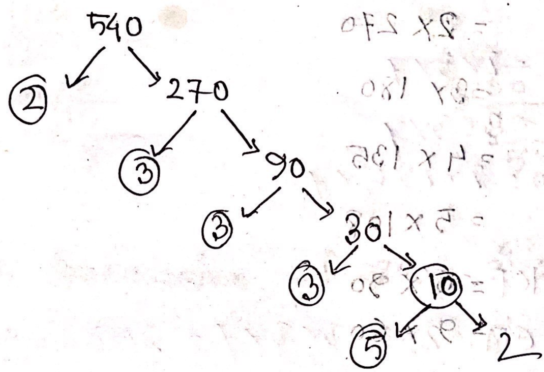
1. Classification of Number System:

- Number
  - Real Number
    - Retional
      - Integer
        - Negative
        - Zero
        - Positive/Natural
          - Prime
          - Neither prime or Composite
          - Composite
      - Fraction
        - Proper
        - Improper
        - Mixed
    - Irrational
  - Complex Number
    - Purely Real
    - Imaginary

2. The tree diagram for the prime factors is as follows:



So, Prime factorization of 540 is,

$$540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 2^2 \cdot 3^3 \cdot 5^1$$

(3) Find the all factor of 540 are

540 = 1 × 540

= 2 × 270

= 3 × 180

= 4 × 135

= 5 × 108

= 6 × 90

= 9 × 60

= 10 × 54

= 12 × 45

= 15 × 36

= 18 × 30

= 20 × 27

Therefore the factors of 540 are: 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 60, 90, 108, 135, 180, 270, 540

4. GCD and LCM of 240 and 540 :

$$\begin{array}{r|l} 2 & 240 \\ \hline 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 540 \\ \hline 3 & 270 \\ \hline 3 & 90 \\ \hline 3 & 30 \\ \hline 2 & 10 \\ \hline & 5 \end{array}$$

$\therefore$ Prime factorization of $240 = 2^4 \cdot 3^1 \cdot 5^1$

Also, $540 = 2^2 \cdot 3^3 \cdot 5^1$

Finally, the GCD of 240 and 540 is:

$2^2 \cdot 3^1 \cdot 5^1 = 4 \cdot 3 \cdot 5 = 60$ and $LCM = 2^4 \cdot 3^3 \cdot 5^1 = 16 \cdot 27 \cdot 5$

$= 2160$ (Ans)

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5.

$$\begin{array}{r|l} 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 2 & 140 \\ \hline 2 & 70 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

$42 = 2 \times 3 \times 7$

$63 = 3 \times 3 \times 7$

$140 = 2 \times 2 \times 5 \times 7$

$\therefore$ HCF $= 7$

$\therefore$ LCM $= 2 \times 2 \times 3 \times 3 \times 7 = 252$

6. $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27}$

Factorization of Numerators:

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

$10 = 2^1 \times 5$

HCF of numerators $= 2$

LCM of numerators $= 80$

Factorization of Denominators :

$3 = 3^1$

$9 = 3^2$

$81 = 3^4$

$27 = 3^3$

HCF of denominators = 3

LCM of denominators = 81

$\therefore$ HCF of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{HCF(2,8,16,10)}{LCM(3,9,81,27)}$

$= \frac{2}{81}$

$\therefore$ LCM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{LCM(2,8,16,10)}{HCF(3,9,81,27)}$

$= \frac{16}{3}$

7. $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}.i}$

$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$

$= \frac{(1+\sqrt{3}i)^2}{(1)^2-(\sqrt{3}i)^2}$

$= \frac{1^2-2\cdot\sqrt{3}i+3i^2}{1+3}$

$= \frac{-2+2\sqrt{3}i}{4}$

$= -\frac{1}{2} + \frac{2\sqrt{3}i}{4}$

$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

$= x+iy$. [where $x=-\frac{1}{2}$ and $y=\frac{\sqrt{3}}{2}$]

now,

$$r = \sqrt{x^2+y^2}$$

$$= \sqrt{\frac{1}{4}+\frac{3}{4}}$$

$$= 1$$

and $\theta = \pi - \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right)$

$$= \pi - \tan^{-1}(-\sqrt{3})$$

$$= \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

So, polar form is, $z = r(\cos\theta + i\sin\theta)$

$$= 1\cdot\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

$$= \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

and exponential form is, $z = e^{i\cdot\frac{2\pi}{3}}$

8. $\sqrt{-16} \times \sqrt{-4}$

$= \sqrt{16}\,i \times \sqrt{4}\,i$

$= 4i \times 2i$

$= 8i^2$

$= -8$

and

$\dfrac{\sqrt{-16}}{\sqrt{-4}}$

$= \dfrac{4i}{2i}$

$= 2$

9

We have, $z = 2+i$

$\therefore 8z - z^2 = 8(2+i) - (2+i)^2$

$= 16 + 8i - (4 + 4i + i^2)$

$= 16 + 8i - 4 - 4i + 1$

$= 13 + 4i$

Modulus $r = \sqrt{(13)^2 + (4)^2}$

$= \sqrt{169 + 16}$

$= \sqrt{185}$

$\theta = \tan^{-1} \dfrac{4}{13}$

$= 17.102$

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10. $1+i\sqrt{3}$

Let, $z = 1+i\sqrt{3}$

$z = x+iy$

$r = \sqrt{(\sqrt{3})^2+1}$

$= 2$

$\theta = \tan^{-1}\left|\frac{\sqrt{3}}{1}\right|$

$= \tan^{-1}\tan\frac{\pi}{3}$

$= \frac{\pi}{3}$

$\therefore r(\cos\theta + i\sin\theta)$ form is

$= 2(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$

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