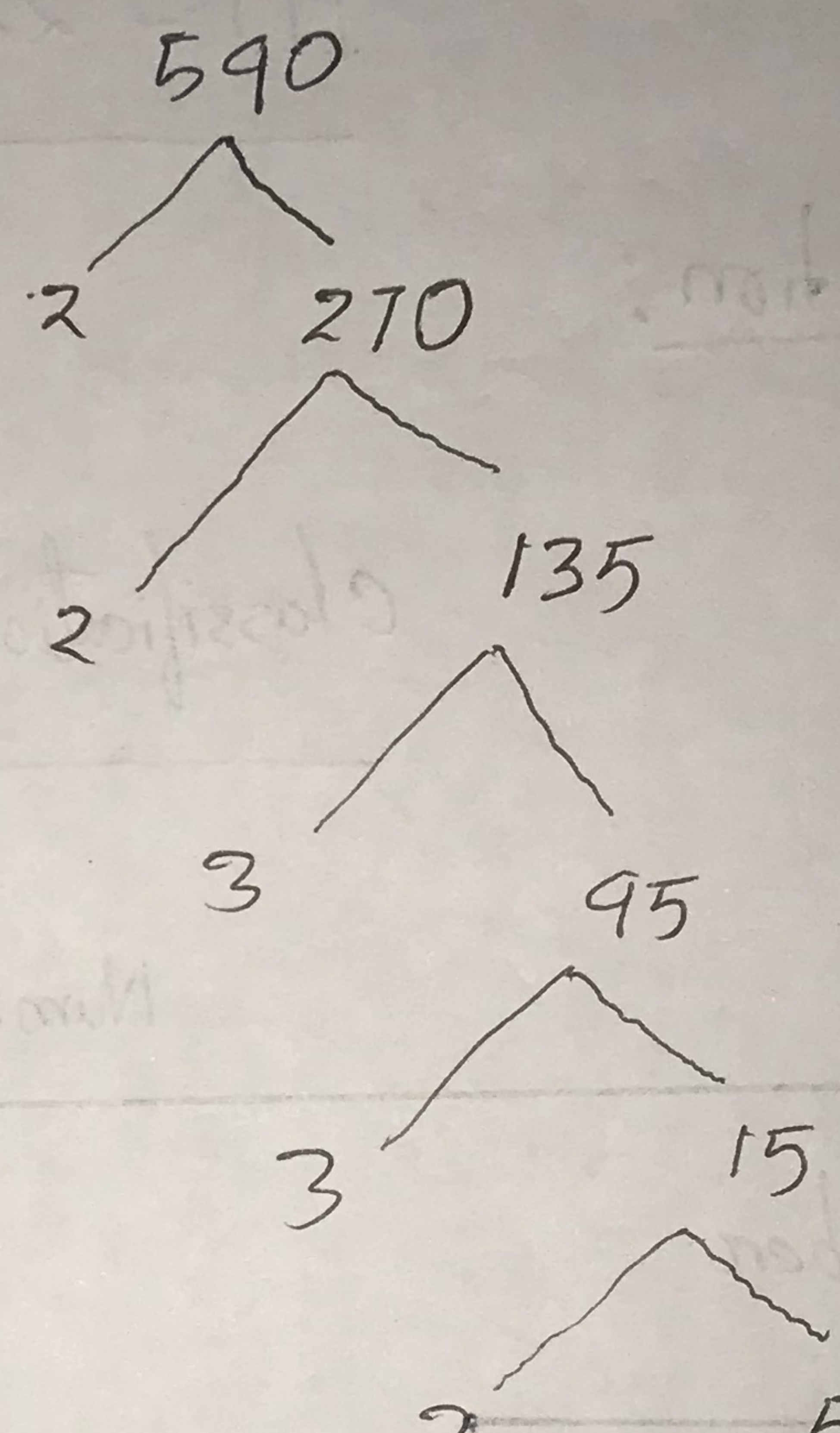


2. Solution:

Division method

$$\begin{array}{r|l} 2 & 540 \\ \hline 2 & 270 \\ \hline 3 & 135 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

Three method



Multiplication Method

$540 = 2 \times 270$

$= 2 \times 2 \times 135$

$= 2 \times 2 \times 3 \times 45$

$= 2 \times 2 \times 3 \times 3 \times 15$

$= 2 \times 2 \times 3 \times 3 \times 3 \times 5$

$= 2^2 \times 3^3 \times 5$

3. Solution ;

$540 = 1 \times 540$

$= 2 \times 270 /$ ~~540~~ 3×180

$= 4 \times 135$

$= 5 \times 108$

$= 6 \times 90$

$= 9 \times 60$

$= 10 \times$ ~~4~~ 54

$= 12 \times 45$

$= 15 \times 36$

$= 18 \times 30$

$= 20 \times 27$

The prime factors are : 1, 2, 3, 4, 5, 6, 9, 10, 15, 18, 20, 27
30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540

9. Solution:

$140 = 2\times 70$

$= 2\times 2\times 35$

$= 2\times 2\times 5\times 7$

$= 2^2\times 5\times 7$

$240 = 2\times 120$

$= 2\times \cancel{60}\ 2\times 60$

$= 2\times 2\times 2\times 30$

$= 2\times 2\times 2\times 2\times 15$

$= 2\times 2\times 2\times 2\times 3\times 5$

$= 2^4\times 3\times 5$

Therefore, the prime factorization of, $240 = 2^4\times 3\times 5$

$140 = 2^2\times 5\times 7$

Finally, the (GCD) of 240 and 140 is $= 2^2\times 5$

$= 50$

and the (LCD) of 240 and 140 is $= 2^4\times 7$

$= 112$

5. Solution :

$42 = 2\times21 = 2\times3\times7$

$63 = 3\times21 = 3\times3\times7 = 3^2\times7$

$140 = 2\times70 = 2\times2\times35 = 2\times2\times5\times7$

Finaly. the L.C.M. of 42, 63 and 140 is $= 3^2\times7\times5\times2^2$

$= \cancel{63}\ 1260$

and the H.C.F of 42, 63 and 140 $= 3\times7$

$= 35$

6. Solution;

Factorization of Numerators

$2 = 2\times1 = 2$

$8 = 2\times2\times2 \times\cancel{2} = 2^3$

$16 = 2\times2\times2\times2 = 2^4$

$10 = 2\times5$

H.C.F of Numerators = 2,

L.C.M of Numerators $= 2^4\times5$

$= 80$

Factorization of Denominators :

$3 = 3$

$9 = 3 \times 3$

$81 = 3 \times 3 \times 3 \times 3$

$27 = 3 \times 3 \times 3$

H.CF of Denomenator $= 3$

L.C.M of Denomenator $= 81$

Finally, the H.C.F of $\frac{2}{3}$, $\frac{8}{9}$, $\frac{16}{81}$ and $\frac{10}{27}$

is $\frac{(H.CF)\ 2}{(L.C.M)\ 81}$ and the L.C.M is $\frac{(L.CM)\ 80}{(H.CF)\ 3}$

7. Solution:

We know, $z = u + yi$ [u is the Real part]
[y is the imaginary part]

Now,
$$Z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$$

$$= \frac{(1+\sqrt{3}i)}{(1-\sqrt{3}i)}$$

$$= \frac{\cancel{1+2\sqrt{3}i}+(-1)}{\cancel{1-}}$$

$$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$$

$$= \frac{1+2\sqrt{3}i-3}{1-(\sqrt{3}i)^2}$$

$$= \frac{1+2\sqrt{3}i-3}{1-3(-1)}$$

$$= \frac{2\sqrt{3}i-2}{4}$$

$$= \frac{\sqrt{3}i-1}{2}$$

$= -\frac{1}{2} + \frac{\sqrt{3}}{2} i$

$= \left(-\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right) i$

Now, $r = \sqrt{a^2 + b^2} = \sqrt{x^2 + y^2}$

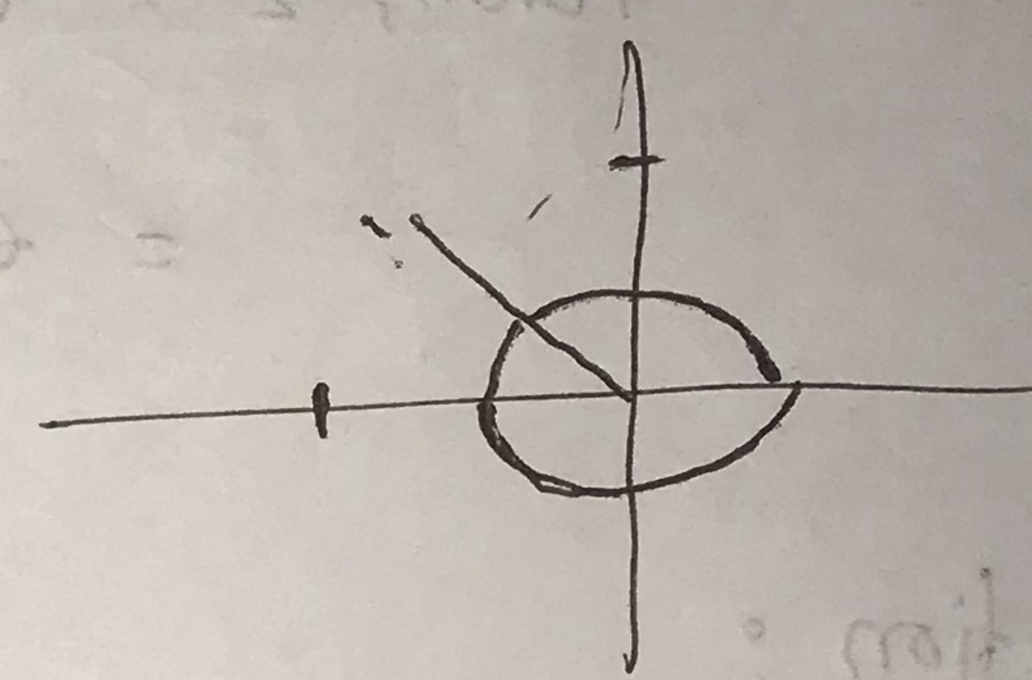
$r = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\sqrt{3}/2\right)^2}$

$= \sqrt{\frac{1}{4} + \frac{3}{4}}$

$= \sqrt{\frac{1+3}{4}}$

$= \sqrt{\frac{4}{4}}$

$= 1$



and $\theta = \tan^{-1} \left| \frac{y}{x} \right|$

$= \tan^{-1} \left| \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} \right|$

$= \tan^{-1} \frac{\sqrt{3}/2}{\frac{1}{2}}$

$= -\tan^{-1} \sqrt{3}$

$= -\frac{\pi}{3}$

$= \frac{2\pi}{3}$

and, $\theta = 180^\circ - \tan^{-1} \left| \frac{y}{x} \right|$

$= 180^\circ - \tan^{-1} \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$= 180^\circ - \tan^{-1} \sqrt{3}$

$= 180 - 60^\circ$

$= 120^\circ$

So, the polar form is $Z = r(\cos\theta + i\sin\theta)$

$= 1(\cos 120^\circ + i\sin 120^\circ)$

$= \cos 120^\circ + i\sin 120^\circ$

Exponential Form, $Z = e^{i\theta}$

$= e^{i\,120^\circ}$

8. Solution :

We have, $i^2 = -1$

Now, $\sqrt{-16} \times \sqrt{-9} \times \frac{\sqrt{-16}}{\sqrt{-9}}$

$= \sqrt{16 i^2} \times \sqrt{-9 i^2} \times \frac{\sqrt{16 i^2}}{\sqrt{9 i^2}}$

$= \sqrt{2^4}\, i \times \sqrt{2^2}\, i \times \frac{\sqrt{2^4}\, i}{\sqrt{2^2}\, i}$

$= \sqrt{(2^2)^2}\, i \times 2i \times \frac{\sqrt{(2^2)^2}}{2}$

$= 2^2 i \times 2 \times \frac{2^2}{2}$

$= 4 \times 2 \times i^2 \times 2 \times 2$

$= 16 i^2$

$= -16$

9 Solution :

Given,

$$z = 2 + i$$

Now,

$$8z - z^2$$

$$= 8(2+i) - (2+i)^2$$

$$= 16 + 8i - (2^2 + 2i \times 2 - 1)$$

$$= 16 + 8i - 4 - 4i + 1$$

$$= 13 + 4i$$

The modulus, $r = \sqrt{a^2 + b^2}$

$$= \sqrt{(13)^2 + (4)^2}$$

$$= \sqrt{185}$$

The Argument is, $\theta = \tan^{-1}\left|\frac{y}{x}\right|$

$$= \tan^{-1}\frac{4}{13}$$

$$= 17.103^\circ$$

10. Solution:

Now, $1+i\sqrt{3}$

We know $z = x+iy$

$\therefore\ r = \sqrt{x^2+y^2}$

$= \sqrt{1^2+(\sqrt{3})^2}$

$= \sqrt{9}$

$= 2$

$\therefore\ \theta = \tan^{-1}\left|\frac{y}{x}\right|$

$= \tan\sqrt{3}$

$= 60^\circ$

$\therefore\ r(\cos\theta + i\sin\theta) = 2(\cos 60^\circ + i\sin 60^\circ)$

$= 2\cos 60^\circ + 2i\sin 60^\circ$