MAT-111

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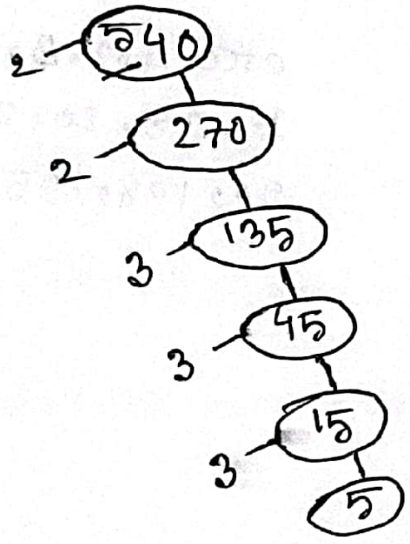
1. Write down the classification of number system.

Ans:

- Number
  - Real Number
    - Rational
      - Integer
        - Negative
        - Zero
        - Positive/ Natural Number
          - Prime
          - Neither Prime nor composite
          - Composite
      - Fraction
        - Proper
        - Improper
        - Mixed
    - Irretional
  - Complex Number
    - Purely real
    - Purely Imaginary

2. Find the prime factorization of 540 using tree.

Ans:



$\therefore$ The prime factorization of 540 $= 2^2 \cdot 3^3 \cdot 5$.

3: Find out all the factors of 540.

Ans: From problem 2 we get the prime factorization of 540 is $= 2^2 \cdot 3^3 \cdot 5$

So, the total number of factors of 540

is $= (2+1)(3+1)(1+1)$

$= 3 \times 4 \times 2$

$= 24$

Now, $540 = 1 \times 540$

$= 2 \times 270$

$= 3 \times 180$

$= 4 \times 135$

$= 5 \times 108$

$= 6 \times 90$

$= 9 \times 60$

$= 10 \times 54$

$= 12 \times 45$

$= 15 \times 36$

$= 18 \times 30$

$= 20 \times 27$

$\therefore$ All factors of 540 are 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 54, 60, 45, 90, 108, 135, 180, 270, 540.

41 What is the GCD and LCM of 240 and 540

Ans:

$$\begin{array}{r|l} 2 & 240 \\ \hline 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

$\therefore 240 = 2^4 . 3 . 5$

$$\begin{array}{r|l} 2 & 540 \\ \hline 2 & 270 \\ \hline 3 & 135 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

$\therefore 540 = 2^2 . 3^3 . 5$

LCM of 240 and $540 = 2^4 . 3^3 . 5 = 2160$

GCD of 240 and $540 = 2^2 . 3 . 5 = 60$

5) Find the HCF and LCM of 42, 63 and 140

Ans:

$$\begin{array}{r|l} 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 2 & 140 \\ \hline 2 & 70 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

$42 = 2 \cdot 3 \cdot 7$ $\qquad 63 = 3^2 \cdot 7$ $\qquad 140 = 2^2 \cdot 5 \cdot 7$

$\therefore$ L.C.M of $(42, 63, 140) = 2^2 \cdot 3^2 \cdot 7 = 1260$

$\therefore$ HCF of $(42, 63, 140) = 7$

6) Find the HCF and LCM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$

Ans:

Calculation of Numerator	Calculation of Denomenator
$2 = 2^1$	$3 = 3^1$
$8 = 2^3$	$9 = 3^3$
$16 = 2^4$	$81 = 3^4$
$10 = 2 \cdot 5$	$27 = 3^3$
$\therefore$ L.C.M of Numerator $= 2^4 \cdot 5 = 80$	$\therefore$ L.C.M of Denominator $= 3^4 = 81$
$\therefore$ H.C.F of Numerator $= 2$	$\therefore$ H.C.F of Denomenator $= 3$

$\therefore$ L.C.M of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{80}{3}$

$\therefore$ H.CF of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{2}{81}$

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7) Find the modulus and Argument of $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$ and also it's polar, expotential form.

Ans: Given that,

$$z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$$

$$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)} = \frac{1+\sqrt{3}i+\sqrt{3}i+3i^2}{1^2+(\sqrt{3}\cancel{i})^2}$$

$$= \frac{2\sqrt{3}i-2}{1+3} = \frac{2\sqrt{3}i-2}{4}$$

$$= \frac{\sqrt{3}}{2}i - \frac{1}{2} = -\frac{1}{2}+\frac{\sqrt{3}}{2}i$$

~~So, polar form $= \frac{1}{2}+\frac{\sqrt{3}}{2}i$~~

$$\therefore z = -\frac{1}{2}+\frac{\sqrt{3}}{2}i$$

$$|z| = \sqrt{\left(-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} \quad \therefore r = \sqrt{\frac{1}{4}+\frac{3}{4}} = \sqrt{\frac{4}{4}} = 1$$

$\therefore$ Modulus of z is $= 1$

And Argument of z is

$$\theta = \tan^{-1}\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \tan^{-1}(-\sqrt{3}) = -\tan^{-1}(\sqrt{3})$$

$$= \frac{\pi}{3} = \frac{2\pi}{3}.$$

So, the polar form is, $z = r(\cos\theta + i\sin\theta)$

$$= 1\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right) = \left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$$

And Expotential forr, $z = re^{i\theta} = e^{\frac{2\pi}{3}i}$

8) Evaluate $\sqrt{-16} \times \sqrt{-4}$ and $\dfrac{\sqrt{-16}}{\sqrt{-4}}$

Ans: We have, $i^2 = -1$

Now, $\sqrt{-16} \times \sqrt{-4}$

$= \sqrt{16i^2} \times \sqrt{4i^2}$

$= \sqrt{4^2 i^2} \times \sqrt{2^2 i^2}$

$= 4i \times 2i$

$= 8i^2$

$= -8$

Again,

$\dfrac{\sqrt{-16}}{\sqrt{-4}}$

$= \dfrac{\sqrt{16i^2}}{\sqrt{4i^2}}$

$= \dfrac{\sqrt{4^2 i^2}}{\sqrt{2^2 i^2}}$

$= \dfrac{4i}{2i} = 2$

9) Evaluate Modulus and Argument of $8z - z^2$ by replacing $z = 2+i$

Ans: Here given that,

$z = 2+i$

$\therefore 8z - z^2 = 8(2+i) - (2+i)^2$

$= 16 + 8i - (4 + 2.2.i + i^2)$

$= 16 + 8i - 4 - 4i + 1$

$= 13 + 4i$

Modulus of $8z - z^2 = \sqrt{(13)^2 + (4)^2} = \sqrt{169 + 16} = \sqrt{185}$. Ans.

Argument of $8z - z^2 = \tan^{-1}\dfrac{4}{13} = 17.1$. Ans.

(10) Express $1+\sqrt{3}i$ in the form of $r(\cos\theta + i\sin\theta)$

Ans: $z = 1+\sqrt{3}i$

Here, Modulus, $r = \sqrt{1^2+(\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$

Argument, $\theta = \tan^{-1}\frac{\sqrt{3}}{1} = \tan^{-1}\tan\frac{\pi}{3}$

$= \frac{\pi}{3}$

Therefore, $r(\cos\theta + i\sin\theta)$ form is

$= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$. Ans.