① Write down the classification of number system:

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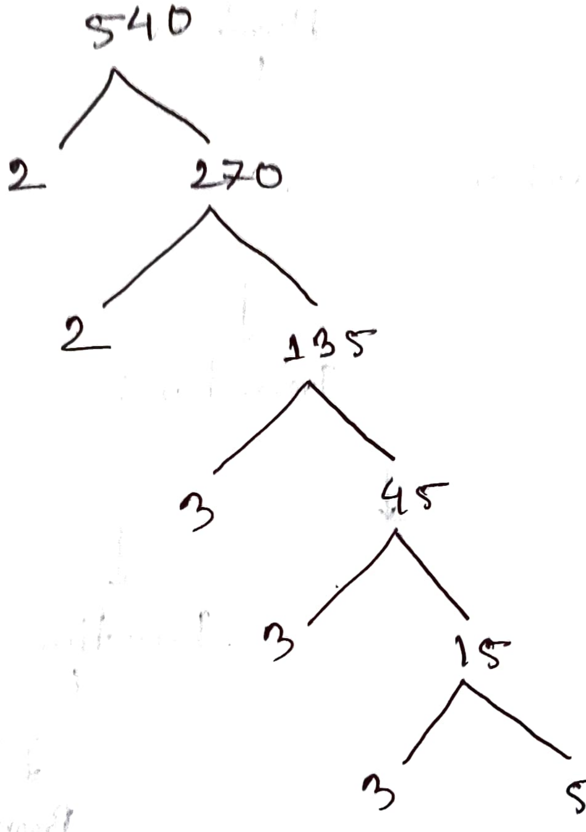
- Numbers
  - Real numbers
    - Rational
      - Integers
        - Negative
        - Zero
        - Positive Numbers
          - Prime
          - Neither prime nor composite.
          - Composite
      - Fraction
        - Proper
        - Improper
        - Mixed
    - Irrational
  - Complex Numbers
    - Purely Real
    - Purely Imaginary

② Find out the prime factorization of 540 using tree :-



Therefore the prime factorization of 1600 is $= 2^2 . 3^3 . 5$.

③ Find the all factors of 540 :-

The prime factorization of 540 is $= 2^2 . 3^3 . 5$

So, the total number of 540 is

$= (2+1)(3+1)(1+1)$

$= 3 \times 4 \times 2$

$= 24$

calculation for all factors:

~~1600 = 1 × 1600~~

$$
\begin{aligned}
540 &= 1 \times 540 \\
&= 2 \times 270 \\
&= 3 \times 180 \\
&= 4 \times 135 \\
&= 5 \times 108 \\
&= 6 \times 90 \\
&= 9 \times 60 \\
&= 10 \times 54 \\
&= 12 \times 45 \\
&= 15 \times 36 \\
&= 18 \times 30 \\
&= 20 \times 27
\end{aligned}
$$

The factors of 540 are: –

$\{1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540\}$

(4) What is the GCD and LCM of 240 and 540 :-

$\therefore 240 = 2 \times 120 = 2 \times 2 \times 60 = 2 \times 2 \times 2 \times 30$

$= 2 \times 2 \times 2 \times 2 \times 3 \times 5$

$= 2^4 . 3 . 5$

$\therefore 540 = 2^2 . 3^3 . 5$ [Following previous ans:]

$\therefore 240 = 2^4 . 3 . 5$

$\therefore LCM = 2^4 . 3^3 . 5 = 2160$. (Ans:)

and $GCD = 2^2 . 3 . 5 = 60$ (Ans:)

(5) Find the H.CF and L.C.M of 42, 63 and 140.

$\therefore 42 = 2 \times 21$

$= 2 \times 3 \times 7$

$\therefore 63 = 3 \times 21$

$= 3 \times 3 \times 7$

$= 3^2 \times 7$

$\therefore 140 = 2 \times 70$

$= 2 \times 2 \times 35$

$= 2^2 \times 5 \times 7$

$\therefore 42 = 2 . 3 . 7$

$\therefore 63 = 3^2 . 7$

$\therefore 140 = 2^2 . 5 . 7$

$\therefore LCM = 2^2 . 3^2 . 5 . 7 = 1260$ (Ans:)

$\therefore HCF = 7$ (Ans:)

⑥ Find the H.C.F and L.C.M of $\frac{2}{3}$, $\frac{8}{9}$, $\frac{16}{81}$ and $\frac{10}{27}$

Calculation for Numerators:

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

$10 = 2^1 \times 5^1$

$\text{Lcm}(2,8,16,10) = 2^4 \cdot 5 = 80$

$\text{HCF}(2,8,16,10) = 2^1 = 2$

Calculation of Denominators:-

$3 = 3^1$

$9 = 3^2$

$81 = 3^4$

$27 = 3^3$

$\text{Lcm}(3,9,81,27) = 3^4 = 81$

$\text{HCF}(3,9,81,27) = 3 = 3$

$\therefore$ Lcm of $\left(\frac{2}{3}, \frac{8}{9}, \frac{16}{81} \text{ and } \frac{10}{27}\right) = \frac{80}{3}$ (Ans:)

$\therefore$ HCF of $\left(\frac{2}{3}, \frac{8}{9}, \frac{16}{81} \text{ and } \frac{10}{27}\right) = \frac{2}{81}$. (Ans:)

(2) Find the modulus and Argument of $z = \dfrac{1+\sqrt{3}i}{1-\sqrt{3}i}$ and also its polar, exponential form :-

$$z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$$

$$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$$

$$= \frac{1+2\sqrt{3}i-3}{1^2-(\sqrt{3}i)^2}$$

$$= \frac{-2+2\sqrt{3}i}{1+3}$$

$$= \frac{\cancel{2}(-1+\sqrt{3}i)}{\cancel{4}\,2}$$

$$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$|z| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\therefore r = \sqrt{\frac{1}{4}+\frac{3}{4}} = \sqrt{\frac{4}{4}} = 1.$$

$\therefore$ Modulus of z is $= 1$

And argument of z will –

$$\theta = \pi - \tan^{-1}\left|\frac{\sqrt{3}/2}{1/2}\right|$$

$$= \pi - \tan^{-1}(\sqrt{3})$$

$$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$\therefore$ Polar form, $z = r(\cos\theta + i\sin\theta)$

$$= 1\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right).$$

$\therefore$ Exponential form $\Rightarrow$ $z = re^{i\theta}$

$$= 1\,e^{i\frac{2\pi}{3}}$$

$$= e^{\frac{2\pi}{3}i}$$

⑧ Evaluate $\sqrt{-16} \times \sqrt{-4}$ and $\frac{\sqrt{-16}}{\sqrt{-4}}$

$\therefore \sqrt{-16} \times \sqrt{-4}$

$= \sqrt{16}\, i \times \sqrt{4}\, i$

$= 4i \times 2i$

$= 8i^2$

$= -8$ (Ans:)

$\frac{\sqrt{-16}}{\sqrt{-4}}$

$= \frac{4i}{2i}$

$= 2.$ (Ans:)

⑨ Evaluate Modulus and Argument of $8z - z^2$ by replacing $z = (2+i)$

$\therefore z = 2+i$

$8z - z^2 = 8(2+i) - (2+i)^2$

$= 16 + 8i - (4 + 4i + i^2)$

$= 16 + 8i - 4 - 4i + 1$

$= 13 + 4i$

Modulus of $8z - z^2 = \sqrt{(13)^2 + (4)^2} = \sqrt{169 + 16} = \sqrt{185}$

Argument of $8z - z^2 = \tan^{-1} \frac{4}{13}$

$= 17.1^\circ$ (Ans:)

(10)

Express $1+\sqrt{3}i$ in the form of $r(\cos\theta + i\sin\theta)$

$z = 1+\sqrt{3}i$

Here, Modulus of

$$|z| = \sqrt{1^2 + (\sqrt{3})^2} = 2$$

Argument of,

$$z = \tan^{-1}\left|\frac{y}{x}\right|$$

$$= \tan^{-1}|\sqrt{3}|$$

$$= \frac{\pi}{3}$$

Therefore,

$r(\cos\theta + i\sin\theta)$ form is $= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$.